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A UNIVERSAL HEREDITARILY INDECOMPOSABLE CONTINUUM

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ABSTRACT. It is proved that there exists a hereditarily indecomposable metric continuum X containing a homeomorphic copy of every hereditarily indecomposable metric continuum. This is a solution of a problem (Problem 125 by H. Cook in *University of Houston Problems Book*) recalled in [**4**, §21]. A similar result was announced by P. Minc.

If a continuum is not a union of two proper subcontinua, then it is called an indecomposable continuum. A hereditarily indecomposable continuum means a continuum of which every subcontinuum is indecomposable.

A collection of all hereditarily indecomposable metric continua is very rich. For example, every metric continuum is a continuous image of a hereditarily indecomposable continuum (see [**4**, (19.3), p. 48]). A natural question of whether there is a universal hereditarily indecomposable continuum (see [**4**, §21, p. 52]) was posed by H. Cook. This paper contains a solution of this problem.

Before we start with a construction, let us recall that a notion of crookedness plays a very important role in a study of hereditarily indecomposable continua. In particular, every hereditarily indecomposable continuum can be approximated by an inverse sequence of polyhedra and crooked bonding maps. Therefore, if we apply McCord's method (see [**6**]) of constructing universal continua and are careful enough to take sufficiently crooked maps, we can obtain a universal hereditarily indecomposable continuum.

If $\mathscr{F}$ is a collection of subsets of a space Y, and $g\colon X \to Y$, then

$$\mathscr{F}^* = \{(A, B, U, V)\colon A, B, U, V \in \mathscr{F}, \mathrm{cl}(A) \subset U, \mathrm{cl}(B) \subset V, \mathrm{cl}(U) \cap \mathrm{cl}(V) = \varnothing\},$$

$$g^{-1}(\mathscr{F}^*) = \{(g^{-1}(A), g^{-1}(B), g^{-1}(U), g^{-1}(V)); (A, B, U, V) \in \mathscr{F}^*\},$$

$$P_g = \{(g, x)\colon x \in X\}, \qquad \pi_g(x) = (g, x) \quad \text{for } x \in X,$$

$$K(X) = \{C\colon C \text{ is a component of } X\},$$

and we say (compare [**2**]) that g is $\mathscr{F}^*$-crooked if for each $(A, B, U, V) \in \mathscr{F}^*$ there exist three closed subsets W_0, W_1, W_2 of X satisfying:

(i) $X = W_0 \cup W_1 \cup W_2$,

(ii) $g^{-1}(A) \subset W_0, g^{-1}(B) \subset W_2$,

(iii) $W_0 \cap W_2 = \varnothing$, $W_0 \cap W_1 \subset g^{-1}(V)$, $W_1 \cap W_2 \subset g^{-1}(U)$.

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As an easy consequence of Theorem 3 in [**2**, p. 677], we obtain

LEMMA 1. *If $f\colon X \to Y$ is a continuous map from an hereditarily indecomposable continuum X into Y and $\mathcal{F}$ is an arbitrary collection of open subsets of Y, then f is $\mathcal{F}^*$-crooked.*

Now we prove

LEMMA 2. *If $\mathcal{F}$ is a finite collection of open subsets of a compactum Y with a metric ρ and a continuous map $f\colon X \to Y$ is $\mathcal{F}^*$-crooked, then there exist $\varepsilon, \delta > 0$ such that if $g\colon X \to Z$ is an ε-map onto Z and $h\colon Z \to Y$ satisfies $\rho(f, hg) < \delta$, then h is $\mathcal{F}^*$-crooked.*

PROOF. Fix $F = (A, B, U, V) \in \mathcal{F}^*$ and choose closed subsets W_0^F, W_1^F, W_2^F of X such that $X = W_0^F \cup W_1^F \cup W_2^F$, $f^{-1}(A) \subset W_0^F$, $f^{-1}(B) \subset W_2^F$, $W_0^F \cap W_2^F = \varnothing$, $W_0^F \cap W_1^F \subset f^{-1}(V)$, $W_1^F \cap W_2^F \subset f^{-1}(U)$. Put $\delta_F = \min\{\rho(A, f(W_1^F \cup W_2^F)), \rho(B, f(W_0^F \cup W_1^F)), \rho(Y \setminus V, f(W_0^F \cap W_1^F)), \rho(Y \setminus U, f(W_1^F \cap W_2^F))\}$ and

$$\delta = \tfrac{1}{4}\min\{\delta_F\colon F \in \mathcal{F}^*\}.$$

Since f is uniformly continuous, we find $\varepsilon' > 0$ such that if $\sigma(x, x') < 2\varepsilon'$, then $\rho(f(x), f(x')) < \delta$, where σ denotes a metric in X. If $F = (A, B, U, V) \in \mathcal{F}^*$, then we put

$$\begin{aligned}\varepsilon''_F = \min\{ &\sigma(W_0^F, W_2^F), \sigma(W_0^F \setminus B(W_0^F \cap W_1^F, \varepsilon'), W_1^F \setminus B(W_0^F \cap W_1^F, \varepsilon')),\\ &\sigma(W_1^F \setminus B(W_1^F \cap W_2^F, \varepsilon'), W_2^F \setminus B(W_1^F \cap W_2^F, \varepsilon'))\}\end{aligned}$$

and

$$\varepsilon = \tfrac{1}{2}\min(\{\varepsilon'\} \cup \{\varepsilon''_F\colon F \in F^*\}),$$

where $B(C, \varepsilon')$ denotes an ε'-ball around C in X.

Let $g\colon X \to Z$ be an arbitrary ε-map into Z, and let $h\colon Z \to Y$ satisfy $\rho(f, hg) < \delta$. If $F = (A, B, U, V) \in \mathcal{F}^*$, then

$$\begin{aligned}\min\{ &\rho(A, hg(W_1^F \cup W_2^F)), \rho(B, hg(W_0^F \cup W_1^F)),\\ &\rho(Y \setminus V, hg(W_0^F \cap W_1^F)), \rho(Y \setminus U, hg(W_1^F \cap W_2^F))\} > 3\delta.\end{aligned}$$

Thus,

$$\begin{aligned}\min\{ &\sigma((hg)^{-1}(A), W_1^F \cup W_2^F), \sigma((fg)^{-1}(B), W_0^F \cup W_1^F),\\ &\sigma((hg)^{-1}(Y \setminus V), W_0^F \cap W_1^F), \sigma((hg)^{-1}(Y \setminus U), W_1^F \cap W_2^F)\} \geqslant 2\varepsilon'.\end{aligned}$$

Therefore,

$$(hg)^{-1}(A) \subset W_0^F, \quad (hg)^{-1}(B) \subset W_2^F, \quad (hg)^{-1}(Y \setminus V) \subset X \setminus B(W_0^F \cap W_1^F, \varepsilon'),$$

and

$$(fg)^{-1}(Y \setminus U) \subset X \setminus B(W_1^F \cap W_2^F, \varepsilon');$$

thus

$$h^{-1}(A) \subset g(W_0^F), \quad h^{-1}(B) \subset g(W_2^F), \quad g(B(W_0^F \cap W_1^F, \varepsilon')) \subset h^{-1}(V),$$

and

$$g\left(B\left(W_1^F \cap W_2^F, \varepsilon'\right)\right) \subset h^{-1}(U).$$

The choice of ε_F'' implies that

$$g\left(W_0^F\right) \cap g\left(W_2^F\right) = \varnothing, \quad g\left(W_0^F\right) \cap g\left(W_1^F\right) \subset g\left(B\left(W_0^F \cap W_1^F, \varepsilon'\right)\right),$$

and

$$g\left(W_1^F\right) \cap g\left(W_2^F\right) \subset g\left(B\left(W_1^F \cap W_2^F, \varepsilon'\right)\right).$$

These relations complete the proof of Lemma 2.

A slight modification of the proof of Lemma 1.13.3 in [**1**, p. 148] gives

LEMMA 3. *For every collection of continuous mappings $f_i\colon X \to Z_i$ of a separable metric space X to compacta Z_i, $i = 1, 2, \ldots, n$, there exists a compactum $\tilde{X}$ containing X as a dense subspace such that* $\dim \tilde{X} \leqslant \dim X$ *and for each $i = 1, 2, \ldots, n$ the mapping f_i is extendable to a continuous mapping $\tilde{f}_i\colon \tilde{X} \to Z_i$.*

We now pass to the main

THEOREM. *There exists a hereditarily indecomposable metric continuum X of dimension $\leqslant d$ containing a homeomorphic copy of every hereditarily indecomposable metric continuum of dimension $\leqslant d$.*

PROOF. Let $\Pi = \{P_1, P_2, \ldots\}$ be a collection of all connected polyhedra of dimension $\leqslant d$. We define a sequence of compacta $\tilde{X}_n$ with countable bases $\mathscr{B}_n = \{B_n^1, B_n^2, \ldots\}$, sets $X_n \subset \tilde{X}_n$, countable collections $G_{C,m}$ of functions from P_m onto $C \in K(X_n)$, and a sequence of continuous mappings $\tilde{f}_n$ from $\tilde{X}_{n+1}$ onto $\tilde{X}_n$ such that if we put

$$\tilde{f}_{m,n} = \tilde{f}_m \cdots \tilde{f}_{n-2}\tilde{f}_{n-1} \quad \text{for } m < n, \qquad \tilde{f}_{m,m} = \operatorname{id}_{\tilde{X}_m}$$

$$\mathscr{B}_{n,m} = \left\{B_n^1, B_n^2, \ldots, B_n^m\right\},$$

$$G_n = \bigcup\left\{G_{C,m}\colon C \in K(X_n), m = 1, 2, \ldots\right\},$$

$$\mathscr{F}_n^* = \bigcup_{m=1}^{n} f_{m,n}^{-1}\left(\mathscr{B}_{m,n}^*\right),$$

and if we take for each $g \in G_{C,m}$ a copy P_g of P_m with a homeomorphic projection π_g from P_m onto P_g, then:

(1) $\tilde{X}_1 = X_1 = P_1$ and $\dim \tilde{X}_n \leqslant d$;

(2) the mapping $\tilde{f}_n$ is $\mathscr{F}_n^*$-crooked;

(3) X_n is an open dense subset of $\tilde{X}_n$;

(4) every component of X_n is homeomorphic to a member of Π and $K(X_n)$ is countable;

(5) $G_{C,m}$ is a countable dense subset of a collection of all continuous mappings from P_m onto C which are $\mathscr{F}_n^*$-crooked;

(6) $X_{n+1} = \bigcup_{g \in G_n} P_g$, $\tilde{f}_n(x) = g\pi_g^{-1}(x)$ for $x \in P_g$, and X_{n+1} has a disjoint union topology.

To obtain X_1, $\tilde{X}_1$, $\mathscr{B}_1$ we take $X_1 = \tilde{X}_1 = P_1$ and fix an arbitrary base $\mathscr{B}_1$ in P_1. When X_i, $\tilde{X}_i$, $\mathscr{B}_i$, $\tilde{f}_{i,j}$, and $G_{C,m}$ (for $C \in K(X_i)$, $i, j = 1, 2, \ldots, n$ and $m = 1, 2, \ldots$) are defined, we put

$$X_{n+1} = \bigcup_{g \in G_n} P_g, \quad \text{and} \quad f_n(x) = g\pi_g^{-1}(x) \quad \text{for } x \in P_g,$$

and we take a disjoint union topology in X_{n+1} such that every π_g is a homeomorphism for $g \in G_n$. Since G_n is countable, we conclude that

(7) X_{n+1} is separable, locally compact, and $\dim X_{n+1} \leqslant d$.

The mapping f_n from X_{n+1} into $\tilde{X}_n$ is continuous because $f_n | P_g$ is continuous for each $g \in G_n$. Obviously, $f(X_{n+1}) = X_n$.

If $g \in G_n$, then the mapping g is $\mathscr{F}_n{}^*$-crooked. Therefore, for each $g \in G_n$ and $F = (A, B, U, V) \in \mathscr{F}_n{}^*$, there exist three closed subsets $W_0^{F,g}$, $W_1^{F,g}$, $W_2^{F,g}$ in $g^{-1}(X_n)$ such that

(8) $g^{-1}(X_n) = W_0^{F,g} \cup W_1^{F,g} \cup W_2^{F,g}$,

(9) $g^{-1}(A) \subset W_0^{F,g}$, $g^{-1}(B) \subset W_2^{F,g}$,

(10) $W_0^{F,g} \cap W_2^{F,g} = \varnothing$, $W_0^{F,g} \cap W_1^{F,g} \subset g^{-1}(V)$, $W_1^{F,g} \cap W_2^{F,g} \subset g^{-1}(U)$.

Put

$$W_j^F = \bigcup_{g \in G_n} \pi_g\big(W_j^{F,g}\big) \quad \text{for } F \in \mathscr{F}_n{}^*, j = 0, 1, 2.$$

Then

(11) W_0^F, W_1^F, W_2^F are closed in X_{n+1}.

Let h_F be an arbitrary continuous mapping from X_{n+1} into $[0, 3]$ such that $h_F(f_{n+1}^{-1}(A)) = 0$, $h_F(f_{n+1}^{-1}(B)) = 3$, $h_F^{-1}([i, i+1]) = W_i^F$ for $i = 0, 1, 2$, $h_F^{-1}(i) = W_{i-1}^F \cap W_i^F$ for $i = 1, 2$, $h_F(f_n^{-1}(U)) \cap [\frac{1}{2}, \frac{3}{2}] = \varnothing$, and $h_F(f_n^{-1}(V)) \cap [\frac{3}{2}, \frac{5}{2}] = \varnothing$, where $F = (A, B, U, V)$ (the existence of h_F is guaranteed by conditions (8)–(11)).

If follows from Lemma 3 that there is a compactification $\tilde{X}_{n+1}$ of X_{n+1} such that $\dim \tilde{X}_{n+1} \leqslant \dim X_{n+1}$, f_n is extendable to a continuous mapping $\tilde{f}_n\colon \tilde{X}_{n+1} \to \tilde{X}_n$, and, for each $F \in \mathscr{F}_n{}^*$, the mapping h_F is extendable to a continuous mapping $\tilde{h}_F\colon \tilde{X}_{n+1} \to [0, 3]$.

Put $\tilde{W}_i^F = \tilde{h}_F^{-1}([i, i+1])$ for $i = 0, 1, 2$. Of course, $\tilde{X}_{n+1} = \tilde{W}_0^F \cup \tilde{W}_1^F \cup \tilde{W}_2^F$, $\tilde{W}_0^F \cap \tilde{W}_2^F = \varnothing$. If $F = (A, B, U, V)$, then

$$\tilde{f}_n^{-1}(A) \subset \operatorname{cl} \tilde{f}_n^{-1}(A) = \operatorname{cl}\big(\tilde{f}_n^{-1}(A) \cap X_{n+1}\big) = \operatorname{cl} f_n^{-1}(A) \subset \tilde{h}_F^{-1}(0) \subset \tilde{W}_0^F;$$

similarly we obtain $\tilde{f}_n^{-1}(B) \subset \tilde{h}_F^{-1}(3) \subset \tilde{W}_2^F$, $\tilde{f}_n^{-1}(V) \subset \tilde{X}_{n+1} \setminus \tilde{h}_F^{-1}(2)$, and $\tilde{f}_n^{-1}(U) \subset \tilde{X}_{n+1} \setminus \tilde{h}_F^{-1}(1)$; thus $\tilde{f}_n$ is $\mathscr{F}_n{}^*$-crooked. In this way we have finished the construction.

Denote the inverse limit of the inverse sequence $\{\tilde{X}_n, \tilde{f}_{m,n}\}$ by $\tilde{X}$ and the natural projections from $\tilde{X}$ onto $\tilde{X}_n$ by α_n. Clearly,

(12) $\tilde{X}$ is a compactum of the dimension $\leqslant d$.

Now we prove

(13) every subcontinuum of $\tilde{X}$ is indecomposable.

In fact, suppose that K and L are subcontinua of $\tilde{X}$ such that $K \cap L \neq \varnothing$ and $K \neq K \cup L \neq L$. Take points a and b such that $a \in K \setminus L$ and $b \in L \setminus K$. There exists a positive integer m such that $\alpha_m(a) \in \alpha_m(K) \setminus \alpha_m(L)$ and $\alpha_m(b) \in \alpha_m(L) \setminus \alpha_m(K)$. Since $\mathscr{B}_m$ is a base of $\tilde{X}_m$, we find sets $A, B, U, V \in \mathscr{B}_m$ such that

$\alpha_m(a) \in A \subset \mathrm{cl}(A) \subset U \subset \tilde{X}_m \setminus \alpha_m(L)$, $\alpha_m(b) \in B \subset \mathrm{cl}(B) \subset V \subset \tilde{X}_m \setminus \alpha_m(K)$, and $\mathrm{cl}(U) \cap \mathrm{cl}(V) = \varnothing$. The definition of $\mathscr{F}_n^*$ implies that there is a positive integer n such that $n \geqslant m$ and $(\tilde{f}_{m,n}^{-1}(A), f_{m,n}^{-1}(B), \tilde{f}_{m,n}^{-1}(U), \tilde{f}_{m,n}^{-1}(V)) \in \mathscr{F}_n^*$. Condition (2) implies that there exist three closed subsets W_0, W_1, W_2 of $\tilde{X}_{n+1}$ satisfying the conditions $\tilde{X}_{n+1} = W_0 \cup W_1 \cup W_2$, $\tilde{f}_{m,n+1}^{-1}(A) \subset W_0$, $\tilde{f}_{m,n+1}^{-1}(B) \subset W_2$, $W_0 \cap W_2 = \varnothing$, $W_0 \cap W_1 \subset \tilde{f}_{m,n+1}^{-1}(V)$, and $W_1 \cap W_2 \subset \tilde{f}_{m,n+1}^{-1}(U)$. Since $\tilde{X} = \alpha_{n+1}^{-1}(W_0) \cup \alpha_{n+1}^{-1}(W_1) \cup \alpha_{n+1}^{-1}(W_2)$, $\alpha_{n+1}^{-1}(W_0) \cap \alpha_{n+1}^{-1}(W_2) = \varnothing$, $a \in \alpha_{n+1}^{-1}(W_0) \cap K$, and $b \in \alpha_{n+1}^{-1}(W_2) \cap L$, we obtain either $\alpha_{n+1}^{-1}(W_1) \cap K \neq \varnothing$ or $\alpha_{n+1}^{-1}(W_1) \cap L \neq \varnothing$. By symmetry we assume $\alpha_{n+1}^{-1}(W_1) \cap K \neq \varnothing$. Since $K = (K \cap \alpha_{n+1}^{-1}(W_0)) \cup (K \cap \alpha_{n+1}^{-1}(W_1)) \cup (K \cap \alpha_{n+1}^{-1}(W_2))$, $K \cap \alpha_{n+1}^{-1}(W_0) \neq \varnothing \neq K \cap \alpha_{n+1}^{-1}(W_1)$, and K is connected, we infer that $K \cap \alpha_{n+1}^{-1}(W_0) \cap \alpha_{n+1}^{-1}(W_1) \neq \varnothing$. But

$$\begin{aligned}\alpha_{n+1}^{-1}(W_0) \cap \alpha_{n+1}^{-1}(W_1) &= \alpha_{n+1}^{-1}(W_0 \cap W_1) \subset \alpha_{n+1}^{-1}\big(\tilde{f}_{m,n+1}^{-1}(V)\big) \\ &\subset \alpha_{n+1}^{-1}\big(\tilde{f}_{m,n+1}^{-1}(\tilde{X}_m \setminus \alpha_m(K))\big) = \alpha_m^{-1}\big(\tilde{X}_m \setminus \alpha_m(K)\big) \subset \tilde{X} \setminus K.\end{aligned}$$

This contradiction completes the proof of (13).

Now, let Z be an arbitrary hereditarily indecomposable continuum of dimension $\leqslant d$. It is known (see [**5**, Theorem 1]) that for each $\varepsilon > 0$ there is an ε-mapping from Z onto $P \in \Pi$. We claim that

(14) Z can be embedded into $\tilde{X}$.

Denote the metric in X_i by ρ_i and the metric in Z by σ. According to Lemma 5 in [**5**, p. 152] we define, by induction on i, the sequences: real numbers $\varepsilon_i > 0$ such that $\lim \varepsilon_i = 0$, ε_i-mappings $\varphi_i: Z \to \tilde{X}_i$ onto a component of X_i, and real numbers $\delta_i > 0$ such that:

(15) for any set $N_j \subset \varphi_j(Z)$ of diameter $\operatorname{diam} N_j \leqslant \delta_j$ we have

$$\operatorname{diam} \tilde{f}_{i,j}(N_j) < \delta_i/2^{j-i} \quad \text{for all } i \leqslant j;$$

(16) $x, x' \in Z$ and $\sigma(x, x) \geqslant 2\varepsilon_i$ imply $\rho_i(\varphi_i(x), \varphi_i(x')) > 2\delta_i$;

(17) $\varphi_i(Z) = \tilde{f}_i \varphi_{i+1}(Z)$;

(18) $\rho_i(\varphi_i, \tilde{f}_i \varphi_{i+1}) \leqslant \delta_i/2$.

First choose an arbitrary $\varepsilon_1 > \operatorname{diam} Z$ and take an arbitrary mapping φ_1 from Z onto P_1. Then φ_1 is an ε_1-map from Z onto X_1. Now assume that we have already defined φ_i, ε_i, and δ_i for all $i < k$ in accordance with (15)–(18) and that $\varepsilon_i < \varepsilon_1/i$. Consider the map $\varphi_{k-1}: Z \to \tilde{X}_{k-1}$ and the numbers ε_{k-1} and δ_{k-1}. By Lemma 1 the map φ_{k-1} is $\mathscr{F}_{k-1}^*$-crooked. From Lemma 2 we find $\varepsilon, \delta > 0$ such that if ψ: $Z \to \psi(Z)$ is an ε-map and β: $\psi(Z) \to \varphi_{k-1}(Z)$ satisfies $\rho_{k-1}(\varphi_{k-1}, \beta\psi) < \delta$, then β is $\mathscr{F}_{k-1}^*$-crooked.

It follows from Lemma 4 in [**5**] that there is an $\varepsilon' > 0$ such that for any polyhedron P and ε'-mapping γ: $Z \to P$ onto P, there exists a mapping ω: $P \to \varphi_{k-1}(Z)$ onto $\varphi_{k-1}(Z)$ such that the distance

$$\rho_{k-1}(\varphi_{k-1}, \omega\gamma) \leqslant \delta' = \min\{\delta_{k-1}/4, \delta/2\}.$$

Take $\varepsilon_k = \min\{\varepsilon_1/k, \varepsilon, \varepsilon'\}$ and fix an ε_k-map ψ: $Z \to P$ onto P, where $P \in \Pi$. There is a mapping β: $P \to \varphi_{k-1}(Z)$ onto $\varphi_{k-1}(Z)$ such that the distance $\rho_{k-1}(\varphi_{k-1}, \beta\psi) < \delta'$. Since $\delta' < \delta$ and $\varepsilon_k < \varepsilon$, the mapping β is $\mathscr{F}_{k-1}^*$-crooked.

Hence, there is $g \in G_{k-1}$ which maps P onto $\varphi_{k-1}(Z)$ and $\rho_{k-1}(\beta, g) < \delta_{k-1}/2$. Since

$$\begin{aligned}\rho_{k-1}(\varphi_{k-1}(x), g\psi(x)) &\leqslant \rho_{k-1}(\varphi_{k-1}(x), \beta\psi(x)) + \rho_{k-1}(\beta\psi(x), g\psi(x)) \\ &< \delta_{k-1}/4 + \delta_{k-1}/4 = \delta_{k-1}/2\end{aligned}$$

for $x \in Z$, we obtain $\rho_{k-1}(\varphi_{k-1}, g\psi) < \delta_{k-1}/2$.

Put $\varphi_k = \pi_g\psi$; then $\tilde{f}_{k-1}\varphi_k = g\psi$; thus, φ_k is an ε_k-map, $\rho_{k-1}(\varphi_{k-1}, \tilde{f}_{k-1}\varphi_k) < \delta_{k-1}/2$, and $\varphi_{k-1}(z) = \tilde{f}_{k-1}\varphi_k(Z)$.

Now consider all the maps $\tilde{f}_{i,k}|\varphi_k(Z)$, where $i \in k$. We have a finite collection of uniformly continuous mappings, and, therefore, it is possible to determine a $\delta_k' > 0$ in such a manner that subsets of $\varphi_k(Z)$ of diameter not greater than δ_k' map under $\tilde{f}_{i,k}$ into subsets of $\varphi_i(Z)$ of diameter not greater than $\delta_i/2^{k-i}$. On the other hand, φ_k being an ε_k-mapping, there is a number $\delta_k'' > 0$ such that $x, x' \in Z$ and $\sigma(x, x') \geqslant 2\varepsilon_k$ imply $\rho_k(\varphi_k(x), \varphi_k(x')) > 2\delta_k''$. If we put $\delta_k = \min\{\delta_k', \delta_k''\}$, we have satisfied (15)–(18). This completes the proof of (14).

Finally, we remark that, by Theorem 15 in **[3]**, the compactum $\tilde{X}$ is contained in a continuum X such that X possesses an atomic mapping η onto a pseudoarc such that $\eta|X \setminus \tilde{X}$ is a homeomorphism and $\eta(\tilde{X})$ is zero dimensional. Hence, by Propositions 11 and 12 in **[3]** X is hereditarily indecomposable and $\dim X = \dim \tilde{X}$; thus, the proof of the Theorem is complete by (14).

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