PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 94, Number 3, July 1985

BOUNDED COHOMOLOGY OF CERTAIN GROUPS OF HOMEOMORPHISMS

SHIGENORI MATSUMOTO AND SHIGEYUKI MORITA

ABSTRACT. We consider the condition when bounded cohomology injects into ordinary cohomology and prove the vanishing of bounded cohomology of the group of all compactly supported homeomorphisms of $\mathbf{R}^n$.

Introduction. In this note we consider relations among bounded cohomology, ordinary real cohomology and l^1 homology of spaces or groups. In particular we present a necessary and sufficient condition under which bounded cohomology injects into ordinary cohomology and by using it prove the vanishing of bounded cohomology and l^1 homology of $\mathrm{Homeo}_K \mathbf{R}^n$, the group of all homeomorphisms of $\mathbf{R}^n$ with compact support. We also determine the second bounded cohomology of $\mathrm{SL}_2 \mathbf{R}$.

1. Bounded cohomology. Let us quickly review the theory of bounded cohomology developed by Gromov [2] (see also Brooks [1] and Mitsumatsu [5]). Let X be a topological space and let $\mathscr{C}_*(X) = \{C_q(X), \partial_q\}$ be the singular chain complex of X with real coefficients. Define a norm on $C_q(X)$ by $\|\sum_{i=1}^n a_i\sigma_i\| = \sum_{i=1}^n |a_i|$. The differentials ∂_q are then bounded linear operators.

Let $\mathscr{C}_*^{l_1}(X) = \{C_q^{l_1}(X), \partial_q\}$ be the norm completion of $\mathscr{C}_*(X)$. Thus $C_q^{l_1}(X) = \{\sum_{i=1}^\infty a_i\sigma_i | \sum_{i=1}^\infty |a_i| < \infty\}$ is a Banach space. Passing to the dual Banach spaces, we obtain a cochain complex $\mathscr{C}_b^*(X) = \{C_b^q(X), \delta_q\}$. It is a subcomplex of the ordinary singular cochain complex consisting of bounded cochains. The homology of $\mathscr{C}_*^{l_1}(X)$, denoted by $H_*^{l_1}(X)$, is called l_1 homology of X and the cohomology of $\mathscr{C}_b^*(X)$, denoted by $H_b^*(X)$, is called bounded cohomology of X. The inclusions induce homomorphisms $H_*(X) \to H_*^{l_1}(X)$ and $H_b^*(X) \to H^*(X)$.

Since the image of a bounded operator is not necessarily a closed subspace, it may happen that the pseudonorms induced on $H_*^{l_1}(X)$ or $H_b^*(X)$ are not norms. Following Mitsumatsu [5], we define $\overline{H}_*^{l_1}(X)$ (resp. $\overline{H}_b^*(X)$) to be the quotient of $H_*^{l_1}(X)$ (resp. $H_b^*(X)$) by the subspace of pseudonorm zero. In other words, $\overline{H}_q^{l_1}(X) = Z_q^{l_1}(X)/\overline{B}_q^{l_1}(X)$ and $\overline{H}_b^q(X) = Z_b^q(X)/\overline{B}_b^q(X)$, where Z or B denotes the spaces of (co)cycles or (co)boundaries of the corresponding complex and $\overline{B}$ denotes the closure of B. Notice that $\overline{H}_q^{l_1}(X)$ and $\overline{H}_b^q(X)$ are Banach spaces. There is a

Received by the editors July 19, 1984.
1980 *Mathematics Subject Classification*. Primary 55N99; Secondary 57T99.
Key words and phrases. Bounded cohomology, l^1 homology, groups of homeomorphisms.

surjective homomorphism $\overline{H}_b^q(X) \to (\overline{H}_q^{l_1}(X))'$, where $'$ denotes the dual Banach space.

Now for a group G, starting from the real chain complex $\mathscr{C}_*(G) = \{C_q(G), \delta_q\}$, similar constructions as above are made, yielding l^1 homology $H_*^{l_1}(G)$ and bounded cohomology $H_b^*(G)$ of G.

2. Uniform boundary condition. A chain complex is called *normed* if each chain group is a normed linear space over $\mathbf{R}$ and each differential is a bounded linear operator.

DEFINITION 2.1. A normed chain complex $\mathscr{C}_*$ is said to satisfy *q uniform boundary condition* (q-UBC, for short) if there exists a number $K > 0$ such that for any boundary $z \in B_q$, there is a chain $c \in C_{q+1}$ satisfying $\partial c = z$ and $\|c\| \leqslant K\|z\|$.

DEFINITION 2.2. (i) A topological space or a group is said to satisfy q-UBC if its ordinary chain complex satisfies q-UBC.

(ii) It is said to satisfy q-UBC^{l_1} if its l^1 chain complex satisfies q-UBC.

THEOREM 2.3. *For spaces or groups, the following conditions are equivalent*:

(i) q-UBC^{l_1}.

(ii) $B_q^{l_1}$ *is closed in* $C_q^{l_1}$.

(iii) $\overline{H}_q^{l_1} = H_q^{l_1}$.

(iv) $\overline{H}_b^{q+1} = H_b^{q+1}$.

(v) *The surjective homomorphism* $H_b^{q+1} \to (\overline{H}_{q+1}^{l_1})'$ *is injective.*

PROOF. (i) is clearly equivalent to that the bijection $C_{q+1}^{l_1}/Z_{q+1}^{l_1} \to B_q^{l_1}$ has a bounded inverse, that is, they are homeomorphic. Since $C_{q+1}^{l_1}/Z_{q+1}^{l_1}$ is a Banach space, it follows that $B_q^{l_1}$ is also a Banach space and hence (ii) holds. Conversely if (ii) is satisfied, then $B_q^{l_1}$ is a Banach space and the open mapping theorem (cf. [**11**]) applied to the above bijection implies (i).

(ii) $\Leftrightarrow$ (iii) is clear.

(iii) $\Leftrightarrow$ (iv) is a direct consequence of the closed range theorem.

(i) $\Rightarrow$ (v). Take $f \in Z_b^{q+1}$ so that $f(z) = 0$ for all $z \in Z_{q+1}^{l_1}$. By (i), the map $B_q^{l_1} \xleftarrow{\sim} C_{q+1}^{l_1}/Z_{q+1}^{l_1} \xrightarrow{\bar{f}} \mathbf{R}$ is bounded and thus by the Hahn-Banach theorem has an extension to $C_q^{l_1}$. This shows (v).

(v) $\Rightarrow$ (iv). It suffices to show $\mathrm{Ker}(H_b^{q+1} \to \overline{H}_b^{q+1}) \subset \mathrm{Ker}(H_b^{q+1} \to (\overline{H}_{q+1}^{l_1})')$. Let $f \in \overline{B}_b^{q+1}$. Then $f = \lim_{i\to\infty} f_i$ ($f_i \in B_b^{q+1}$). Thus for any $z \in Z_{q+1}^{l_1}$, we have $f(z) = \lim_{i\to\infty} f_i(z) = 0$. This completes the proof. Q.E.D.

COROLLARY 2.4. (i) *If* $\overline{H}_q^{l_1} = H_q^{l_1}$ *and* $\overline{H}_{q+1}^{l_1} = 0$, *then* $H_b^{q+1} = 0$.

(ii) *If* $\overline{H}_b^{q+1} = H_b^{q+1}$ *and* $\overline{H}_b^q = 0$, *then* $H_q^{l_1} = 0$.

(iii) *The reduced bounded cohomology* $\tilde{H}_b^*$ *vanishes if and only if the reduced* l^1 *homology* $\tilde{H}_*^{l_1}$ *also vanishes.*

In Brooks [**1**] and Gromov [**2**], it is shown that the reduced bounded cohomology vanishes for spaces with amenable π_1. This, combined with the above corollary, gives

COROLLARY 2.5. *If* $\pi_1(X)$ *is amenable, then* $\tilde{H}_*^{l_1}(X) = 0$.

REMARK 2.6. It seems plausible that the l^1 homology of a space depends only on its fundamental group. But we do not have a proof.

COROLLARY 2.7. *For any space or a group, we have*
(i) $H_1^{l_1} = H_b^1 = 0$.
(ii) $H_b^{-2} = H_b^2$. *Equivalently, H_b^2 is a Banach space.*

PROOF. First we deal with a group G. In [5], Mitsumatsu constructed for $g \in g$,

$$S(g) = \sum_{k=1}^{\infty} \frac{1}{2^k}(g^k, g^k) \in C_2^{l_1}(G).$$

Clearly, $\|S(g)\| = 1$ and $\partial S(g) = g$, showing that $H_1^{l_1}(G) = 0$ and that G satisfies 1-UBC^{l_1}. From this follows the corollary. Notice that 0-UBC^{l_1} is always satisfied.

For spaces, according to Gromov [2], $H_b^2(X)$ is isometric to $H_b^2(\pi_1(X))$. This shows (ii). Also it is known that $H_b^1(X) = 0$ [2]. Thus (i) follows from Corollary 2.4. Q.E.D.

We have shown that 1-UBC^{l_1} is always true. It would be interesting to determine whether q-UBC^{l_1} always holds or not.

Next, we investigate q-UBC for spaces or groups.

THEOREM 2.8. *The following conditions are equivalent*:
(i) q-UBC.
(ii) q-UBC^{l_1} *and Z_{q+1} is dense in $Z_{q+1}^{l_1}$.*
(iii) *The homomorphism $H_b^{q+1} \to H^{q+1}$ is injective.*

PROOF. (i) $\Rightarrow$ (ii). We first prove q-UBC^{l_1}. Clearly B_q is dense in $B_q^{l_1}$. Further a standard argument shows that for any $z \in B_q^{l_1}$, there exist $z_i \in B_q$ such that $\sum_{i=1}^{\infty} z_i = z$ and $\sum_{i=1}^{\infty}\|z_i\| \leqslant (1+\varepsilon)\|z\|$. Now by q-UBC, one can choose $c_i \in C_{q+1}$ such that $\partial c_i = z_i$ and $\|c_i\| \leqslant K\|z_i\|$. Let $c = \sum_{i=1}^{\infty} c_i \in C_{q+1}^{l_1}$. Then $\partial c = z$ and $\|c\| \leqslant (1+\varepsilon)K\|z\|$.

Next we prove that Z_{q+1} is dense in $Z_{q+1}^{l_1}$. Take $z \in Z_{q+1}^{l_1}$ and let $z = \lim_{i\to\infty} c_i$ ($c_i \in C_{q+1}$). Choose an element $d_i \in C_{q+1}$ such that $\partial d_i = -\partial c_i$ and $\|d_i\| \leqslant K\|\partial c_i\|$. Then we have

$$\|\partial c_i\| = \|\partial(c_i - z)\| \leqslant \|\partial\|\,\|c_i - z\| = (q+2)\|c_i - z\|.$$

Hence $\|d_i\| \leqslant (q+2)K\|c_i - z\|$. Now $c_i + d_i \in Z_{q+1}$ and $c_i + d_i \to z$.

(ii) $\Rightarrow$ (i). This is left to the reader.

(ii) $\Rightarrow$ (iii). Take $f \in Z_b^{q+1}$ such that $[f] = 0$ in H^{q+1}, that is, $f(z) = 0$ for any $z \in Z_{q+1}$. Then $f(z) = 0$ for any $z \in Z_{q+1}^{l_1}$. Thus f induces a bounded map $\bar{f}$: $C_{q+1}^{l_1}/Z_{q+1}^{l_1} \to \mathbf{R}$. Now q-UBC^{l_1} implies that the bijection $C_{q+1}^{l_1}/Z_{q+1}^{l_1} \to B_q^{l_1}$ has a bounded inverse. Compose it with $\bar{f}$ and extend to the whole of $C_q^{l_1}$ by the Hahn-Banach theorem. This shows (iii).

(iii) $\Rightarrow$ (ii). Notice that the map $H_b^{q+1} \to H^{q+1}$ is a composite of maps $H_b^{q+1} \to \overline{H}_b^{q+1} \to H^{q+1}$. Injectivity of the first map implies q-UBC^{l_1} by Theorem 2.3. Next, clearly (iii) implies that the image of the map $H_{q+1} \to \overline{H}_{q+1}^{l_1}$ is dense. From this the denseness of Z_{q+1} follows easily. Q.E.D.

DEFINITION 2.9. A group G is said to be *uniformly perfect* if for some $N > 0$, any $g \in G$ can be represented as a product of at most N commutators.

LEMMA 2.10. *If a group G is uniformly perfect, then it satisfies* 1-UBC.

PROOF. Notice that for f_i, g, $h \in G$,

$$\begin{aligned}\partial((f_1, f_2) + (f_1 f_2, f_3) + \cdots + (f_1 f_2 \cdots f_{N-1}, f_N)) \\ = (f_1) + (f_2) + \cdots + (f_N) - (f_1 f_2 \cdots f_N)\end{aligned}$$

and

$$\partial(([g, h], h) + (ghg^{-1}, g) - (g, h)) = ([g, h])$$

where ( ) denotes a chain and [ ] a commutator. This shows for all $f \in G$ there exists $c \in C_2(G)$ such that $\partial c = (f)$ and $\|c\| \leqslant 4N - 1$. That is, G satisfies 1-UBC. Q.E.D.

COROLLARY 2.11. *If G is uniformly perfect, then the map $H_b^2(G) \to H^2(G)$ is injective.*

As applications, we shall compute H_b^2 for some groups.

EXAMPLE 2.12. The group of all orientation preserving homeomorphisms of S^1, denoted by $\mathrm{Homeo}_+(S^1)$, is uniformly perfect. In fact any element is a product of two homeomorphisms with compact support, which are commutators by Mather [**4**]. Thus H_b^2 injects into H^2. Now it is a consequence of Thurston's general result [**8**] that $H^*(\mathrm{Homeo}_+(S^1); \mathbf{Z}) \cong \mathbf{Z}[\chi]$, where $\chi \in H^2$ is the Euler class. Now H_b^* is mapped onto real cohomology, because χ can be represented by a bounded cocycle (see Morita [**6**]). Hence we have $H_b^2(\mathrm{Homeo}_+(S^1)) \cong \mathbf{R}$.

EXAMPLE 2.13. Sah and Wagoner [**7**] have calculated second homology of certain Lie groups (considered to be discrete groups). Combined with our result, this gives information about H_b^2. For example, $\mathrm{SL}_2\mathbf{R}$ is uniformly perfect (see Wood [**10**]) and $H_2(\mathrm{SL}_2\mathbf{R}; \mathbf{Z})$ is isomorphic to $\mathbf{Z} \oplus A$, where A is a certain $\mathbf{Q}$-vector space. $\mathbf{Z}$ is detected by the "volume class" $\in H^2(\mathrm{SL}_2\mathbf{R})$, which is a bounded cohomology class. Any element of A is supported on a torus (see [**7**] and Tsuboi [**9**]). From these, we can conclude $H_b^2(\mathrm{SL}_2\mathbf{R}) \cong H_b^2(\mathrm{PSL}_2\mathbf{R}) \cong \mathbf{R}$.

3. Bounded cohomology of $\mathrm{Homeo}_K(\mathbf{R}^n)$. In this section we prove the vanishing of bounded cohomology and l^1 homology of $\mathrm{Homeo}_K(\mathbf{R}^n)$, the group of all homeomorphisms of $\mathbf{R}^n$ with compact support.

THEOREM 3.1. *For $q > 0$,*

$$H_b^q(\mathrm{Homeo}_K(\mathbf{R}^n)) = H_q^{l_1}(\mathrm{Homeo}_K(\mathbf{R}^n)) = 0.$$

Our argument is a refinement of Mather's proof of the acyclicity of $\mathrm{Homeo}_K(\mathbf{R}^n)$. In the sequel we follow Mather [**4**]. We write $G = \mathrm{Homeo}_K(\mathbf{R}^n)$ and $G^i = \{g \in G: \operatorname{supp} g \subset \operatorname{Int} iD^n\}$ $(i = 1, 2, 3)$, where D^n is the unit ball. Inclusions are denoted by ι^1: $G^1 \to G^2$, ι^2: $G^2 \to G^3$, $\bar{\iota} = \iota^2\iota^1$ and ι: $G^1 \to G$. Let C_q and C_q^i be the chain complex of G and G^i. Z_q and Z_q^i (resp. B_q and B_q^i) denote the cycle group (resp.

boundary group) of the corresponding complexes. In fact, $Z_q = B_q$ and $Z_q^i = B_q^i$ by the acyclicity of the group.

We shall prove inductively the existence of bounded linear operators S_q: $B_q^1 \to C_{q+1}$ such that $\partial_{q+1} S_q = \iota_*$. Let us show first that this suffices for our purpose. We have only to show q-UBC for G, because the acyclicity of G, together with Theorem 2.8 and Corollary 2.4 yields Theorem 3.1. Take $z \in B_q^1$. Choose $\varphi \in G$ such that φ is the identity on $\operatorname{supp} z$ and maps $\operatorname{supp} S_q(z)$ into $\operatorname{Int} D^n$. Define I_φ: $G \to G$ by $I_\varphi(g) = \varphi g \varphi^{-1}$. Then we have $I_{\varphi*}(S_q(z)) \in C_{q+1}^1$, $\|I_{\varphi*}(S_q(z))\| = \|S_q(z)\| \leqslant \|S_q\| \, \|z\|$ and $\partial(I_{\varphi*}(S_q(z))) = I_{\varphi*}(\partial S_q(z)) = I_{\varphi*}(z) = z$. This proves q-UBC for G^1, hence for G.

S_1 is constructed in an elementary fashion as follows. Choose $k \in G$ such that $k(3D^n) \cap 3D^n = \varnothing$ and that $k^i(3D^n)$ tends to one point as $i \to \infty$. Define ψ_1: $G^3 \to G$ by $\psi_1(g) = \sum_{i=1}^{\infty} k^i g k^{-i}$ and let $\psi_0(g) = k^{-1}\psi_1(g)k$. Then for any $g \in G^3$, $\operatorname{supp} g \cap \operatorname{supp} \psi_1(g) = \varnothing$ and $\psi_0(g) = g\psi_1(g)$. The restrictions of ψ_i to G^1 are denoted by the same letter. Now we define a bounded linear map S_1: $B_1^1 \to C_2$ by

$$S_1(g) = (k, g) - (\psi_1(g), k) + (kg, \psi_1(g)).$$

Direct computation shows $\partial_2 S_1 = \iota_*$.

Next we assume there exist S_j^1: $B_j^1 \to C_{j+1}^2$ and S_j^2: $B_j^2 \to C_{j+1}^3$ for $0 \leqslant j \leqslant q-1$ and construct S_q: $B_j^1 \to C_{j+1}$. Let α: $C_*(G \times G) \to C_* \otimes C_*$ (resp. β: $C_* \otimes C_* \to C_*(G \times G)$) be the Alexander-Whitney map (resp. Eilenberg-Mac Lane map) (see Mac Lane [3]). Those maps for G^i are also denoted by the same letters. They are functorial and if we give a norm to $C_* \otimes C_*$ in a canonical manner, they are bounded linear.

For each $z \in B_q^1$, define $D(z) = \alpha\Delta_* z - (z \otimes 1 + 1 \otimes z)$, where Δ: $G^1 \to G^1 \times G^1$ is the diagonal map. Then $D(z) \in Z_q'(C^1 \otimes C^1) = Z_q(C^1 \otimes C^1) \cap \sum_{i=1}^{q-1} C_i^1 \otimes C_{q-}^1$. D is bounded linear. Now let Z^1 and $\overline{B}^1$ be the chain complexes (with trivial differential) defined by $(Z^1)_q = Z_q^1$ and $(\overline{B}^1)_q = B_{q-1}^1$. Then we have the following commutative diagram, whose horizontal sequences are all exact.

$$\begin{array}{ccccccccc}
0 & \to & (C^1 \otimes Z^1)_{q+1} & \to & (C^1 \otimes C^1)_{q+1} & \xrightarrow{1\otimes\partial} & (C^1 \otimes \overline{B}^1)_{q+1} & \to & 0 \\
 & & \downarrow\partial & & \downarrow\partial & & \downarrow\partial & & \\
0 & \to & (C^1 \otimes Z^1)_q & \to & (C^1 \otimes C^1)_q & \xrightarrow{1\otimes\partial} & (C^1 \otimes \overline{B}^1)_q & \to & 0 \\
 & & \downarrow\partial & & \downarrow\partial & & \downarrow\partial & & \\
0 & \to & (C^1 \otimes Z^1)_{q-1} & \to & (C^1 \otimes F^1)_{q-1} & \xrightarrow{1\otimes\partial} & (C^1 \otimes \overline{B}^1)_{q-1} & \to & 0
\end{array}$$

Analogous diagrams are considered for G^2 and G^3. They are combined by ι_*^1 and ι_*^2. Now because $\partial(D(z)) = 0$, $(1 \otimes \partial)D(z)$ is contained in $Z_q'(C^1 \otimes \overline{B}^1)$. Notice that $Z_q'(C \otimes \overline{B}^1) = (Z^1 \otimes \overline{B}^1)_q'$. Hence by the induction assumption we can consider

$$(S^1 \otimes S^1)(1 \otimes \partial)D(z) = (S^1 \otimes S^1\partial)D(z) \in (C^2 \otimes C^2)_{q+1}.$$

Let $u = (\iota_*^1 \otimes \iota_*^1)D(z) - \partial(S^1 \otimes S^1\partial)D(z) \in (C^2 \otimes C^2)_q$. Direct computation shows that $(1 \otimes \partial)u = 0$. Thus we have $u \in (C^2 \otimes Z^2)_q'$. Also we have $\partial u = 0$. That is,

$$u \in Z_q'(C^2 \otimes Z^2) = (Z^2 \otimes Z^2)_q'.$$

Therefore by the induction assumption, we can consider

$$(S^2 \otimes (\iota_*^2 - S^2\partial))u \in (C^3 \otimes C^3)_{q+1}.$$

Again direct computation shows that

$$\partial\big(S^2 \otimes (\iota_*^2 - S^2\partial)\big)u = (\iota_*^2 \otimes \iota_*^2)u.$$

Hence we have $(\bar{\iota}_* \otimes \bar{\iota}_*)D(z) = \partial(ED(z))$, where

$$E = (\iota_*^2 S^1 \otimes \iota_*^2 S^1\partial) + \big(S^2 \otimes (\iota_*^2 - S^2\partial)\big)(\iota_*^1 - \partial(S^1 \otimes S^1\partial)).$$

E: $Z_q'(C^1 \otimes C^1) \to (C^3 \otimes C^3)_{q+1}$ is a bounded linear map. We can now write

$$(\bar{\iota}_* \otimes \bar{\iota}_*)\alpha\Delta_* z = (\bar{\iota}_* \otimes \bar{\iota}_*)z \otimes 1 + \partial(ED(z)) + (\bar{\iota}_* \otimes \bar{\iota}_*)(1 \otimes z).$$

Let η: $G^3 \otimes G^3 \to G$ be the homomorphism given by $\eta(g, h) = g\psi_1(h)$. Applying $\eta_*\beta$, we get

$$\eta_*(\bar{\iota} \times \bar{\iota})_*\beta\alpha\Delta_* z = \iota_* z + \eta_*\beta\partial(ED(z)) + \chi_{1*} z.$$

As is well known, $\beta\alpha$ is chain homotopic to the identity. Namely there is a linear map Φ: $C_*(G^1 \times G^1) \to C_{*+1}(G^1 \times G^1)$ such that $\beta\alpha - \mathrm{id} = \partial\Phi + \Phi\partial$. It is easy to show that we can choose Φ as a bounded linear map. Now $\beta\alpha\Delta_* z = \Delta_* z + \partial\Phi(\Delta_* z)$. Because $\eta\Delta = \psi_0$, we have

$$\psi_{0*} z + \eta_*(\bar{\iota} \times \bar{\iota})_*\partial\Phi\Delta_* z = \iota_* z + \eta_*\beta\partial ED(z) = \psi_{1*} z.$$

Two homomorphisms ψ_0 and ψ_1 are conjugate and thus ψ_{0*} is chain homotopic to ψ_{1*}. Namely there is a linear map Θ: $C_*^1 \to C_{*+1}$ such that $\psi_{1*} - \psi_{0*} = \partial\Theta + \Theta\partial$. Here we can also choose Θ to be bounded linear. Finally we obtain $\iota_* = \partial_{q+1} S_q$, where

$$S_q = \eta_*(\bar{\iota} \times \bar{\iota})_*\Phi\Delta_* - \Theta - \eta_*\beta ED.$$

This completes the proof.

References

1. R. Brooks, *Some remarks on bounded cohomology*, Ann. of Math. Stud., no. 97, Princeton Univ. Press, Princeton, N. J., 1981, pp. 53–63.
2. M. Gromov, *Volume and bounded cohomology*, Inst. Hautes Études Sci. Publ. Math. **56** (1982), 5–99.
3. S. Mac Lane, *Homology*, Springer-Verlag, Berlin and New York, 1963.
4. J. Mather, *The vanishing of homology of certain groups of homeomorphisms*, Topology **10** (1973), 297–298.
5. Y. Mitsumatsu, *Bounded cohomology and l^1 homology of surfaces*, Topology (to appear).
6. S. Morita, *Characteristic classes of surface bundles*. I, preprint.
7. C-H. Sah and J. B. Wagoner, *Second homology of Lie groups made discrete*, Comm. Algebra **5** (1977), 611–642.
8. W. Thurston, *Foliations and groups of diffeomorphisms*, Bull. Amer. Math. Soc. **80** (1974), 304–307.
9. T. Tsuboi, *Foliated cobordism classes of certain foliated S^1-bundles over surfaces*, Topology **23** (1984), 233–244.
10. J. Wood, *Bundles with totally discontinuous structure group*, Comm. Math. Helv. **46** (1971), 257–273.
11. K. Yosida, *Functional analysis*, Springer-Verlag, Berlin and New York, 1960.

Department of Mathematics, College of Science and Technology, Nihon University, 101 Kanda - Surugadai, Chiyoda - ku, Tokyo, Japan

Department of Mathematics, College of General Education, Tokyo University, 153 Komaba, Meguro - ku, Tokyo, Japan