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SHORTER NOTES

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INVARIANCE OF COMPLEMENTARY DOMAINS OF A FIXED POINT SET

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Introduction. The following useful result seems not to be in the literature. It has a simple but perhaps nonobvious proof.

PROPOSITION. *Let f be a homeomorphism of a connected topological manifold M with fixed point set F. Then either* (1) *f is invariant on each* (*connected*) *component of $M - F$ or* (2) *there are exactly two components and f interchanges them.*

PROOF. Suppose f is not invariant on a component U of $M - F$. Let g be the involution on M defined by

$$g(x) = \begin{cases} f(x), & x \in U, \\ f^{-1}(x), & x \in f(U), \\ x, & x \in M - [U \cup f(U)]. \end{cases}$$

It is an elementary exercise to verify that g is continuous. By a well-known theorem of M. H. A. Newman [**3**] (cf. also [**1**, p. 157]) g cannot be the identity on a nonempty open set. Hence U and $f(U)$ are the only components of $M - F$ and $f(f(U)) = U$, thus proving the Proposition.

In the case of (2) the above argument shows that F cannot contain an open set, hence $\dim F \leq (\dim M) - 1$, and since F separates M we have $\dim F = (\dim M) - 1$. In a refinement of Smith Theory, G. Bredon [**2**] has shown that if M is also orientable then any involution with an odd codimensional fixed point set must reverse the orientation; hence we obtain

COROLLARY. *Let f be an orientation-preserving homemorphism of an orientable manifold M; then f is invariant on each component of $M - F$.*

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