

ERRATUM TO "MEASURE-THEORETIC QUANTIFIERS AND HAAR MEASURE"

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Dr. Henry Schaerf has kindly pointed out the following gap in the application of Proposition 1 of [1] to the main example of a measurable predicate, " $st \notin E$ ", where E is a Borel subset of a topological group G . Namely, this predicate is measurable with respect to the Borel subsets $\mathcal{B}(G \times G)$ of $G \times G$, but not necessarily with respect to $\mathcal{B}(G) \times \mathcal{B}(G)$, the smallest σ -algebra containing $\{E \times F: E, F \in \mathcal{B}(G)\}$. Hence Proposition 1 does not apply. (An example where $\mathcal{B}(X \times X) \neq \mathcal{B}(X) \times \mathcal{B}(X)$ for a topological space (though, admittedly, not for a group) is given in [2, p. 222, (17-17)].) Since this mistake is clearly easy to make (see also [4]) and yet the needed form of Fubini's theorem is difficult to find, it seems worthwhile to make a careful correction.

The easiest rectifying assumption to make is that all groups in [1] satisfy the second axiom of countability. It is then easy to show that $\mathcal{B}(G \times G) = \mathcal{B}(G) \times \mathcal{B}(G)$ and a standard form of the Fubini-Tonelli theorem [3] applies.

Another way to rectify [1] is to assume that all spaces are locally compact Hausdorff and that all measures on them are complete and regular (as well as σ -finite). Here, we are using the following

DEFINITION [2, p. 109]. If X is a locally compact Hausdorff space, \mathcal{M} a σ -algebra in X , and μ a positive measure on \mathcal{M} , then μ is called *regular* if the following conditions hold:

- (i) \mathcal{M} contains all open sets;
- (ii) $\mu F < \infty$ if F is compact;
- (iii) if G is open, $\mu G = \sup\{\mu F: F \subset G, F \text{ compact}\}$;
- (iv) if $A \in \mathcal{M}$, $\mu A = \inf\{\mu G: A \subset G, G \text{ open}\}$.

A complex measure μ is *regular* if $|\mu|$ is.

We call a function f on a positive measure space (X, \mathcal{M}, μ) μ -*summable* if f is \mathcal{M} -measurable and $\int_X |f| d\mu < \infty$. From [2, Theorems 17.12 and 17.13 on p. 215, Theorem 17.8 on p. 212, and pp. 199-200], we have the following form of

THE FUBINI-TONELLI THEOREM. *Let X_i ($i = 1, 2$) be locally compact Hausdorff spaces and $X = X_1 \times X_2$ with the product topology. Let μ_i be positive complete regular σ -finite measures on X_i for $i = 1, 2$. Then there exists a unique complete regular σ -finite measure ν on X such that:*

- (i) *If f is a ν -summable function on X , then $\forall^e y [\mu_2] f(x, y)$ is μ_1 -summable as a function of x ; $\int_{X_1} f(x, y) d\mu_1(x)$, which is defined a.e. $[\mu_2]$, is μ_2 -summable; likewise with the roles of μ_1 and μ_2 reversed; and*

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$$\begin{aligned}
 (*) \quad \int_X f \, d\nu &= \int_{X_2} \left(\int_{X_1} f(x, y) \, d\mu_1(x) \right) d\mu_2(y) \\
 &= \int_{X_1} \left(\int_{X_2} f(x, y) \, d\mu_2(y) \right) d\mu_1(x).
 \end{aligned}$$

(ii) If f is a nonnegative ν -measurable function on X , then $\forall^e y[\mu_2]$ $f(x, y)$ is μ_1 -measurable as a function of x ; $\int_{X_1} f(x, y) \, d\mu_1(x)$, which is defined as an extended real number a.e. $[\mu_2]$, is μ_2 -measurable; likewise with the roles of μ_1 and μ_2 reversed; and $(*)$ holds in the extended reals.

This form of the Fubini-Tonelli theorem gives a new form of Proposition 1 of [1] which, with the above assumption of having complete regular measures on locally compact groups, validates the other results of [1].

Finally, we note a typographical error in the statement of Proposition 1 of [1]: “ $\forall^e[\mu]$ ” should be “ $\forall^e x[\mu]$ ”. Also, “left” should be “right” in lines 3 and 5 of p. 69 of [1].

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