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DERIVATIVES OF H^p FUNCTIONS

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ABSTRACT. We prove that if $\{z_n\}$ is uniformly separated and $f \in H^p$, then $\{f^{(k)}(z_n)(1 - |z_n|^2)^{k+1/p}\}_{n=1}^{\infty} \in l^p$ for $k = 1, 2, \ldots$.

We give a simple proof of

LEMMA. *Let $\{z_n\}$ be uniformly separated and $f \in H^p$. For $k = 1, 2, \ldots$ we have $\{f^{(k)}(z_n)(1 - |z_n|^2)^{k+1/p}\}_{n=1}^{\infty} \in l^p$.*

H^p is the Hardy space of the unit disc D. A sequence $\{z_n\} \in D$ is called uniformly separated if

$$\inf_n \prod_{m \neq n} \left| \frac{z_n - z_m}{1 - \bar{z}_n z_m} \right| > 0.$$

A technical proof of the lemma was given in **[2]**. There it was also proved that every l^p sequence is obtained in this way. When **[2]** was published, the result was already known in the Soviet Union (see, for instance, F. A. Shamoian's paper **[3]**). Inspired by this paper we prove the lemma.

For small τ let $D_n = \{z: |z - z_n| \leqslant \tau(1 - |z_n|)\}$. A simple computation using the pseudohyperbolic metric $\psi(a, b) = |(a - b)/(1 - \bar{a}b)|$ proves that $z_n^* \in D_n \Rightarrow \{z_n^*\}$ is uniformly separated. By Cauchy's formula

$$\begin{aligned} |f^{(k)}(z_n)| &= \left| \frac{k!}{2\pi i} \int_{\partial D_n} \frac{f(\xi)}{(\xi - z_n)^{k+1}}\, d\xi \right| \leqslant A\left(1 - |z_n|^2\right)^{-k} \max_{\xi \in D_n} |f(\xi)| \\ &= A\left(1 - |z_n|^2\right)^{-k} |f(z_n^*)|. \end{aligned}$$

Hence

$$\begin{aligned} \left| f^{(k)}(z_n)\left(1 - |z_n|^2\right)^{k+1/p} \right| &\leqslant A|f(z_n^*)|\left(1 - |z_n|^2\right)^{1/p} \\ &\leqslant A \cdot B|f(z_n^*)|\left(1 - |z_n^*|^2\right)^{1/p} \end{aligned}$$

where B is seen to be independent of n. Since $\{z_n^*\}$ is uniformly separated, the lemma follows from the well-known interpolation theorem of Shapiro and Shields **[1]**.

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