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## SHORTER NOTES

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# AN EXAMPLE OF FRÉCHET SPACE, NOT MONTEL, WITHOUT INFINITE DIMENSIONAL NORMABLE SUBSPACES

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ABSTRACT. Given $X$ a Fréchet Montel space, no infinite dimensional subspace of $X$ is normable. We show that the converse implication is not true in general. In fact we provide here a Fréchet space $X$ (moreover, $X$ is a perfect Fréchet space), which is not Montel but does not contain an infinite dimensional normable subspace.

Clearly, given $X$ a Fréchet Montel space, none of the infinite dimensional subspaces of $X$ is normable. The converse implication is true for certain classes of Fréchet spaces (e.g. echelon sequence spaces, $X$-Köthe sequence space [**2**], closed subspaces of the reduced projective limits of $\mathfrak{L}_{p,\lambda}$-spaces $p \geq 2$ [**1**],...). However, we give here a counterexample in order to prove that the converse implication is not true in general.

In fact, let $\rho \in \mathbf{R}$, $\rho \geq 1$, and let us consider the following perfect Fréchet space

$$\lambda_\rho = \bigcap_{p>\rho} l^p = \bigcap_{n\in\mathbf{N}} l^{\rho+1/n} = \underleftarrow{\operatorname{proj}}(l^{\rho+1/n}, I_{nm}),$$

$I_{nm}: l^{\rho+1/m} \to l^{\rho+1/n}$, $n \leq m$, denoting the inclusion mapping. According to the results of Dubinsky [**3**], $\lambda_\rho$ is a, not Montel, perfect Fréchet space. On the other hand, if $\lambda_\rho$ contains an infinite dimensional normable subspace, say $B$, then it is easy to check that there is $k_0 \in \mathbf{N}$ so that $B$ is isomorphic to a subspace of $l^{\rho+1/k}$ for every $k \geq k_0$, but this is not possible since $l^p$ and $l^r$ are totally incomparable spaces if $p \neq r$ [**4**]. Thus $\lambda_\rho$ is a not Montel, Fréchet space, without infinite dimensional normable subspaces.

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