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A NOTE ON DISJOINTNESS PRESERVING OPERATORS

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ABSTRACT. In this paper we present some results concerning the automatic order boundedness of disjointness preserving operators on Riesz spaces (vector lattices).

Let L and M be Archimedean Riesz spaces. The linear mapping T from L into M is called *disjointness preserving* if $Tf \perp Tg$ whenever $f \perp g$ in L. Observe that a positive linear mapping T is disjointness preserving iff T is a Riesz homomorphism.

The purpose of this note is to prove a basic result (Theorem 2) concerning disjointness preserving operators, which has many applications to the automatic order boundedness problem for disjointness preserving operators. First of all it provides a short and simple proof of a recent result of Yu. A. Abramovich [**1**] to the effect that a disjointness preserving operator T with the additional property that $\inf_n(|Tf_n| + |Tg_n|) = 0$ in M whenever $f_n, g_n \to 0$ (r.u.) in L, is order bounded. We note that a similar result for band preserving operators on Archimedean Riesz spaces was proved by S. J. Bernau [**6**] (recall that the linear operator T from L into itself is called band preserving if $f \perp g$ implies that $Tf \perp g$).

Furthermore, Theorem 2 has as an immediate corollary that any order bounded disjointness preserving operator T can be written as the difference of two Riesz homomorphisms, a result due to M. Meyer [**10**].

Finally we shall use Theorem 2 to show that any disjointness preserving operator from L into M is order bounded on some order dense ideal in L, whenever L is uniformly complete and M satisfies some additional conditions (e.g. if M is a normed Riesz space). In particular, it follows from a combination of Proposition 6 and Theorem 8 that any band preserving operator on a Banach lattice is order bounded (and hence norm bounded), which is a result of Yu. Abramovich, A. I. Veksler and A. V. Koldunov [**2**].

For terminology used and properties of Riesz spaces not explained or proved in this paper, we refer to [**4** or **9**]. We start with a lemma.

LEMMA 1. *Let L be an Archimedean Riesz space and let n be a fixed natural number. If $0 \leqslant u \leqslant e$ in L, then there exist $0 \leqslant p_k \in L$ $(k = 0, 1, \ldots, 2n)$ and $0 \leqslant v$, $w \in L$ such that*

(i) $\sum_{k=0}^{2n} p_k = u$,

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(ii) $|v-u|\leqslant e/n$ *and* $|w-u|\leqslant e/n$,

(iii) $v-ke/n\perp p_{2k}$ *for* $k=0,1,\ldots,n$ *and* $w-(2k-1)e/2n\perp p_{2k-1}$ *for* $k=1,\ldots,n$.

PROOF. For $k=1,2,\ldots,2n$ put $y_k=[4nu-(2k-1)e]^+\wedge e$. Define

$$v=\frac{1}{n}\sum_{k=1}^{n}y_{2k},\qquad w=\frac{1}{2n}y_1+\frac{1}{n}\sum_{k=2}^{n}y_{2k-1},$$

and $p_k=[4nu-2ke]^+\wedge u-[4nu-2(k+1)e]^+\wedge u$ $(k=0,1,\ldots,2n)$. We shall show that $\{p_k\colon k=0,1,\ldots,2n\}$, v and w fulfill the requirements. First of all, since $p_0=u-(4nu-2e)^+\wedge u$ and $p_{2n}=0$, it is clear that (i) holds.

In order to prove (ii), observe that

$$u=\sum_{k=1}^{n}\left[u-\left(\frac{k-1}{n}\right)e\right]^+\wedge\left(\frac{1}{n}e\right),$$

and so

$$|v-u|\leqslant\sum_{k=1}^{n}\left|\frac{1}{n}y_{2k}-\left[u-\left(\frac{k-1}{n}\right)e\right]^+\wedge\left(\frac{1}{n}e\right)\right|. \tag{1}$$

Since each term on the right-hand side is majorized by e/n, it is sufficient to show that these terms are mutually disjoint. Take $1\leqslant k<l\leqslant n$. Since

$$y_{2l}\leqslant 4n[u-((l-1)/n)e]^+,$$

it is sufficient to show that

$$\frac{1}{n}y_{2k}-\left[u-\left(\frac{k-1}{n}\right)e\right]^+\wedge\left(\frac{1}{n}e\right)\perp\left[u-\left(\frac{l-1}{n}\right)e\right]^+,$$

and hence we have to prove that

$$[4nu-(4k-1)e]^+\wedge e-[nu-(k-1)e]^+\wedge e\perp(nu-ke)^+.$$

Note that for $f,g\in L$ we have $|f\wedge e-g\wedge e|\leqslant(e-f)^+\vee(e-g)^+$, $(e-f^+)^+=(e-f)^+\wedge e$. Now

$$\left(e-[4nu-(4k-1)e]^+\right)^+=(4ke-4nu)^+\wedge e\perp(nu-ke)^+,$$

and similarly

$$\left(e-[nu-(k-1)e]^+\right)^+=(ke-nu)^+\wedge e\perp(nu-ke)^+.$$

Hence the terms in the sum (1) are mutually disjoint and therefore $|v-u|\leqslant e/n$. The proof that $|w-u|\leqslant e/n$ follows the same lines.

In order to prove (iii) we first show that $e-y_l\perp p_k$ for $l\leqslant k$ and that $y_l\perp p_k$ for $l\geqslant k+2$. Suppose that $l\leqslant k$, then

$$\begin{aligned}e-y_l&=e-[4nu-(2l-1)e]^+\wedge e=\left(e-[4nu-(2l-1)e]^+\right)^+\\&=(2le-4nu)^+\wedge e\leqslant(2ke-4nu)^+\perp(4nu-2ke)^+\wedge u,\end{aligned}$$

and hence $e - y_l \perp p_k$. Now suppose that $l \geq k+2$, then

$$\begin{aligned} p_k &\leq u - [4nu - 2(k+1)]^+ \wedge u = \left(u - [4nu - 2(k+1)e]^+\right)^+ \\ &= [2(k+1)e - (4n-1)u]^+ \wedge u \leq [(2l-2)e - (4n-1)u]^+ \\ &\leq [(2l-1)e - 4nu]^+ \perp [4nu - (2l-1)e]^+ , \end{aligned}$$

and hence $p_k \perp y_l$. Now

$$v - \frac{k}{n}e = \frac{1}{n}\sum_{l=1}^{k}(y_{2l} - e) + \frac{1}{n}\sum_{l=k+1}^{n} y_{2l},$$

and by the above $y_{2l} - e \perp p_{2k}$ $(1 \leq l \leq k)$ and $y_{2l} \perp p_{2k}$ $(k+1 \leq l \leq n)$. Hence $v - ke/n \perp p_{2k}$ for $k = 0, 1, \dots, n$. In like manner it is shown that $w - (2k-1)e/2n \perp p_{2k-1}$ $(k = 1, 2, \dots, n)$.

THEOREM 2. *Let L and M be Archimedean Riesz spaces and T a disjointness preserving operator from L into M. If* $0 \leq u \leq e$ *in L, then there exist two sequences* $\{f_n\}_{n=1}^{\infty}$ *and* $\{g_n\}_{n=1}^{\infty}$, *contained in the ideal generated by e in L, such that* $f_n, g_n \to 0$ (*e-uniformly*) *and* $(|Tu| - |Te|)^+ \leq |Tf_n| + |Tg_n|$ *for all n.*

PROOF. Take $0 \leq u \leq e$ in L. By the above lemma there exist for each $n = 1, 2, \dots$ elements $0 \leq p_{nk} \in L$ $(k = 0, 1, \dots, 2n)$ and $0 \leq v_n, w_n \in L$ such that v_n and w_n are contained in the ideal generated by e, and

(i) $\sum_{k=0}^{2n} p_{nk} = u$,

(ii) $|v_n - u| \leq e/n$, $|w_n - u| \leq e/n$,

(iii) $v_n - ke/n \perp p_{n,2k}$ $(k = 0, 1, \dots, n)$ and $w_n - (2k-1)e/2n \perp p_{n,2k-1}$ $(k = 1, 2, \dots, n)$.

Since T is disjointness preserving, it follows from (iii) that

$$Tv_n - \frac{k}{n}Te \perp Tp_{n,2k} \quad \text{and} \quad Tw_n - \frac{2k-1}{2n}Te \perp Tp_{n,2k-1} \qquad (0 \leq k \leq n).$$

Furthermore, $(|Tv_n| - |Te|)^+ \leq |Tv_n - kTe/n|$ and $(|Tw_n| - |Te|)^+ \leq |Tw_n - (2k-1)Te/2n|$, which implies that $(|Tv_n| - |Te|)^+ \wedge (|Tw_n| - |Te|)^+ \perp Tp_{nk}$, i.e., that

$$\left(|Tv_n| \wedge |Tw_n| - |Te|\right)^+ \wedge |Tp_{nk}| = 0 \quad \text{for all } k = 0, 1, \dots, 2n.$$

Now it follows from (i) that $(|Tv_n| \wedge |Tw_n| - |Te|)^+ \wedge |Tu| = 0$.

For all $n = 1, 2, \dots$ we now have

$$\left(|Tu| - |Te|\right)^+ \leq |T(u - v_n)| + \left(|Tv_n| - |Te|\right)^+$$

and

$$\left(|Tu| - |Te|\right)^+ \leq |T(u - w_n)| + \left(|Tw_n| - |Te|\right)^+ ,$$

hence

$$\begin{aligned} \left(|Tu| - |Te|\right)^+ &\leq \left\{|T(u - v_n)| + \left(|Tv_n| - |Te|\right)^+\right\} \\ &\qquad \wedge \left\{|T(u - w_n)| + \left(|Tw_n| - |Te|\right)^+\right\} \\ &\leq |T(u - v_n)| + |T(u - w_n)| + \left(|Tv_n| \wedge |Tw_n| - |Te|\right)^+ . \end{aligned}$$

Since $(|Tu|-|Te|)^{+} \leq |Tu|$, this implies that $(|Tu|-|Te|)^{+} \leq |T(u-v_n)| + |T(u-w_n)|$, and so we can take $f_n = u - v_n$ and $g_n = u - w_n$ for $n = 1,2,\ldots$.

The next corollary includes Abramovich's results [**1**, Proposition B].

COROLLARY 3. *Let L and M be Archimedean Riesz spaces and T a disjointness preserving operator from L into M. If T has the property that* $\inf_n(|Tf_n|+|Tg_n|) = 0$ *whenever f_n, $g_n \to 0$ (r.u.) in L, then $0 \leq u \leq e$ in L implies that $|Tu| \leq |Te|$ (and hence T is order bounded).*

COROLLARY 4 (SEE [**10** AND **11**]). *Let L and M be Archimedean Riesz spaces and let T be an order bounded disjointness preserving operator from L into M. Then $T = T^{+} - T^{-}$, where T^{+} and T^{-} are Riesz homomorphisms and $T^{+}u = (Tu)^{+}$, $T^{-}u = (Tu)^{-}$ for all $0 \leq u \in L$.*

PROOF. Let $M^{\wedge}$ be the Dedekind completion of M and consider T as an operator from L into $M^{\wedge}$. Since T is order bounded, the absolute value $|T|$ exists as mapping from L into $M^{\wedge}$. For each $0 \leq u \in L$ we have $|T|(u) = \sup\{|Tf| : |f| \leq u\}$. Since T is order bounded it is clear that the conditions of Corollary 3 are fulfilled, and so $|Tf| = |T|f|| \leq |Tu|$ for all $f \in L$ with $|f| \leq u$. Hence $|T|(u) = |Tu|$, which shows that $|T|$ maps L into M. Moreover, it follows from $T^{+} = \frac{1}{2}(|T|+T)$ and $T^{-} = \frac{1}{2}(|T|-T)$ that $T^{+}u = (Tu)^{+}$ and $T^{-}u = (Tu)^{-}$ for all $0 \leq u \in L$. Finally it is clear that $|T|$, T^{+} and T^{-} are Riesz homomorphisms.

REMARK 5. Let L be a Riesz space with the principal projection property and let T be a disjointness preserving operator from L into an Archimedean Riesz space M. If $0 \leq u \leq e$ in L then there exists a sequence $\{f_n : n = 1,2,\ldots\}$ in L such that $f_n \to 0$ (r.u.) and $(|Tu|-|Te|)^{+} \leq |Tf_n|$ for all n. Indeed, by the Freudenthal spectral theorem [**9**, Theorem 40.2] there exists a sequence $\{s_n : n = 1,2,\ldots\}$ in L such that $0 \leq s_n \uparrow u$ (e-uniformly), where each s_n is a sum of the form $\sum_{i=1}^{k} \alpha_i p_i$ with $0 \leq \alpha_i \leq 1$ $(i = 1,\ldots,k)$ and $p_1,\ldots,p_k$ mutually disjoint components of e. Using that T is disjointness preserving, it follows easily that $|Ts_n| \leq |Te|$ and so $(|Tu|-|Te|)^{+} \leq |T(u-s_n)|$ $(n = 1,2,\ldots)$. Hence we can take $f_n = u - s_n$. It follows, in particular, that if in this case T has the additional property that $\inf|Tf_n| = 0$ whenever $f_n \to 0$ (r.u.) in L, then T is order bounded (and $0 \leq u \leq e$ in L implies that $|Tu| \leq |Te|$).

As observed by Abramovich [**1**, §2], if L and M are normed Riesz spaces and T is a norm bounded disjointness preserving operator from L into M, then f_n, $g_n \to 0$ (r.u.) in L implies that $\inf_n(|Tf_n|+|Tg_n|) = 0$ in M, and so T is order bounded. This was also proved in the case when L is a Dedekind σ-complete Banach lattice by W. Arendt [**5**, Theorem 2.5].

A linear operator T from an Archimedean Riesz space L into itself is called *band preserving* if $T(B) \subset B$ for all bands B in L (equivalently, $Tf \perp g$ whenever $f \perp g$ in L). Since a band preserving operator is clearly disjointness preserving, all of the above results apply to band preserving operators. An order bounded band preserving operator is called an *orthomorphism*. Band preserving operators and orthomorphisms have been studied extensively (see e.g. [**2**, **6**, **7**, **8**, **12**, **13** and **14**]).

We shall present some other applications of Theorem 2. In particular we will prove that under certain conditions on the spaces L and M, any disjointness preserving operator from L into M is order bounded on some order dense ideal in L.

Let L and M be Archimedean Riesz spaces and let T be a linear operator from L into M. If A and B are ideals in L such that the restrictions $T|A$ and $T|B$ are order bounded, then $T|(A+B)$ is order bounded as well. Hence, if A_T is the union of all ideals A in L with the property that $T|A$ is order bounded, then A_T is an ideal in L and $T|A_T$ is order bounded. Clearly, A_T is the largest ideal in L on which T is order bounded. If T is disjointness preserving, then it follows immediately from Corollary 3 that $|Tu| \leqslant |Tv|$ whenever $0 \leqslant u \leqslant v \in A_T$. For band preserving operators we have the following interesting result.

PROPOSITION 6. *If T is a band preserving operator on the Archimedean Riesz space L, then A_T is a band.*

PROOF. Take $0 \leqslant u \leqslant e \in A_T^{dd}$ and suppose that $(|Tu|-|Te|)^+ > 0$. Then there exists $v \in A_T$ such that $0 < v \leqslant (|Tu|-|Te|)^+$. Let n be a natural number such that $(nv-e)^+ > 0$ and put $p = nv \wedge u$ and $q = nv \wedge e$. Then $0 \leqslant p \leqslant q \in A_T$ and so $|Tp| \leqslant |Tq|$. Since T is band preserving, it follows from $u - p \perp (nv-e)^+$ and $e - q \perp (nv-e)^+$ that $Tu - Tp \perp (nv-e)^+$ and $Te - Tq \perp (nv-e)^+$. Now it is easy to see that

$$\big(|Tu|-|Te|\big)^+ - \big(|Tp|-|Tq|\big)^+ \perp (nv-e)^+$$

and hence $(|Tu|-|Te|)^+ \perp (nv-e)^+$. By the choice of v this implies that $(nv-e)^+ = 0$, which is a contradiction. We thus have proved that $|Tu| \leqslant |Te|$ whenever $0 \leqslant u \leqslant e \in A_T^{dd}$. Therefore T is order bounded on A_T^{dd}. Now it follows from the definition of A_T that $A_T = A_T^{dd}$; hence A_T is a band.

In order to prove the next result we need a lemma.

LEMMA 7. *Let L and M be Archimedean Riesz spaces and assume that L is uniformly complete. Let T be a disjointness preserving operator from L into M. Suppose $0 \leqslant u_n \leqslant e_n \leqslant e$ $(n = 1,2,\ldots)$ in L such that $\{e_n\}_{n=1}^\infty$ is a disjoint sequence and put $w_n = (|Tu_n|-|Te_n|)^+$ for all n. Then $\{w_n\}_{n=1}^\infty$ is a disjoint sequence in M with the property that $\{\lambda_n w_n\}_{n=1}^\infty$ is order bounded in M for any sequence $\{\lambda_n\}_{n=1}^\infty$ of positive real numbers.*

PROOF. By Theorem 2 there exist for each n two sequences $\{f_{nk}\}_{k=1}^\infty$ and $\{g_{nk}\}_{k=1}^\infty$ in the ideal generated by e_n, such that $f_{nk}, g_{nk} \to 0$ (e-uniformly) as $k \to \infty$ and $0 \leqslant w_n \leqslant |Tf_{nk}| + |Tg_{nk}|$ $(k = 1,2,\ldots)$ for all n. Take positive real numbers $\{\lambda_n\}_{n=1}^\infty$. For each n there exists a natural number k_n such that

$$|f_{nk_n}| \leqslant \frac{1}{n^2\lambda_n} e \quad \text{and} \quad |g_{nk_n}| \leqslant \frac{1}{n^2\lambda_n} e.$$

Define

$$f = \sum_{n=1}^\infty \lambda_n f_{nk_n} \quad \text{and} \quad g = \sum_{n=1}^\infty \lambda_n g_{nk_n}$$

(e-uniformly convergent series). Since the terms in the series defining f are mutually disjoint and since T is disjointness preserving, it is easy to see that $0 \leqslant \lambda_n |Tf_{nk_n}| \leqslant |Tf|$ for all n. Similarly we get that $0 \leqslant \lambda_n |Tg_{nk_n}| \leqslant |Tg|$ ($n = 1, 2, \ldots$). Therefore

$$0 \leqslant \lambda_n w_n \leqslant \lambda_n |Tf_{nk_n}| + \lambda_n |Tg_{nk_n}| \leqslant |Tf| + |Tg|$$

for all n. Hence $\{\lambda_n w_n\}_{n=1}^{\infty}$ is order bounded.

THEOREM 8. *Let L and M be Archimedean Riesz spaces such that L is uniformly complete and for each disjoint sequence $0 < w_n \in M$ ($n = 1, 2, \ldots$) there exist positive real numbers $\{\lambda_n\}_{n=1}^{\infty}$ such that $\{\lambda_n w_n\}_{n=1}^{\infty}$ is not order bounded in M. Then each disjointness preserving operator T from L into M is order bounded on some order dense ideal in L.*

PROOF. Let A_T be the largest ideal in L on which T is order bounded. We claim that $A_T^d = \{0\}$. Indeed, suppose $A_T^d \neq \{0\}$ and take $0 < e \in A_T^d$. Since the ideal generated by e is not finite dimensional, there exist mutually disjoint elements $0 < e_n \leqslant e$ ($n = 1, 2, \ldots$). Since T is not order bounded on the ideal generated by e_n, there exists $0 \leqslant u_n \leqslant e_n$ such that $w_n = (|Tu_n| - |Te_n|)^+ > 0$ ($n = 1, 2, \ldots$). Now it follows from Lemma 7 that $\{\lambda_n w_n\}_{n=1}^{\infty}$ is order bounded for any sequence $\{\lambda_n\}_{n=1}^{\infty}$ of positive real numbers, which contradicts the assumption on M. Hence $A_T^d = \{0\}$, i.e., $A_T^{dd} = L$.

The above theorem has the following immediate corollary.

COROLLARY 9. *If L is a Banach lattice and M is a normed Riesz space, then any disjointness preserving operator from L into M is order bounded on some order dense ideal in L.*

It follows from a combination of Proposition 6 and Theorem 8 that if L is a uniformly complete Riesz space with the property that for each disjoint sequence $0 < w_n \in L$ ($n = 1, 2, \ldots$) there exist positive real numbers $\{\lambda_n\}_{n=1}^{\infty}$ such that $\{\lambda_n w_n\}_{n=1}^{\infty}$ is not order bounded, then any band preserving operator in L is an orthomorphism. This implies in particular that any band preserving operator on a Banach lattice is order bounded, and hence norm bounded (as any order bounded operator on a Banach lattice is norm bounded), which is a result of Yu. A. Abramovich, A. I. Veksler and A. V. Koldunov [**2**, **3**]. We note, however, that using Lemma 7 it is not difficult to prove the following result, which is due to A. W. Wickstead (presented at the meeting on Riesz spaces and operator theory in Oberwolfach, 1982): If L is a uniformly complete Riesz space with the property that for each $0 < e \in L$ such that the order interval $[0, e]$ does not contain any atoms, there exists a disjoint sequence $\{e_n\}_{n=1}^{\infty}$ in $[0, e]$ such that $\{\lambda_n e_n\}_{n=1}^{\infty}$ is not bounded in L for some sequence $\{\lambda_n\}_{n=1}^{\infty}$ of positive real numbers, then each band preserving operator in L is automatically order bounded. The case when L is in addition Dedekind σ-complete was proved by W. A. J. Luxemburg [**8**, Theorem 9.9].

The condition that every disjointness preserving operator from an Archimedean Riesz space L into itself is order bounded is quite strong. Indeed, let P be a prime ideal in L which is not uniformly closed. Then there exists a sequence $\{f_n\}_{n=1}^{\infty}$ in P

and $f_0 \in L \backslash P$ such that $f_n \to f_0$ (r.u.). Let φ be any linear functional on L with $\varphi(f) = 0$ for all $f \in P$ and $\varphi(f_0) = 1$. Take $0 < e \in L$ and define the disjointness preserving operator T by $Tf = \varphi(f)e$ for all $f \in L$. Since $f_n \to f_0$ (r.u.), $Tf_n = 0$ $(n = 1, 2, \ldots)$ and $Tf_0 = e > 0$, T is not order bounded. Furthemore we note that a proper prime ideal P in L is uniformly closed iff P is a maximal ideal (e.g., this follows from a combination of [**9**, Theorems 27.1, 33.2 and 60.2]). Therefore, if every disjointness preserving operator in L is order bounded, then every proper prime ideal in L is a maximal ideal. By [**9**, Theorem 37.6], this last condition is equivalent to the property that the quotient Riesz space L/A is Archimedean for every ideal A in L. Riesz spaces with this property are called hyper-Archimedean. We thus have the following result.

THEOREM 10. *If L is an Archimedean Riesz space such that any disjointness preserving operator in L is order bounded, then L is hyper-Archimedean.*

Finally, we note that if L is uniformly complete and hyper-Archimedean, then every principal ideal in L is finite dimensional [**9**, Theorem 61.4] and hence any linear operator on L is order bounded.

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