DERIVATIVES OF $H^p$ FUNCTIONS

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Abstract. We prove that if $\{z_n\}$ is uniformly separated and $f \in H^p$, then 
$\left\{ f^{(k)}(z_n)(1 - |z_n|^2)^{k+1/p} \right\}_{n=1}^\infty \in \ell^p$ for $k = 1, 2, \ldots$.

We give a simple proof of

Lemma. Let $\{z_n\}$ be uniformly separated and $f \in H^p$. For $k = 1, 2, \ldots$ we have 
$\left\{ f^{(k)}(z_n)(1 - |z_n|^2)^{k+1/p} \right\}_{n=1}^\infty \subset \ell^p$.

$H^p$ is the Hardy space of the unit disc $D$. A sequence $\{z_n\} \subset D$ is called 
uniformly separated if 
$$\inf_n \prod_{m \neq n} \left| \frac{z_n - z_m}{1 - \overline{z}_n z_m} \right| > 0.$$ 

A technical proof of the lemma was given in [2]. There it was also proved that 
every $\ell^p$ sequence is obtained in this way. When [2] was published, the result was 
already known in the Soviet Union (see, for instance, F. A. Shamoian’s paper [3]). 
Inspired by this paper we prove the lemma.

For small $\tau$ let $D_n = \{z : |z - z_n| < \tau(1 - |z_n|)\}$. A simple computation using the 
pseudohyperbolic metric $\varrho(a, b) = |(a - b)/(1 - \overline{a}b)|$ proves that $z_n^* \in D_n \Rightarrow \{z_n^*\}$ 
is uniformly separated. By Cauchy’s formula 
$$|f^{(k)}(z_n)| = \left| \frac{k!}{2\pi i} \int_{\partial D_n} \frac{f(\xi)}{(\xi - z_n)^{k+1}} d\xi \right| \leq A(1 - |z_n|^2)^{-k} \max_{\xi \in D_n} |f(\xi)|$$ 
$$= A(1 - |z_n|^2)^{-k} |f(z_n^*)|.$$ 

Hence 
$$|f^{(k)}(z_n)(1 - |z_n|^2)^{k+1/p}| \leq A |f(z_n^*)|(1 - |z_n|^2)^{1/p}$$ 
$$\leq A \cdot B |f(z_n^*)|(1 - |z_n^*|^2)^{1/p}$$

where $B$ is seen to be independent of $n$. Since $\{z_n^*\}$ is uniformly separated, the 
lemma follows from the well-known interpolation theorem of Shapiro and Shields [1].

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