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SOME REMARKS ON BRAUER'S THIRD MAIN THEOREM

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ABSTRACT. We consider two classes of p-blocks of a finite group G which have the property that for every block B of them and every subgroup H of G, H has only a small number of admissible blocks b with $b^G = B$. In this they are similar to the principal block of G. These blocks are described by means of certain modules they contain.

Introduction. Let G be a finite group of order $|G|$ and χ a complex character of G in a p-block B of G, $P \mid |G|$. If u is an element of G of order a power of p and r is an element of G of order prime to p which commutes with u, then by [3, 1.1]

$$(*) \qquad \chi(ur) = \sum_{b\in\beta} \sum_{\phi\in[b]} d(\chi,\phi)\phi(r)$$

where β is the set of all the admissible blocks of $C_G(u)$ with $b^G = B$, $d(\chi,\phi)$ the generalized decomposition numbers and $[b]$ is a basic set for b. (See [3].) Here, following Brauer [3, 2C], we call a block b of a subgroup H of G admissible if the centralizer of one of its defect groups is contained in H.

When using $(*)$, one has to have some information on β. The aim of this work is to supply sufficient conditions for β to be "small". A typical result of this kind is Brauer's Third Main Theorem which states that if B is the principal block of G and H is any subgroup of G, b an admissible block of H, then $b^G = B$ if and only if b is the principal block of H. For $H = C_G(u)$ this implies, of course, that β in $(*)$ consists only of the principal block of H. Later, Brauer showed in [2] that, in general, blocks which contain a linear representation have a similar property. We shall describe more cases like this in terms of modular representations:

Call a block B of KG a *quasi-principal* block if every subgroup of G which has admissible blocks, has exactly one admissible block b with $b^G = B$. Call B a *weak-principal* block if for every subgroup H of G which has admissible blocks and for every p-subgroup Q of H there is at most one admissible block b of H with $b^G = B$.

In these terms our main results are the following.

THEOREM 1. *Let B be a block of KG and S a Sylow p-subgroup of G. Assume that K is a splitting field for the subgroups of G and B contains an indecomposable KG-module M of K-dimension $d \le p-1$. If $(d, |N_G(S)/SC_G(S)|) = 1$ then B is quasi-principal. Moreover, let H be a subgroup of G which has admissible blocks. Then H has exactly one admissible block b with $b^G = B$ and it contains all the components of M_H.*

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THEOREM 2. *Let K be a splitting field for the subgroups of G and let B be a block of KG with a defect group D. If B contains an indecomposable KG-module M such that all the components of $M_{DC_G(D)}$ belong to the same block b, then $b^G = B$, $DC_G(D)$ contains a Sylow p-subgroup of G and B is weak-principal. If D is a Sylow p-subgroup of G then B is quasi-principal.*

The proof of Theorem 2 provides a sufficient (and necessary) condition for a block B of KG to be the principal block (Theorem 3). As a corollary we get a result of Cassey and Gaschütz [**5**] concerning certain elementary abelian sections of G of order a power of p.

In §1 we quote the necessary results and fix the notation. In §2 we investigate weak-principal blocks while in §3 we deal with quasi-principal blocks.

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I am indebted to Professor Walter Feit for drawing my attention to [**2**]. I also wish to express my thanks to the referee for his useful suggestions.

1. Notation and preliminary results. In what follows fix the following notation: G a finite group, H a subgroup of G, S a Sylow p-subgroup of G, K a field of characteristic p, $p \mid |G|$, which is a splitting field for G and its subgroups, B a block of KG with idempotent E and M an indecomposable KG-module in B. Write "component" for "indecomposable direct summand" and for every subgroup X of G and block c of X denote by (c, M_X) the fact that c contains a component of M_X. Other notation is standard, see [**6**].

We recall some definitions and results from Brauer's work [**3**]. Denote by $\underline{P}$ the set of all pairs (Q, b), where Q is a p-subgroup of G and b is a block of $QC(Q)$, with the defect group Q. Here $C(Q)$ stands for $C_G(Q)$.

DEFINITION 1.1 [**3**, 3.1]. Two pairs (P, b^*) and (Q, b^{**}) of $\underline{P}$ are *linked* if Q is a normal subgroup of P, $Q \neq P$, and if $(b^*)^{PC(Q)} = (b^{**})^{PC(Q)} = b$, b a block of $PC(Q)$.

LEMMA 1.2 [**3**, 3A]. *If the pairs (P, b^*) and (Q, b^{**}) of $\underline{P}$ are linked and $b = b^{*PC(Q)}$, then*

(a) *b and b^* have the same defect group P.*

(b) *b and b^{**} have the same corresponding central idempotent e.*

(c) $C_P(Q) \subseteq Q$.

LEMMA 1.3. *Let B be a block of KG with defect group D and $\tilde{b}$ an admissible block of H with a defect group Q, $Q \leq D$, such that $\tilde{b}^G = B$. Then*

(a) [**3**, J] *There is a pair (Q, b) in $\underline{P}$ with $b^H = \tilde{b}$ (hence $b^G = B$).*

(b) [**3**, 3E, 3F] *If $(Q, b^*) \in \underline{P}$ and b^{*G} has a defect group D with $D \neq Q$, then there exists a p-subgroup P of G with $|P : Q| = p$ and a pair (P, b^{**}) in $\underline{P}$ such that (Q, b^*) and (P, b^{**}) are linked.*

Finally, we recall some results from [**8**]

PROPOSITION 1.4 [**8**, 3(a)]. *If B has a defect group D then every admissible block b of H which satisfies $b^G = B$ and has a defect group D contains a component of M_H.*

PROPOSITION 1.5 [**8**, PROPOSITION]. *Let D be a p-subgroup of G which is not normal in any Sylow p-subgroup of G and let H be a normal subgroup of $N_G(D)$ which contains $DC_G(D)$ and a Sylow p-subgroup of $N_G(D)$. If B is a block of KG with defect group D and M is an indecomposable KG-module in B, then M_H has a component in a block b of KH with $b^G \neq B$, provided K is a splitting field for $DC_G(D)$.*

PROPOSITION 1.6 [**10**, 1.6(a)]. *Let β be the set of all the defect groups of B and denote $\mathcal{D} = \{D \cap H | D \in \beta,\ C_G(D \cap H) \subseteq H\}$. Let $\mathcal{D}_0$ be the set of all the maximal elements of $\mathcal{D}$ with respect to " $\leq_H$ ". Then*

(a) *If b is an admissible block of H with $b^G = B$ then b has a defect group in $\mathcal{D}$.*

(b) *For every $Q \in \mathcal{D}_0$ there exists a block b of H with $b^G = B$ having defect group Q.*

2. Weak-principal blocks.

LEMMA 2.1. *Let B be a block of KG with a defect group D and let b be a block of $DC(D)$ having defect group D with $b^G = B$.* (*See Lemma* 1.3(a).) *Let P be a Sylow p-subgroup of $N_G(D)$, and assume that b is the only block of $DC(D)$ with $b^G = B$. Then*

(a) $P = DC_P(D)$.

(b) *D is a Sylow p-subgroup of G if and only if $Z(D)$ is a Sylow p-subgroup of $C_G(D)$.*

In particular, if B is weak-principal, then (a) *and* (b) *hold.*

PROOF. (a) By [**3**, 4K] the set of all the blocks b_i of $DC(D)$ with $b_i^G = B$ forms an $N_G(D)$-conjugacy class of blocks. Therefore, if $T(b) = \{x \in N_G(D) | x^{-1}bx = b\}$, then $|N_G(D) : T(b)| = 1$ by assumption, i.e. $N_G(D) = T(b)$. On the other hand, $DC_P(D)$ is a Sylow p-subgroup of $T(b)$, by [**12**]. Thus $P = DC_P(D)$.

(b) If D is a Sylow p-subgroup of G then certainly $Z(D)$ is a Sylow p-subgroup of $C_G(D)$.

If $Z(D)$ is a Sylow p-subgroup of $C_G(D)$ then $P = D$ by part (a), hence D is the Sylow p-subgroup of $N_G(D)$. Therefore D is a Sylow p-subgroup of G.

LEMMA 2.2. *Let D be a defect group of B and set $H_0 = DC(D)$. If all the components of M_{H_0} belong to the same block b of H_0, then*

(a) $b^G = B$.

(b) *$DC_G(D)$ contains a Sylow p-subgroup of G.*

(c) *For every subgroup Q of D there is at most one block $\tilde{b}$ of $QC(Q)$ having defect group Q and satisfying $\tilde{b}^G = B$. $\tilde{b}$ contains all the components of $M_{QC(Q)}$.*

PROOF. (a) Follows by Proposition 1.4.

(b) Follows by Lemma 2.1(a), Propositions 1.4 and 1.5.

(c) By induction on $|D : Q|$. For $|D : Q| = 1$ this is just the assumption, by Proposition 1.4. Let Q be a proper subgroup of D and assume that $\tilde{b}$ is a block of $QC(Q)$ with $\tilde{b}^G = B$ having defect group Q. Then by Lemma 1.3(b) there is a subgroup P of D containing Q such that $|P : Q| = p$ and a block b^{**} of $PC(P)$ such that $(Q, \tilde{b})$ and $(P, \tilde{b}^{**})$ are linked. By Lemma 1.2(b) $b^{**G} = B$ and by the induction hypothesis b^{**} contains all the components of $M_{PC(P)}$. Let L be a component of $M_{PC(Q)}$. By the induction hypothesis b^{**} contains all the components of $L_{PC(P)}$, as $L|M_{PC(Q)}$. Since b and b^{**} have the same defect group P by Lemma 1.2(a), L must belong to b by Proposition 1.4. Consequently $M_{PC(Q)}$ belongs to $\tilde{b}$, hence $M_{QC(Q)}$ belong to b by Lemma 1.2(b), since $Me = M$, e as in Lemma 1.2(b). This obviously proves part (c).

PROPOSITION 2.3. *Let B be a block of KG with a defect group D. If D is a Sylow p-subgroup of G then B is quasi-principal if and only if B is weak-principal.*

PROOF. Let $\mathcal{H} = \{H \leq G | H$ has admissible blocks b with $b^G = B\}$. If B is quasi-principal then it is weak-principal, by definition. Let B be a weak-principal block of KG. Let $H \in \mathcal{H}$ and assume that b is an admissible block of KH with $b^G = B$, having a defect group Q. Let S and P be Sylow p-subgroups of G and H respectively with $Q \leq P \leq S$. We show by induction on $|S:Q|$ that $Q = P$. Then the result follows, since B is weak-principal. For $S = Q$ this is obvious. Assume that $|S:Q| > 1$ and the result holds for every subgroup H^* of G and admissible block b^* of H^* with $b^{*G} = B$ having defect group $Q^* \gneqq Q$. By assumption B has maximal defect, hence by Proposition 1.6 KH has a block $\tilde{b}$ with $\tilde{b}^G = B$ having defect group P. Since $Q \leq P$, it follows by Bauer's First Main Theorem that $QC(Q)$ has blocks β and $\tilde{\beta}$ with $\beta^H = b$ and $\tilde{\beta}^H = \tilde{b}$, respectively. Let $H^* = N_G(Q)$. Then $b^* = \beta^{H^*}$ and $\tilde{b}^{**} = \tilde{\beta}^{H^*}$ have defect groups $\gneqq Q$ by Brauer's First Main Theorem, hence $\tilde{\beta}^{H^*} = \beta^{H^*}$ by the induction hypothesis since B is weak-principal and $\tilde{\beta}^G = \beta^G = B$. So β and $\tilde{\beta}$ are conjugate in H^* hence both have defect group Q. But then $\beta = \tilde{\beta}$, since B is weak-principal and $\tilde{b} = \tilde{\beta}^H = \beta^H = b$. Hence $Q = P$, and $\tilde{b}$ is the only admissible block of H with $\tilde{b}^G = B$.

We now prove the main result of this section.

THEOREM 2. *Let K be a splitting field for the subgroups of G and let B be a block of KG with a defect group D. If B contains an indecomposable KG-module M such that all the components of $M_{DC_G}(D)$ belong to the same block b, then $b^G = B$, $DC_G(D)$ contains a Sylow p-subgroup of G and B is weak-principal. If D is a Sylow p-subgroup of G then B is quasi-principal.*

PROOF. $b^G = B$ by Lemma 2.2(a), $DC_G(D)$ contains a Sylow p-subgroup of G by Lemma 2.2(b) and B is weak-principal by Lemma 2.2(c). If D is a Sylow p-subgroup of G then B is quasi-principal by Proposition 2.3.

COROLLARY 1. *Let K be a splitting field for the subgroups of G, B a block of KG, M an indecomposable KG-module in B and S a Sylow p-subgroup of G. If all the components of $M_{SC(S)}$ belong to the same block then each of the following conditions on M implies that B is quasi-principal:*

(a) *S is contained in the kernel of the representation afforded by M.*

(b) *M has K-dimension prime to p.*

PROOF. In case (a) M has vertex S by [**6**, 53.8] and in case (b) M has vertex S by [**6**, 52.4]. So in both cases B has defect group S. Consequently B is quasi-principal by Theorem 2.

The proof of Theorem 2 yields

THEOREM 3. *Let B be a block of KG and M an indecomposable KG-module in B. Let D be a defect group of B. Then each one of the following conditions implies that B is the principal block of KG.*

(a) *$C_G(D)$ is contained in the kernel of the representation afforded by M.*

(b) *$C_G(D)$ has a normal p-complement T and T is contained in the kernel of the representation afforded by M.*

PROOF. Note that in Theorem 4 there is no restriction on the field K.

(a) Assume first that K is a splitting field for the subgroups of G. Since $C_G(D)$ acts trivially on M by assumption, $M_{C(D)}$ is the direct sum of trivial one-dimensional $KC(D)$ modules, hence $M_{C(D)}$ belongs to the principal block of $KC(D)$. On the other hand, it is elemenary and well known (see e.g. [**1**, 2.9(1)]) that every central primitive idempotent of $DC(D)$ lies in $C(D)$ and every central primitive idempotent of $C(D)$ is a central idempotent of $DC(D)$, as D centralizes $C(D)$. Consequently, the principal blocks of $C(D)$ and $DC(D)$ have the same idempotent e and so $M_{DC(D)}$ belongs to the principal block of $DC(D)$, as $Me = M$. Hence by Theorem 3 and Brauer's Third Main Theorem (or Theorem 1) M belongs to the principal block of KG.

Assume now the general case and let $K \subseteq \overline{K}$ be a splitting field for the subgroup of G. Let $M \otimes \overline{K} = L_1 \oplus \cdots \oplus L_r$, L_i indecomposable $\overline{K}G$-modules. Since $C_G(D)$ acts trivially on the L_i, $1 \le i \le r$, $M \otimes \overline{K}$ belong to the principal block of $\overline{K}G$, by the above. Assume $E \otimes 1 = e_1 + \cdots + e_t$, e_i central primitive idempotents of $\overline{K}G$. Then $(M \otimes 1)(E \otimes 1) = M \otimes 1$ implies that $(M \otimes 1)e_i \neq 0$ for some i, $1 \le i \le t$. Therefore e_i is the idempotent of the principal block of $\overline{K}G$ and moreover $(M \otimes \overline{K})e_i = M \otimes \overline{K}$, by the above. We want to show that E is the principal block of KG. Let L be the trivial representation of KG. Then $L \otimes \overline{K}$ is the trivial representation of $\overline{K}G$; hence $(L \otimes \overline{K})e_i = L \otimes \overline{K}$. Consequently, $(L \otimes \overline{K})(E \otimes 1) = L \otimes \overline{K}$. This easily implies $LE = L$, i.e. M belongs to the principal block of KG.

(b) By [**4** or **11**] every idempotent of $DC(D)$ lies in T. Therefore, as in part (a) $M_{DC(D)}$ belongs to the principal block of $DC(D)$. The rest is the same as in part (a).

COROLLARY 2 [**5**]. *Let R_1 and R_2 be two normal p-subgroups of G such that $R_1 \subseteq R_2$ and R_2/R_1 is elementary abelian. Then the $\mathbf{Z}_P G$-module R_2/R_1 belongs to the principal block.*

PROOF. Assume first that R_2/R_1 is indecomposable and belongs to B with defect group D. Then $R_2 \le D$ by [**3**, 2]. Therefore $C(D) \le C(R_2)$, hence $C(D)$ is contained in the kernel of the representation afforded by R_2/R_1. Consequently R_2/R_1 belongs to the principal block of KG, by Theorem 3(a). If R_2/R_1 decomposes, then the same argument for each component of it shows that R_2/R_1 belongs to the principal block.

COROLLARY 3. *Let S be a Sylow p-subgroup of G and T a normal p-complement of $C_G(S)$. Then the principal block of G contains at least $|G : TG'|$ linear characters.*

PROOF. Every linear character of G which contains $G'T$ in its kernel belongs to the principal block of KG, by Theorem 3(b). Hence the result follows.

COROLLARY 4. *Let D be a normal p-subgroup of G. Then the KG-module $K(G/C_G(D))$ belongs to the principal block of KG.*

PROOF. Every component of $K(G/C_G(D))$ contains $C_G(D)$ in its kernel. Since D is a normal p-subgroup of G, every block of KG has defect group containing D. Consequently, the centralizer of every defect group is contained in the kernel of every component of $K(G/C_G(D))$, hence the result follows by Theorem 3(a).

3. Quasi-principal blocks.

LEMMA 3.1. *Let B be a block of KG with a defect group D and b an admissible block of KH. Assume that B has an indecomposable KG-module M which satisfies*
$(**)$ *Every component of $M_{D^*C(D^*)}$ has vertex $\geq D^*$, for every $D^* < D$.*
Then $b^G = B$ if and only if b contains a component of M_H

PROOF. Let $L_1, \dots, L_t$ be all the components of M_H and let b_i be the block of H which contains L_i, b_i with defect group D_i, $1 \leq i \leq t$. Assume that b has defect group $D_0 \leq D$. Then we claim

(1) $b_i^G = B$ for $1 \leq i \leq t$. (In other words (b, M_H) implies $b^G = B$.)

(2) If $D_0 \lhd G$, $H = D_0C(D_0)$ and $b^G = B$ then $b = b^i$ for some i, $1 \leq i \leq t$.

(3) Let β be a block of $D_0C(D_0)$ with $\beta^H = b$. Then $(\beta^{N_G(D_0)}, M_{N_G(D_0)})$ implies $b = b_i$ for some i, $1 \leq i \leq t$, (i.e. (b, M_H)).

Assume we have proved claims (1)–(3). Then in view of (1) it remains only to show that $b^G = B$ implies (b, M_H). Thus assume $b^G = B$ and prove (b, M_H) by induction on $|D : D_0|$.

If $D = D_0$, then our assertion is Proposition 1.4.

Therefore assume $D_0 \neq D$ and suppose we have proved the assertion for every subgroup H' and admissible block b' of H with defect group $D_0' > D_0$ and prove for H and b. Since H is arbitrary, this will do. Let β be as in (3). Then by Brauer's First Main Theorem $\beta^{N_G(D_0)}$ has a defect group D', strictly containing D_0, as $D_0 \neq D$. Therefore $N_G(D_0)$ satisfies the induction hypothesis with $N_G(D_0)$ in place of H' and $\beta^{N_G(D_0)}$ in place of b', hence $(\beta^{N_G(D_0)}, M_{N_G(D_0)})$. By (3) this proves the lemma. Now we prove claims (1)–(3).

(1) Since every component of $M_{D_iC(D_i)}$ has vertex $\geq D_i$, obviously every L_i has vertex $V_i \geq D_i$, hence $C(V_i) \leq C(D_i)$ and $b^G = B$ by Nagao's theorem [**6**, 56.5].

(2) All block b' of H with $b'^G = B$ are conjugate in G. Since every such b' contains a component of M_H, every such b' is one of the b_i, $1 \leq i \leq t$.

(3) By (2) $(\beta, M_{D_0C(D_0)})$. Therefore (b, M_H) by (1), as $b = \beta^H$. Here we used the fact that if M satisfies $(**)$ then every component of M_H satisfies it.

We come to the main result of this section:

THEOREM 1. *Let B be a block of KG and S a Sylow p-subgroup of G. Assume that K is a splitting field for the subgroups of G and B contains an indecomposable KG-module M of K-dimension $d \leq p-1$. If $(d, |N_G(S)/SC_G(S)|) = 1$ then B is quasi-principal. Moreover, let H be a subgroup of G which has admissible blocks. Then H has exactly one admissible block b with $b^G = B$ and it contains all the componets of M_H.*

PROOF. Since $\dim_K M \leq p-1$, M and all the components of M_H have vertex a Sylow p-subgroup of G and H respectively, by [**7**, 22.6]. Therefore, every block b of KH with $b^G = B$ contains a component of M_H by Lemma 3.1 and has maximal defect by [**6**, 54.10]. It suffices to show that B is weakly-principal. To this end, we may assume $H = SC(S)$, S a Sylow p-subgroup of G, by Theorem 2.

Let $b_1, \dots, b_r$ be all the block of H with $b_i^G = B$. Then by Lemma 3.1 $\dim_K M = \sum_{i=1}^r \dim_K Me_i$ where e_i is the central idempotent of b_i, $1 \leq i \leq r$. Since $\dim_K Me_i = \dim_K Mx^{-1}e_ix$ for every i, $1 \leq i \leq r$, and every $x \in N_G(S)$ and since the b_i, $1 \leq i \leq r$, form a $N_G(S)$ conjugacy class of blocks of H, $d = \dim_K M = r \dim Me_i$. Hence $r \mid d$. On the other hand, $r = |N_G(S) : T(b_1)|$ hence $r \| N_G(S) : SC(S)|$.

Consequently, $r|(|N_G(S):SC(S)|,d)=1$ and the result follows by Theorem 2. Here $T(b_1)=\{g\in N_G(S)|g^{-1}b_1g=b_1\}$.

The following corollaries are immediate from Theorem 1.

COROLLARY 5 [**2**, 4E, 4D]. *If B contains a linear representation then B is quasi-principal.*

COROLLARY 6. *If $N_G(S)=SC_G(S)$, then every block B which contains a representation of degree $\leq p-1$ is quasi-principal.*

REMARKS. 1. Lemma 3.1 provides a short module-theoretic proof to Brauer's Third Main Theorem and to Corollary 1. This proof uses Brauer's First Main Theorem and Nagao's theorem and does not require that K is a splitting field for the subgroups of G [**9**].

2. All the results quoted from [**3**] have short module-theoretic proofs [**10**].

3. Theorem 3 for the special case $\dim M=1$ was proved in [**2**, 4E]. Also it was proved there that the exact number of linear characters in the principal block is $|G:TG'|$.

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