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CORRECTIONS AND ADDITIONS TO "A GENERALIZATION OF A THEOREM OF AYOUB AND CHOWLA"

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Let χ_1 and χ_2 be characters modulo q_1 and q_2, respectively, and let

$$f(n) = \sum_{d|n} \chi_1(d)\chi_2(n/d).$$

In [3], I estimated the sum

$$\sum_{n \leqslant x} f(n)\log(x/n).$$

Unfortunately, as was pointed out to me in a letter by A. Ivić of Beograd, the error term claimed in [3] is incorrect. The error lies in the estimate (3.14) and the best that one can claim is

$$\sum_{n \leqslant x} f(n)\log(x/n) = C_1(\chi_1, \chi_2)x\log x + C_2(\chi_1, \chi_2)x + C_3(\chi_1, \chi_2)\log x$$
$$+ C_4(\chi_1, \chi_2) + O(x^{-1/4}),$$

as $x \to +\infty$, where the constants $C_j(\chi_1, \chi_2)$, $1 \leqslant j \leqslant 4$, are as stated in [3]. This error term is the same as obtained in [1 and 2], however, we still have achieved a uniform calculation of the constants $C_j(\chi_1, \chi_2)$, $1 \leqslant j \leqslant 4$.

Let $k \geqslant 2$ be a positive integer. Then another generalization is to consider k characters χ_j of modulus q_j, $1 \leqslant j \leqslant k$, and let

$$f_k(n) = \sum_{d_1 \cdots d_k = n} \chi_1(d_1) \cdots \chi_k(d_k).$$

Then, in the same way as above, I obtain

$$\sum_{n \leqslant x} f_k(n)\log^{k-1}(x/n) = xP_{1,k-1}(\log x) + P_{2,k-1}(\log x) + O(x^{-(k-1)/2k}),$$

where $P_{1,k-1}(u)$ and $P_{2,k-1}(u)$ are polynomials of degree $k-1$, which arise is the calculation of the residues. Indeed, if

$$F_k(s) = \sum_{n=1}^{+\infty} f_k(n)n^{-s},$$

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then

$$xP_{1,k-1}(\log x) = \operatorname{res}\left(x^s F_k(s) s^{-k}, s = 1\right)$$

and

$$P_{2,k-1}(\log x) = \operatorname{res}\left(x^s F_k(s) s^{-k}, s = 0\right).$$

The actual calculation would be carried out in the same manner as in §§3 and 4 of [**3**].

The case $q_1 = \cdots = q_2 = 1$, that is, when $f_k(n) = d_k(n)$, the k-fold divisor function, was obtained by A. Ivić and mentioned in the letter referred to above. It was his result that suggested this generalization to me.

References

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