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AN OPEN BOOK DECOMPOSITION FOR $RP^2 \times S^1$

ERIC KLASSEN

ABSTRACT. In this paper an open book decomposition for $RP^2 \times S^1$ is exhibited.

In a recent paper [1], Berstein and Edmonds prove the existence of open book decompositions for compact, nonorientable 3-manifolds. Their method is to pull back an open book decomposition of $RP^2 \times S^1$ via a branched cover. Their demonstration that $RP^2 \times S^1$ has such a decomposition is algebraic; they define a homomorphism $\pi_1(RP^2 \times S^1\text{-knot}) \to Z$ and apply Stallings' fibering theorem [2]. In this paper I explicitly exhibit an open book decomposition of $RP^2 \times S^1$ via a map $RP^2 \times S^1$-same knot $\to S^1$, inducing the same homomorphism on π_1.

Let $D^2 = \{z \in C\colon |z| \leqslant 1\}$. Think of RP^2 as D^2 with antipodal points of the boundary circle identified. Define $\bar\varphi\colon D^2 \to D^2$ by $\bar\varphi(z) = -z$. Note that $\bar\varphi$ induces a map $\varphi\colon RP^2 \to RP^2$ which is isotopic to the identity (just rotate through 180°). Thus we may take $D^2 \times I$ with the identifications

$$D^2 \times I \to RP^2 \times I \to RP^2 \times I/(x,0) \sim (\varphi(x),1)$$

as our model of $RP^2 \times S^1$. The image of $\{-\frac{1}{2}\} \times I \cup \{\frac{1}{2}\} \times I$ is a circle C in $RP^2 \times S^1$.

Let $p\colon (D^2 - \{-\frac{1}{2}, \frac{1}{2}\}) \times I \to S^1$ by

$$p(z,t) = \frac{z^2 - 1/4}{|z^2 - 1/4|} e^{2\pi i t}.$$

An open book decomposition of $RP^2 \times S^1$ is given by taking as binding the circle C and as pages the surfaces F_θ in $RP^2 \times S^1$ represented by $(\{-\frac{1}{2}, \frac{1}{2}\} \times I) \cup p^{-1}(e^{2\pi i\theta})$ in $D^2 \times I$.

In Figure 1 I exhibit the surface $F = F_1$ in $RP^2 \times S^1$ by drawing its intersection with each $D^2 \times \{t\}$ in $D^2 \times I$. The boundary of F is C.

$F \cap D^2 \times \{\frac{1}{2}\}$ is not a manifold, because F has a saddle at this level (i.e., a critical point with respect to the Morse function proj: $RP^2 \times S^1 \to S^1$). Because the only critical point is of index one, $\chi(F) = -1$. Thus F, being nonorientable, is a punctured Klein bottle. Because F is the graph of a function from $D^2 - \{-\frac{1}{2}, \frac{1}{2}\} \to [0,1)$, we see that $RP^2 \times S^1 - C$ is foliated by leaves which are simply copies of

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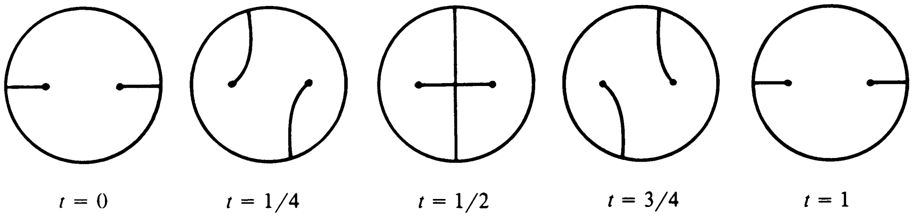

FIGURE 1

F translated in the S^1 direction. Indeed these are the surfaces F_θ, where $F = F_1$. This gives an open book decomposition of $RP^2 \times S^1$ with binding C. The intersection of this foliation with an arbitrary $D^2 \times \{t\}$ is shown in Figure 2.

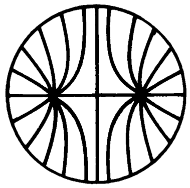

FIGURE 2

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DEPARTMENT OF MATHEMATICS, CORNELL UNIVERSITY, ITHACA, NEW YORK 14850