PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 86, Number 2, October 1982

THE MINIMAL NORMAL FILTER ON $P_\kappa\lambda$

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ABSTRACT. Let κ be an uncountable regular cardinal, let CF_κ be the cub filter on κ and let FSF_κ be the filter generated by $\{\{\beta < \kappa : \beta > \alpha\} : \alpha < \kappa\}$. It is well known that CF_κ is normal, that $CF_\kappa = \Delta FSF_\kappa$ and hence that every normal filter on κ extends CF_κ.

Jech extended some of these results to the context of $P_\kappa\lambda$. Let λ be a cardinal $\geqslant \kappa$ and let $CF_{\kappa\lambda}$ denote the cub filter on $P_\kappa\lambda$ as defined by Jech; he showed that $CF_{\kappa\lambda}$ is normal and that every normal *ultra*filter on $P_\kappa\lambda$ extends $CF_{\kappa\lambda}$.

In this paper we extend these results further. In particular, we show that $CF_{\kappa\lambda} = \Delta\Delta FSF_{\kappa\lambda}$ where $FSF_{\kappa\lambda}$ is the filter generated by $\{\{y \in P_\kappa\lambda : x \subset y\} : x \in P_\kappa\lambda\}$, and that every normal *filter* on $P_\kappa\lambda$ extends $CF_{\kappa\lambda}$.

Finally, we show that for any $\lambda \geqslant \kappa$ and any ideal I on $P_\kappa\lambda$, $\nabla\nabla\nabla I = \nabla\nabla I$.

1. Introduction and notation.

1.1 Unless specified otherwise, κ denotes an uncountable regular cardinal and λ is a cardinal $\geqslant \kappa$.

$P_\kappa\lambda$ denotes the set $\{x \subset \lambda: |x| < \kappa\}$, and for each $x \in P_\kappa\lambda$, $\hat{x}$ is the set $\{y \in P_\kappa\lambda : x \subset y\}$. Notice that the family $\{\hat{x} : x \in P_\kappa\lambda\}$ generates a proper, nonprincipal, κ-complete filter over $P_\kappa\lambda$. We denote this filter by $FSF_{\kappa\lambda}$ (the "final segment filter") and its dual by $I_{\kappa\lambda}$.

By a *filter on* $P_\kappa\lambda$ we mean a proper, nonprincipal, κ-complete filter on $P_\kappa\lambda$ extending $FSF_{\kappa\lambda}$. Dually, an *ideal on* $P_\kappa\lambda$ is a proper, nonprincipal, κ-complete ideal on $P_\kappa\lambda$ extending $I_{\kappa\lambda}$.

1.2 As in Jech [3] we say that $X \subset P_\kappa\lambda$ is *unbounded* iff $(\forall y \in P_\kappa\lambda)(X \cap \hat{y} \neq 0)$. Thus $I_{\kappa\lambda}$ is the ideal of "*not unbounded*" subsets of $P_\kappa\lambda$.

$C \subset P_\kappa\lambda$ is said to be *closed* iff $(\forall X \subset C)(|X| < \kappa$ & X is directed $\Rightarrow \bigcup X \in C)$. Note that by a result of Solovay (e.g. see [6]), $C \subset P_\kappa\lambda$ is closed iff $(\forall X \subset C)(|X| < \kappa$ & X is a chain $\Rightarrow \bigcup X \in C)$. Finally $C \subset P_\kappa\lambda$ is called a *cub* iff it is both closed and unbounded.

We denote the family of all cub subsets of $P_\kappa\lambda$ by $C_{\kappa\lambda}$, and say that $S \subset P_\kappa\lambda$ is *stationary* iff $(\forall C \in C_{\kappa\lambda})(S \cap C \neq 0)$.

$C_{\kappa\lambda}$ is easily seen to generate a filter on $P_\kappa\lambda$ (e.g. see Jech [3]). We denote this filter by $CF_{\kappa\lambda}$ and call it the *cub filter* on $P_\kappa\lambda$. Its dual $NS_{\kappa\lambda}$ is the *nonstationary* ideal on $P_\kappa\lambda$.

Received by the editors February 9, 1981 and, in revised form, October 14, 1981.

1980 *Mathematics Subject Classification.* Primary 03E05.

[1]The main results of this paper were first reported in [2] and were presented at the CMS Summer Research Workshop on Set Theory and Set-Theoretic Topology, Erindale College, University of Toronto, July-August 1980. These results are included in the author's Ph.D. dissertation (McMaster University) written under the direction of Donald H. Pelletier to whom the author is grateful. The author also wishes to thank the referee for his/her comments on a preliminary version of this paper.

1.3 $C \subset P_\kappa\lambda$ is said to be *strongly closed* iff $(\forall X \subset C)(|X| < \kappa \Rightarrow \bigcup X \in C)$. Thus $C \subset P_\kappa\lambda$ is called a *strong cub* iff it is both unbounded and strongly closed. Notice that Menas in **[6]** used the term "strongly closed" for a different but related concept. See 2.4 below for particulars of his concept.

It is easy to see that the family $SC_{\kappa\lambda}$ of strong cub subsets of $P_\kappa\lambda$ generates a filter on $P_\kappa\lambda$. We call this the *strong cub filter* and denote it by $SCF_{\kappa\lambda}$. Its dual $SNS_{\kappa\lambda}$ is called the *strongly nonstationary ideal.*

It is easy to see that $SCF_{\kappa\kappa} = CF_{\kappa\kappa}$. But this is *not* the case if $\lambda > \kappa$; in §2 below we will use an argument due to Menas **[6]** to show that $(\forall \lambda > \kappa)(SCF_{\kappa\lambda} \subsetneqq CF_{\kappa\lambda})$.

1.4 The *diagonal intersection* $\Delta(X_\alpha : \alpha < \lambda)$ and the *diagonal union* $\nabla(X_\alpha : \alpha < \lambda)$ of a λ-sequence $(X_\alpha : \alpha < \lambda)$ of subsets of $P_\kappa\lambda$ are defined by $\Delta(X_\alpha : \alpha < \lambda) = \{x \in P_\kappa\lambda : (\forall \alpha \in x)(x \in X_\alpha)\}$ and $\nabla(X_\alpha : \alpha < \lambda) = \{x \in P_\kappa\lambda : (\exists \alpha \in x)(x \in X_\alpha)\}$.

A filter F (an ideal I) on $P_\kappa\lambda$ is said to be *normal* iff F (I) is closed under diagonal intersections (diagonal unions).

1.5 A generalization of some notation developed by Baumgartner, Taylor and Wagon in **[1]** will be useful.

For any filter F on $P_\kappa\lambda$, ΔF denotes the set $\{X \subset P_\kappa\lambda : (\exists (X_\alpha : \alpha < \lambda) \in {}^\lambda F)(X = \Delta(X_\alpha : \alpha < \lambda))\}$. It is easy to see that ΔF is a (not necessarily proper) filter extending F, and that F is normal iff $F = \Delta F$. The dual definition and facts for ideals are clear.

1.6 In this paper we will use some results of Menas **[6]** to prove the following

THEOREM. (i) $(\forall \lambda \geqslant \kappa)(\Delta FSF_{\kappa\lambda} = SCF_{\kappa\lambda})$,
(ii) $(\forall \lambda > \kappa)(SCF_{\kappa\lambda} \subsetneqq CF_{\kappa\lambda})$,
(iii) $(\forall \lambda \geqslant \kappa)(CF_{\kappa\lambda} = \Delta\Delta FSF_{\kappa\lambda})$,
(iv) $(\forall \lambda \geqslant \kappa)$($CF_{\kappa\lambda}$ *is the smallest normal filter on* $P_\kappa\lambda$),
(v) $(\forall \lambda > \kappa)$($SCF_{\kappa\lambda}$ *is not normal*).

The main results are (ii), (iii) and (iv) which appear below in 2.7, 2.10 and 2.11 respectively.

2. The strong cub filter and the minimality of $CF_{\kappa\lambda}$. $SCF_{\kappa\lambda}$ is easily obtained from $FSF_{\kappa\lambda}$ as we now show.

2.1 THEOREM. $(\forall \lambda \geqslant \kappa)(SCF_{\kappa\lambda} = \Delta FSF_{\kappa\lambda})$.

PROOF. First, pick $(x_\alpha : \alpha < \lambda) \in {}^\lambda P_\kappa\lambda$ and set $C = \Delta(\hat{x}_\alpha : \alpha < \lambda) = \{x \in P_\kappa\lambda : (\forall \alpha \in x)(x_\alpha \subset x)\}$. Clearly C is a cub. Let $X \in [C]^{<\kappa}$. Clearly $\bigcup X \in P_\kappa\lambda$. Now let $\alpha \in \bigcup X$ and pick $x \in X \subset C$ such that $\alpha \in x$. Then $x_\alpha \subset x \subset \bigcup X$, so $\bigcup X \in C$.

Conversely, let $C \subset P_\kappa\lambda$ be a strong cub. For each $\alpha < \lambda$ pick $x_\alpha \in C$ such that $\alpha \in x_\alpha$. We show that $\Delta(\hat{x}_\alpha : \alpha < \lambda) \subset C$. Pick $x \in \Delta(\hat{x}_\alpha : \alpha < \lambda)$. Since $(\forall \alpha \in x)(x_\alpha \subset x)$ and since $x \subset \bigcup\{x_\alpha : \alpha \in x\}$ it is clear that $x = \bigcup\{x_\alpha : \alpha \in x\}$. Then since C is strongly closed, it follows that $x \in C$. □

Note that our proof of Theorem 2.1 yields the following useful fact.

2.2 *For any λ-sequence $(x_\alpha : \alpha < \lambda)$ of elements of $P_\kappa\lambda$, $\Delta(\hat{x}_\alpha : \alpha < \lambda)$ is a strong cub.* □

It is clear that $(\forall\lambda \geqslant \kappa)(SCF_{\kappa\lambda} \subset CF_{\kappa\lambda})$ and that this inclusion reverses if $\lambda = \kappa$. If $\lambda > \kappa$ however, then as a careful examination of Menas' proof of 1.7 in **[6]** reveals, $SCF_{\kappa\lambda} \subsetneq CF_{\kappa\lambda}$. For the sake of completeness, we will give all of the particulars here (2.6, 2.7 below). This requires two easy preliminaries (2.3, 2.5).

2.3 LEMMA. *For any* λ*-sequence* $(x_\alpha : \alpha < \lambda)$ *of elements of* $P_\kappa\lambda$, $C = \Delta(\hat{x}_\alpha : \alpha < \lambda)$ *has the property that* $(\forall X \subset C)(X \neq 0 \Rightarrow \bigcap X \in C)$. $\square$

2.4 REMARK. In **[6]** Menas called a closed subset C of $P_\kappa\lambda$ strongly closed iff it has the property given in the preceding lemma. We call these sets *Menas closed*. Thus we call $C \subset P_\kappa\lambda$ a *Menas cub* iff it is a cub and has the property $(\forall X \subset C)(X \neq 0 \Rightarrow \bigcap X \in C)$.

It is easy to see that the intersection of any $< \kappa$ sequence of Menas cubs is a Menas cub, and that the diagonal intersection of any λ sequence of Menas cubs is a Menas cub. Thus the *Menas cub filter* $MCF_{\kappa\lambda}$ is a normal filter on $P_\kappa\lambda$. In fact, Menas proved in **[6]** that $MCF_{\kappa\lambda} = CF_{\kappa\lambda}$. This will also follow as a corollary to our 2.12 below.

2.5 LEMMA. (1) *For any* $f\colon \lambda \times \lambda \to \lambda$, $C_f = \{x \in P_\kappa\lambda : f''(x \times x) \subset x\}$ *is a Menas cub.*

(2) *For any* $f\colon \lambda \to \lambda$, $C_f = \{x \in P_\kappa\lambda : f''(x) \subset x\}$ *is a strong cub.* $\square$

2.6 LEMMA. *For any* $\lambda > \kappa$ *and any bijection* $f\colon \lambda \times \lambda \leftrightarrow \lambda$, $C_f \in CF_{\kappa\lambda} - SCF_{\kappa\lambda}$.

PROOF. In view of 2.5(1) above, it will suffice to prove that $C_f \notin SCF_{\kappa\lambda}$.

We will show that $C_p \notin SCF_{\kappa\lambda}$ where $p\colon \lambda \times \lambda \leftrightarrow \lambda$ is the canonical bijection. In view of 2.5(2) above, this will suffice; if $f\colon \lambda \times \lambda \leftrightarrow \lambda$ is any (other) bijection, then there is a bijection $h\colon \lambda \leftrightarrow \lambda$ (namely $h = p \circ f^{-1}$) such that $p = h \circ f$ and $C_f \cap C_h \subset C_p$.

Thus let $p\colon \lambda \times \lambda \leftrightarrow \lambda$ be the canonical bijection, and notice that $q \restriction \kappa^+ \times \kappa^+$ is the canonical bijection on $\kappa^+ \times \kappa^+$. We will show that $C_q \notin SCF_{\kappa\kappa^+}$; this will suffice since for any strong cub subset C of $P_\kappa\lambda$, $\{y \cap \kappa^+ : y \in C\}$ is easily seen to be a strong cub in $P_\kappa\kappa^+$, and since $C_q = \{y \cap \kappa^+ : y \in C_p\}$.

Suppose by way of contradiction that $C_q \in SCF_{\kappa\kappa^+} = \Delta FSF_{\kappa\kappa^+}$, and let $(z_\alpha : \alpha < \kappa^+)$ be a κ^+-sequence of elements of $P_\kappa\kappa^+$ such that $C = \Delta(\hat{z}_\alpha : \alpha < \kappa^+) \subset C_q$. We construct a regressive function $g\colon (\kappa^+ - \kappa) \to \kappa^+$ and then use this to obtain the required contradiction. This will require a few preliminaries.

For each $\alpha, \beta < \kappa^+$ define $x_\alpha = \bigcap\{x \in C : \alpha \in x\}$, $x_\beta = \bigcap\{x \in C : \beta \in x\}$, $x_{\alpha\beta} = \bigcap\{x \in C : \{\alpha, \beta\} \subset x\}$. By 2.3, C is Menas closed so x_α, x_β, $x_{\alpha\beta}$ are all in $C \subset C_q$. And by 2.2, C is also strongly closed so $x_\alpha \cup x_\beta \in C$. Thus $x_{\alpha\beta} = x_\alpha \cup x_\beta$.

Now pick $\alpha \in \kappa^+ - \kappa$, and note that since q is one-one, $|\{q(\alpha, \beta) : \beta < \alpha\}| = \kappa$. But $|x_\alpha| < \kappa$, so $(\exists \beta < \alpha)(q(\alpha, \beta) \notin x_\alpha)$. For each $\alpha \in \kappa^+ - \kappa$, pick $\beta_\alpha < \alpha$ such that $q(\alpha, \beta) \notin x_\alpha$, and then set $g(\alpha) = \beta_\alpha$. Clearly g is regressive.

We can now obtain the required contradiction. Pick $\beta < \kappa^+$ such that $X = g^{-1}(\{\beta\}) \in NS^+_{\kappa^+}$. The definition of g guarantees that $(\forall \alpha \in X)(q(\alpha, \beta) \notin x_\alpha)$. But $(\forall \alpha \in X)(q(\alpha, \beta) \in x_{\alpha\beta} = x_\alpha \cup x_\beta)$ since $\{\alpha, \beta\} \subset x_{\alpha\beta} = x_\alpha \cup x_\beta$, and since $C \subset C_q$. This means that $(\forall \alpha \in X)(q(\alpha, \beta) \in x_\beta)$ thus contradicting the one-oneness of q since $|x_\beta| < \kappa < \kappa^+ = |X|$. $\square$

2.7 THEOREM. *For every* $\lambda > \kappa$, $SCF_{\kappa\lambda} \subsetneq CF_{\kappa\lambda}$.

PROOF. Immediate by 2.6. $\square$

The minimality of $CF_{\kappa\lambda}$. In view of 2.1 and 2.7 above we know that $(\forall\lambda > \kappa)(\Delta FSF_{\kappa\lambda} \subsetneq CF_{\kappa\lambda})$. In 2.10 below we use a result of Menas to show that $(\forall\lambda \geqslant \kappa)(CF_{\kappa\lambda} = \Delta\Delta FSF_{\kappa\lambda})$. We start with the following definition which is due to Menas [6].

2.8 DEFINITION. For any finite $n \geqslant 1$ and any $w\colon \lambda^n \to P_\kappa\lambda$ define $\mathcal{C}(\{w\}) \subset P_\kappa\lambda$ by

$$\mathcal{C}(\{w\}) = \{x \in P_\kappa\lambda : (\forall \vec{\alpha} \in x^n)(w(\vec{\alpha}) \subset x)\}.$$

Menas proved in [6] that *for any cub subset* C *of* $P_\kappa\lambda$, *there is a* $w\colon \lambda^2 \to P_\kappa\lambda$ *such that* $\mathcal{C}(\{w\}) \subset C$. We use this result together with the following simple lemma to prove that $CF_{\kappa\lambda} = \Delta\Delta FSF_{\kappa\lambda}$.

2.9 LEMMA. *For any* $n \in \{1,2\}$ *and any* $w\colon \lambda^n \to P_\kappa\lambda$,

$$\mathcal{C}(\{w\}) = \begin{cases} \Delta(\widehat{w(\alpha)}: \alpha < \lambda) & \text{if } w\colon \lambda \to P_\kappa\lambda, \\ \Delta(\Delta(\widehat{w(\alpha,\beta)}: \alpha < \lambda): \beta < \lambda) & \text{if } w\colon \lambda^2 \to P_\kappa\lambda. \end{cases}$$

PROOF. It is clear that for any $w\colon \lambda \to P_\kappa\lambda$, $\Delta(\widehat{w(\alpha)} : \alpha < \lambda) = \{x \in P_\kappa\lambda : (\forall\alpha \in x)(w(\alpha) \subset x)\} = \mathcal{C}(\{w\})$. Now let $w\colon \lambda^2 \to P_\kappa\lambda$. Then for any $x \in P_\kappa\lambda$, $x \in \mathcal{C}(\{w\})$ iff $(\forall\alpha,\beta \in x)(w(\alpha,\beta) \subset x)$ iff $(\forall\alpha \in x)(\forall\beta \in x)(w(\alpha,\beta) \subset x)$ iff $(\forall\beta \in x)(x \in \Delta(\widehat{w(\alpha,\beta)} : \alpha < \lambda))$ iff $x \in \Delta(\Delta(\widehat{w(\alpha,\beta)} : \alpha < \lambda) : \beta < \lambda)$. $\square$

2.10 THEOREM. *For every* $\lambda \geqslant \kappa$, $CF_{\kappa\lambda} = \Delta\Delta FSF_{\kappa\lambda}$.

PROOF. Since $FSF_{\kappa\lambda} \subset CF_{\kappa\lambda}$ and since $CF_{\kappa\lambda}$ is normal, it is clear that $\Delta\Delta FSF_{\kappa\lambda} \subset CF_{\kappa\lambda}$.

Now let $C \subset P_\kappa\lambda$ be a cub and let $w\colon \lambda^2 \to P_\kappa\lambda$ be such that $\mathcal{C}(\{w\}) \subset C$. Then by 2.9 above, $\Delta(\Delta(\widehat{w(\alpha,\beta)} : \alpha < \lambda) : \beta < \lambda) \subset C$, so $C \in \Delta\Delta FSF_{\kappa\lambda}$. $\square$

2.11 COROLLARY. *For every* $\lambda \geqslant \kappa$, $CF_{\kappa\lambda}$ *is the smallest normal filter on* $P_\kappa\lambda$.

PROOF. This is immediate from 2.10 since every normal filter on $P_\kappa\lambda$ must extend $\Delta\Delta FSF_{\kappa\lambda}$. $\square$

2.12 COROLLARY. *For every* $\lambda > \kappa$, $SCF_{\kappa\lambda}$ *is not normal.*

PROOF. This is immediate from 2.7 and 2.10 for if $\lambda > \kappa$, then $\Delta FSF_{\kappa\lambda} \subsetneq \Delta\Delta FSF_{\kappa\lambda}$. $\square$

REMARK. An immediate consequence of 2.12 is that the family of strong cub subsets of $P_\kappa\lambda$ $(\lambda > \kappa)$ is not closed under diagonal intersections. In 1978, Jech [4] provided a direct proof of this fact for $P_{\aleph_0}\aleph_1$.

3. Some additional remarks.

3.1 We denote the dual of $SCF_{\kappa\lambda}$ by $SNS_{\kappa\lambda}$ and call it the *strongly nonstationary* ideal on $P_\kappa\lambda$. Notice that in view of Theorem 2.1, $(\forall\lambda \geqslant \kappa)(SNS_{\kappa\lambda} = \nabla I_{\kappa\lambda})$.

It is easy to see that *for any ideal I* on $P_\kappa\lambda$ and any $X \subset P_\kappa\lambda$, $X \in \nabla I$ iff there is an I-small regressive function on X, i.e. a function f: $X \to \lambda$ with the properties (i) $(\forall x \in X)(f(x) \in x)$ and (ii) $(\forall \alpha < \lambda)(f^{-1}(\{\alpha\}) \in I)$.

Finally, notice that the "dual" of Theorem 2.7 is $(\forall \lambda > \kappa)(SNS_{\kappa\lambda} \subsetneq NS_{\kappa\lambda})$. Thus we obtain the result expressed in Menas' Proposition 1.7 in [**6**].

It is well known that for any ideal I on κ, $\nabla\nabla I = \nabla I$ (e.g. see [**1**]). A $P_\kappa\lambda$ version of the argument used to prove this shows that for any ideal I on $P_\kappa\lambda$, if $SNS_{\kappa\lambda} \subset I$ then $\nabla\nabla I = \nabla I$ (3.2 below). An immediate consequence of this is that for any ideal I on $P_\kappa\lambda$, $\nabla\nabla\nabla I = \nabla\nabla I$ (3.3 below). Notice that in view of the fact that $(\forall \lambda > \kappa)(\nabla I_{\kappa\lambda} = SNS_{\kappa\lambda} \subsetneq NS_{\kappa\lambda} = \nabla\nabla I_{\kappa\lambda})$, these results are the best we can expect.

3.2 THEOREM. *For every $\lambda \geqslant \kappa$ and any ideal I on $P_\kappa\lambda$, if $SNS_{\kappa\lambda} \subset I$, then $\nabla\nabla I = \nabla I$.*

PROOF. Clearly $\nabla I \subset \nabla\nabla I$, so it remains to prove the reverse inclusion.

Pick $X \in \nabla\nabla I$ and let f: $X \to \lambda$ be a ∇I-small regressive function on X. For each $\alpha < \lambda$ set $X_\alpha = f^{-1}(\{\alpha\})$ and recall that $(\forall \alpha < \lambda)(X_\alpha \in \nabla I)$. Thus for each $\alpha < \lambda$ let f_α: $X_\alpha \to \lambda$ be an I-small regressive function on X_α.

Now let p: $\lambda \times \lambda \leftrightarrow \lambda$ be any bijection, and set $C = \{x \in P_\kappa\lambda : p''(x \times x) \subset x\}$. Since $X - C \in NS_{\kappa\lambda} = \nabla SNS_{\kappa\lambda} \subset \nabla I$, we can complete the proof by showing that $X \cap C \in \nabla I$.

Define g: $X \cap C \to \lambda$ by $g(x) = p(f(x), f_{f(x)}(x))$. We show that g is I-small and regressive. It is clear that g is regressive on $X \cap C$ since f is regressive on X, since $f_{f(x)}$ is regressive on $X_{f(x)} \subset X$ and since $X \cap C \subset C$. Now pick $\beta < \lambda$ and let $\beta_0, \beta_1 < \lambda$ be such that $p(\beta_0, \beta_1) = \beta$. Then $x \in g^{-1}(\{\beta\}) \Rightarrow g(x) = \beta \Rightarrow f(x) = \beta_0$ & $f_{f(x)}(x) = f_{\beta_0}(x) = \beta_1 \Rightarrow x \in f_{\beta_0}^{-1}(\{\beta_1\})$. Thus $g^{-1}(\{\beta\}) \subset f_{\beta_0}^{-1}(\{\beta_1\})$, so $g^{-1}(\{\beta\}) \in I$. □

3.3 COROLLARY. *For every $\lambda \geqslant \kappa$ and any ideal I on $P_\kappa\lambda$, $\nabla\nabla\nabla I = \nabla\nabla I$.*

PROOF. Since $I_{\kappa\lambda} \subset I$ and since $SNS_{\kappa\lambda} = \nabla I_{\kappa\lambda}$, it is clear that $SNS_{\kappa\lambda} \subset \nabla I$. It now follows by 3.2 that $\nabla\nabla\nabla I = \nabla\nabla I$. □

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