

REMARK. This is well known, at least when A is taken to be a field. The theorem above permits a one-line proof.

PROOF. $R(f, fr + gs) = \overline{N(fr + gs)} = \overline{N(gs)} = N(\bar{g})N(\bar{s}) = R(f, g)R(f, s)$.

COROLLARY. Let f and g be as in the Theorem above. Let $J = J(f, g)$ be the ideal generated by f and g in $A[x]$, and let c be an element of $J \cap A$. Then $R(f, g)$ divides c^n , where n is the degree of f .

REMARK. That $R(f, g)$ divides some power of c is known (see e.g. [2, Lemma 11.3]). In transcendence theory it is often helpful to have bounds for a nonzero resultant (see, e.g. [1]). We present the Corollary in the spirit of such bounds.

PROOF. Let f, g and c be as given. Then there exist polynomials r and s with coefficients in A such that $c = fr + gs$. Then $c^n = R(f, c) = R(f, fr + gs) = R(f, g)R(f, s)$, whence $R(f, g)$ divides c^n .

This Corollary is sharp, in the following sense. Given any nonunit c in A , and any positive integer n , there exist f and g in $A[x]$, f monic of degree n , such that c is in $J(f, g)$, and $R(f, g)$ does not divide c^{n-1} . For example, take $f(x) = x^n$, $g(x) = x^n + c$; then $R(f, g) = c^n$.

Moreover, it is not possible, in general, to remove the condition that f be monic. Take for A the integers, let $f(x) = 2x + 1$, and let $g(x) = 2x + 17$. Then 1 is in $J(f, g)$, since

$$x^4g(x) - (x^4 + 8x^3 - 4x^2 + 2x - 1)f(x) = 1.$$

However, $R(f, g) = 32$. The construction exemplified here is clearly quite general.

If it is required that the leading coefficients of f and g be relatively prime, it can be shown that if c is in $A \cap J(f, g)$ then $R(f, g)$ divides c^k , where $k = \max(m, n)$.

We note as a further corollary that, under the assumption that f is monic, $J(f, g)$ contains A if and only if $R(f, g)$ is a unit. We would like to propose the problem of characterizing those pairs f, g for which $R(f, g)$ is a unit or, more generally, for which $R(f, g)$ divides every element of $A \cap J(f, g)$.

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REFERENCES

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