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SHORTER NOTES

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FOR ANY X, THE PRODUCT $X \times Y$ IS HOMOGENEOUS FOR SOME Y

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ABSTRACT. We prove that for every topological space X there exists a cardinal k and a nonempty subspace $Y \subseteq X^k$ such that the product $X \times Y$ is homogeneous. This answers a question of A. V. Arhangel'skiĭ.

A topological space in which every point can be mapped to every other point by a homeomorphism of the space onto itself is called homogeneous. Answering a question of A. V. Arhangel'skiĭ [**A**], Jan van Mill [**vM**] has constructed an example of a rigid (= no autohomeomorphisms beyond the identity) compact space X such that $X \times X$ is homogeneous. It is not known whether for every compact space X there exists a nonempty compact space Y such that $X \times Y$ is homogeneous (cf. [**DvM**, Question 6.3]). We show if the requirement of compactness is omitted, such a Y always does exist.

THEOREM. *For every nonempty topological space X there exists a nonempty topological space Y such that $X \times Y$ and Y are homeomorphic and homogeneous.*

PROOF. Let A be an infinite set of cardinality $k \geqslant |X|$. In the cube X^A, consisting of all functions $f\colon A \to X$, consider the subspace $Y = \{f \in X^A\colon |f^{-1}(x)| = k$ for every $x \in X\}$. Clearly $X \times Y$ and Y are homeomorphic. Let $g \in Y$ and $h \in Y$. Since $|g^{-1}(x)| = |h^{-1}(x)|$ for each $x \in X$, there exists a permutation p of the set A such that $p(h^{-1}(x)) = g^{-1}(x)$ for each $x \in X$, which means $h = g \circ p$. The mapping $f \mapsto f \circ p$ $(f \in Y)$ is an autohomeomorphism of Y which maps g to h. Hence Y is homogeneous, and so is $X \times Y$.

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