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A NEGATIVE ANSWER TO THREE QUESTIONS ON K-PRIMITIVE RINGS

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ABSTRACT. It is shown that three questions on K-primitive rings posed by Kezlan in [3] have a negative answer.

We shall use the terminology of [3, 4]. Kezlan has noticed [3] that every strongly K-primitive ring is a right Ore domain. In fact the following characterization of strongly K-primitive rings is true.

PROPOSITION 1. *A ring R is (right) strongly K-primitive if and only if R is a (right) Ore domain and R is (right) bounded.*

Let us notice that Proposition 1 corresponds to the second part of Question 3 of [3] and the counterexample given there.

If σ is an automorphism of a field D then $D[[t, \sigma]]$ denotes a σ-twisted power series algebra and $D((t, \sigma))$ denotes a twisted Laurent series algebra (see e.g. [2]).

THEOREM 2. *Let σ be an automorphism of infinite order of a field D. Then $D[[t, \sigma]]$ is a left and right strongly K-primitive ring which together with the center of its quotient ring Q does not generate Q.*

PROOF. Since every one-sided ideal of a domain $D[[t, \sigma]]$ is a two-sided ideal, $D[[t, \sigma]]$ is left and right strongly K-primitive. It is well known and easy to see that $D((t, \sigma))$ is a left and right quotient ring of $D[[t, \sigma]]$ and its center F is the fixed subfield of σ acting on D. Hence $FD[[t, \sigma]] = D[[t, \sigma]] \neq D((t, \sigma))$.

The above theorem gives us a negative answer to Question 2 of [3].

Let J denote the Jacobson radical. We shall need the following

THEOREM 3 [1]. *Let k be a field and R a J-radical k-algebra contained in a skew field K. Then K can be embedded in a skew field L which contains a simple J-radical k-subalgebra containing R.*

The next theorem gives the negative answer to the first part of Question 3 and hence also to Question 4 posed by Kezlan.

THEOREM 4. *For any field k there exists a k-algebra which is a right and left Ore domain and is not K-primitive.*

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PROOF. Let $K_0 = k((t))$, $R_0 = k[[t]]t$. Notice that R_0 is a right and left Ore domain. Since R_0 is J-radical then, by Theorem 3, there exists a skew field K_1 containing K_0 and simple J-radical k-algebra R_1 containing R_0. Let $K_2 = K_1((t))$, $R_2 = R_1 + tK_1[[t]]$. Then R_2 is a J-radical right and left Ore domain. Continuing in this way we get an ascending sequence $(K_i)_{i=0}^{\infty}$ of division algebras and an ascending sequence $(R_i)_{i=0}^{\infty}$ of their J-radical subalgebras such that R_i is a right and left Ore domain for even i and R_i is a simple ring for odd i. Then $\bigcup R_i \subseteq \bigcup K_i$ is a right and left Ore domain which is a simple J-radical ring. From [4] it follows that $\bigcup R_i$ is not a K-primitive ring.

REFERENCES

1. P. M. Cohn, *The embedding of radical rings in simple radical rings*, Bull. London Math. Soc. **3** (1971), 185–188.

2. N. Jacobson, *PI algebras, an introduction*, Lecture Notes in Math., Vol. **441**, Springer-Verlag, Berlin and New York, 1975.

3. T. P. Kezlan, *On K-primitive rings*, Proc. Amer. Math. Soc. **74** (1979), 24–28.

4. A. H. Ortiz, *On the structure of semiprime rings*, Proc. Amer. Math. Soc. **38** (1973), 22–26.

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