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BOUNDARY CONTINUITY OF HOLOMORPHIC FUNCTIONS IN THE BALL

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ABSTRACT. It is shown that any holomorphic function on the unit ball of $\mathbf{C}^n$ with nth partial derivatives in the Hardy class H^1 has a continuous extension to the closed unit ball, and that the restriction to any real analytic curve in the boundary which is nowhere complex tangential is absolutely continuous.

Introduction. A well-known result of Privalov asserts that any holomorphic function f on the unit disk satisfying

$$\sup_{0<r<1} \int_0^{2\pi} |f'(re^{i\theta})|\, d\theta < \infty \tag{1}$$

has a continuous extension to the closed unit disk, and, moreover, the boundary value function is absolutely continuous on the unit circle. The purpose of this note is to address the analogous problem in the unit ball $B = B_n$ in complex n-space. This problem has been considered previously by Graham [**5**, **6**] and Krantz [**8**], who considered the following analogue of condition (1):

$$\sup_{0<r<1} \int_{\partial B} |\mathcal{R}f(rz)|^p\, d\sigma(z) < \infty, \tag*{$(2)_p$}$$

where $\mathcal{R}$ denotes the radial derivative operator defined by $\mathcal{R} = \sum z_j \partial_j$ and σ denotes the normalized surface measure on the unit sphere. It has been shown by Graham [**6**] and Krantz [**8**] that any holomorphic function on B_n satisfying $(2)_{n+\varepsilon}$ for some $\varepsilon > 0$ is in the Lipschitz class Λ_ε. However, in [**5**], Graham showed that there are unbounded holomorphic functions on B_2 satisfying $(2)_2$, and conjectured that holomorphic functions on B_n satisfying $(2)_n$ must be in the class BMOA of functions with bounded mean oscillation, which was later verified by Krantz [**8**].

In the present paper, we consider a somewhat different analogue of condition (1) than that considered by Graham and Krantz. To be precise, we consider, for $0 < p < \infty$ and $s \geq 0$, functions f satisfying the condition

$$\sup_{0<r<1} \int_{\partial B} |\mathcal{R}^s f(rz)|^p\, d\sigma(x) < \infty. \tag*{$(3)_{p,s}$}$$

(For noninteger values of s, the fractional derivatives can be interpreted in terms of power series.) It can be shown (see [**2**]) that for $s > n/p$, any holomorphic function f satisfying $(3)_{p,s}$ is in the Lipschitz class $\Lambda_{s-n/p}$, and that in the limiting case $s = n/p$, any function satisfying $(3)_{p,s}$ is in the class BMOA. Moreover, we will show (see Theorem 1.3 below) that for any $p > 1$ there are unbounded holomorphic functions on B_n satisfying $(3)_{p,n/p}$.

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The preceding discussion suggests condition $(3)_{1,n}$ as a reasonable analogue of condition (1) when $n \geq 2$, and, indeed, a special case of the main result of this paper (Theorem 1.4) is a several variable analogue of the Privalov result for holomorphic functions satisfying $(3)_{1,n}$. More precisely, we will show that any holomorphic function on B_n which satisfies condition $(3)_{1,n}$ has a continuous extension to the closed unit ball and that moreover, the restriction of such a function to any real analytic curve in ∂B which is not an integral curve of the complex structure on ∂B is absolutely continuous.

Throughout this paper we will use the convention of denoting by C any positive constant which is independent of the relevant parameters in the expression in which it occurs. The value of C may change from one occurrence to the next. In addition we will use the notation $A \approx B$ to mean $C^{-1}A \leq B \leq CB$ for some positive constant C.

1. Preliminaries. We begin by introducing a family of weighted Sobolev norms on the space $\mathcal{O}(B)$ of holomorphic functions on the unit ball. Following [**2**] we define for each $q > 0$ a measure dV_q on $\overline{B}$ by $dV_q(z) = C_{n,q}(1-|z|^2)^{q-1}dV(z)$, where dV is the usual Lebesgue measure on $\mathbf{C}^n$ and the constant $C_{n,q}$ is chosen so that dV_q is a probability measure on $\overline{B}$. For $q > 0$, $0 < p < \infty$ and s a nonnegative integer, let

$$|||f|||_{p,q,s} = \left[\sum_{|\alpha|\leq s} \int |\partial^\alpha f|^p \, dV_q\right]^{1/p}.$$

Here we have used the usual multi-index conventions: $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbf{Z}_+^n$, $|\alpha| = \sum \alpha_j$ and $\partial^\alpha = \partial_1{}^{\alpha_1} \cdots \partial_n{}^{\alpha_n}$.

It follows by integration in polar coordinates that as $q \to 0^+$, the measures dV_q on $\overline{B}$ converge weakly to the normalized surface measure $d\sigma$ on ∂B. Thus we define dV_0 to be $d\sigma$ and we set

$$|||f|||_{p,0,s} = \sup_{0<r<1} \left[\sum_{|\alpha|\leq s} \int |\partial^\alpha f(rz)|^p \, dV_0\right]^{1/p}.$$

It should be noted that the norm $|||\ |||_{p,0,0}$ coincides with the usual Hardy class norm.

Although the definitions of Sobolev norms given above are standard, it will be convenient for our purposes to use equivalent norms which involve differentiation in only the radial direction. We will denote by $\mathcal{R}$ the *radial derivative operator* defined by $\mathcal{R} = \sum z_j \partial_j$, and by $\mathcal{D}$ the operator $\mathcal{R} + 1$. It follows that for any holomorphic function $f = \sum a_\alpha z^\alpha$ on B and any nonnegative integer s we have $\mathcal{D}^s f(z) = \sum (1+|\alpha|)^s z^\alpha$. Thus we may use this formula as the definition of $\mathcal{D}^s f$ for an arbitrary real number s. By analogy with the above definitions, we define

$$\|f\|_{p,q,s} = \left[\int |\mathcal{D}^s f|^p \, dV_q\right]^{1/p}, \qquad q > 0, s \in \mathbf{R},$$

and

$$\|f\|_{p,0,s} = \sup_{0<r<1} \left[\int |\mathcal{D}^s f(rz)|^p \, dV_0\right]^{1/p}, \qquad s \in \mathbf{R}.$$

The proof of the following result can be found in [**2**].

(1.1) THEOREM. *For $0 \le q$, $0 < p < \infty$ and any nonnegative integer s the norms $\| \, \|_{p,q,s}$ and $||| \, |||_{p,q,s}$ are equivalent on $\mathcal{O}(B)$.*

We will denote by $A^p_{q,s}$ the space of all holomorphic functions f on B with $\|f\|_{p,q,s} < \infty$. In the case $s = 0$ we will drop the s and write simply A^p_q and $\| \, \|_{p,q}$.

For $s > 0$, the *Lipschitz class* $\Lambda_s = \Lambda_s(B)$ is defined as follows. For $0 < s < 1$, Λ_s is the class of functions on B satisfying $|f(z+\varsigma) + f(z-\varsigma) - 2f(z)| \le C|\varsigma|^s$ whenever $z, z+\varsigma$ and $z-\varsigma$ are in B. For arbitrary $s > 0$, let m be the greatest integer satisfying $m < s$, and define Λ_s to be the class of functions f on B with $\partial^\alpha f \in \Lambda_{s-m}$ whenever $|\alpha| \le m$.

We shall also need some properties of the space BMOA consisting of holomorphic functions of bounded mean oscillation. We will use the conformally invariant BMOA norm. Let P_a be the *invariant Poisson kernel* defined by

$$P_a(z) = (1-|a|^2)^n / |1 - \langle z, a \rangle|^{2n},$$

and for $u \in L^1(\partial B; d\sigma)$ we define the *invariant Poisson integral* of u to be the function $\tilde{u}$ on B defined by $\tilde{u}(z) = \int u P_z \, d\sigma$. The BMOA *norm* of an invariant Poisson integral is defined by

$$\begin{aligned} \|\tilde{u}\|_{\mathrm{BMOA}} &= |\tilde{u}(0)| + \sup\left\{ \int |u - \tilde{u}(a)| P_a \, d\sigma \colon a \in B \right\} \\ &= |\tilde{u}(0)| + \sup\left\{ \int |u \circ \varphi_a - \tilde{u} \circ \varphi_a(0)| \, d\sigma \colon a \in B \right\}, \end{aligned}$$

where φ_a is any automorphism of B which interchanges 0 and a. Then BMOA $= \{f \in A^1_0 \colon \|f\|_{\mathrm{BMOA}} < \infty\}$ is a Banach space which is isomorphic to the dual of A^1_0, with the duality realized by the pairing $\langle f, g \rangle = \lim_{r \to 1^-} \int_{\partial B} f(z) \overline{g(rz)} \, d\sigma$ for $f \in A^1_0$ and $g \in$ BMOA. For more details on these matters we refer to [**3**].

The next theorem summarizes some known results concerning the boundary behavior of functions in $A^p_{q,s}$. Special cases of this result can be found in [**6** and **8**]. For the general case, see [**2**].

(1.2) THEOREM. *Let $0 < p < \infty$ and $q \ge 0$.*

(i) *For $s > (n+q)/p$, the space $A^p_{q,s}$ is contained in the Lipschitz class $\Lambda_{s-(n+q)/p}$.*

(ii) *For $s = (n+q)/p$, the space $A^p_{q,s}$ is contained in* BMOA.

The principal purpose of the present paper is to refine part (ii) of the above result, obtaining better boundary behavior when $p \le 1$.

In [**5**], Graham constructed an example in the unit ball in $\mathbf{C}^2$ of an unbounded function in $A^2_{0,1}$, thus showing that in general one cannot expect much improvement in (ii). The following may be viewed as a refinement of the result of Graham.

(1.3) THEOREM. *Assume $q \ge 0$, $s = (n+q)/p$ and $p > 1$. Then $A^p_{q,s}$ contains unbounded functions.*

PROOF. For $s > 0$, define a holomorphic function h_s on the unit disk Δ by

$$h_s(\lambda) = \frac{1}{(1-\lambda)^s} \left[\frac{1}{\lambda} \log \frac{1}{1-\lambda} \right]^{-1} = \sum a_{s,j} \lambda^j.$$

It follows that the coefficients $a_{s,j}$ satisfy

$$a_{s,j} \approx j^{s-1} / \log j \quad \text{as } j \to \infty, \tag{4}$$

and that

$$(5) \qquad \int_0^{2\pi} |h_s(re^{i\theta})|^p \, d\theta \approx \begin{cases} (1-r)^{1-sp}\left[\dfrac{1}{r}\log\dfrac{1}{1-r}\right]^{-p}, & sp > 1, \\ C_{s,p}, & sp = 1. \end{cases}$$

For these estimates, we refer to [**9**, pp. 93–96].

Assume that p, q and s are as in the statement of the theorem, and define a function f on B by $f(z) = \sum(1+j)^{-s} a_{s,j} z_1{}^j$. It is immediate from (4) that $\lim_{r\to 1^-} f(r, 0, \dots, 0) = \infty$, so f is unbounded on B. Moreover, $\mathcal{D}^s f = h_s \circ \pi_1$, where π_1 is the projection of $\mathbf{C}^n$ on the first coördinate. But it follows from Fubini's Theorem that $\|h_s \circ \pi_1\|_{p,q} = \|h_s\|_{p,q+n-1}$ (see [**12**, pp. 127–128]). Thus, whenever $sp > 1$, it follows from (5) and integration in polar coordinates that

$$\begin{aligned} \|f\|_{p,q,s}{}^p &= C \int |h_s(\lambda)|^p (1-|\lambda|^2)^{q+n-2} \, dA(\lambda) \\ &\approx \int_0^1 (1-r)^{1-sp} \left[\frac{1}{r}\log\frac{1}{1-r}\right]^{-p} (1-r^2)^{q+n-2} r \, dr \\ &\approx \int_0^1 (1-r)^{-1} \left[\frac{1}{r}\log\frac{1}{1-r}\right]^{-p} dr < \infty \end{aligned}$$

and so $f \in A^p_{q,s}$ in the case $sp > 1$. In the remaining case, $sp = 1$, we must have $n = 1$ and $q = 0$, and in this case it follows immediately from (5) that the function $\mathcal{D}^s f = h_s$ is in the Hardy space A^p_0, and we are done.

We are now in a position to formulate our main result. We will say that a C^1 curve $\gamma\colon (a,b) \to \partial B$ is *nontangential* if $\langle \gamma(t), \gamma'(t)\rangle \neq 0$ for every $t \in (a,b)$.

(1.4) THEOREM. *Let $f \in A^p_{q,s}$ with $s = (n+q)/p$ and $0 < p \leq 1$. Then f has a continuous extension to $\overline{B}$ (which we continue to denote by f), and moreover, if γ is any real analytic, nontangential curve in ∂B, then $f \circ \gamma$ is absolutely continuous.*

2. Proof of the main theorem. The continuity portion of Theorem 1.4 is based on the following representation formula which is a special case of Theorem 1.9 and Corollary 2.4 of [**2**].

(2.1) THEOREM. *There is a holomorphic function $g(\lambda) = \sum a_j \lambda^j$ on the unit disk Δ such that*

(i) *$a_j > 0$ for $j = 0, 1, \dots$;*

(ii) *$g(\lambda) = -\lambda^{-1}\log(1-\lambda) + F(\lambda)$ with $F \in \Lambda_1(\Delta)$;*

(iii) *the kernel $K(z,\varsigma) = g(\langle z, \varsigma\rangle)$ (where $\langle\ ,\ \rangle$ denotes the usual hermitian inner product on $\mathbf{C}^n$) has the reproducing property*

$$f(z) = \int_{\partial B} \mathcal{D}^n f(\varsigma) K(z,\varsigma) \, d\sigma(\varsigma)$$

whenever $f \in A^1_{0,n}$ and $z \in B$.

Note that the integral in (iii) is well defined since the function $\mathcal{D}^n f$ is in the Hardy class A^1_0, and hence it has a boundary value function in $L^1(\partial B; \sigma)$.

The next lemma provides a uniform bound for the BMOA norm of the functions $K(\cdot, \varsigma)$, $\varsigma \in \overline{B}$. The result is well known. We include the proof for the sake of completeness.

(2.2) LEMMA. *There is a constant* $C > 0$ *such that for any* $f \in \mathcal{O}(D)$ *we have* $\|\mathrm{Re}\, f\|_{\mathrm{BMOA}} \leq C\|\mathrm{Im}\, f\|_\infty$.

PROOF. We may assume without loss of generality that $f(0) = 0$. Write $u = \mathrm{Re}\, f$ and $v = \mathrm{Im}\, f$. We first observe that we have $\|u\|_{2,0} \leq \|v\|_{2,0}$. In the one variable case this follows from Parseval's Theorem, and the general case can be obtained from the slice integration formula (see [**12**, p. 15]):

$$\begin{aligned}\|u\|_{2,0}^2 = \int |u|^2\, d\sigma &= (2\pi)^{-1} \int\int |u(\varsigma e^{i\theta})|^2\, d\theta\, d\sigma(\varsigma) \\ &\leq (2\pi)^{-1} \int\int |v(\varsigma e^{i\theta})|^2\, d\theta\, d\sigma(\varsigma) = \|v\|_{2,0}^2.\end{aligned}$$

Thus for any $a \in B$ we have

$$\begin{aligned}\left[\int |u \circ \varphi_a - u(a)|\, d\sigma\right]^2 &\leq \int |u \circ \varphi_a - u(a)|^2\, d\sigma \leq \int |v \circ \varphi_a - v(a)|^2\, d\sigma \\ &\leq C\|v \circ \varphi_a - v(a)\|_\infty \leq 2C\|v\|_\infty.\end{aligned}$$

Taking the supremum over $a \in B$ gives the desired result.

(2.3) COROLLARY. *There is a constant* $C > 0$ *such that the kernel* $K(z, \varsigma)$ *of Theorem* 2.1 *satisfies* $\|K(\cdot, \varsigma)\|_{\mathrm{BMOA}} \leq C$ *for all* $\varsigma \in \overline{B}$.

PROOF. Since the function F of Theorem 2.1 is continuous on the closed unit disk, it follows that $F(\langle z, \varsigma\rangle)$ is continuous on $\overline{B} \times \overline{B}$, so it suffices to consider the logarithmic term of K. But the function $\lambda^{-1} \log((1 - \lambda)^{-1})$ has bounded imaginary part on the unit disk, so the logarithmic term of the kernel K has bounded imaginary part, and the result follows from Lemma 2.2.

(2.4) COROLLARY. *The mapping* $\varsigma \to K(\cdot, \varsigma)$ *is weak* continuous as a mapping from* $\overline{B}$ *into* BMOA.

PROOF. Let ς_j be a sequence of points in $\overline{B}$ with $\varsigma_j \to \varsigma$. By Corollary 2.3 and the Banach-Alaoglu Theorem, the sequence $K(\cdot, \varsigma_j)$ has a weak* convergent subsequence. To complete the proof, it is only necessary to show that any weak* convergent subsequence must converge to $K(\cdot, \varsigma)$. Thus, since $K(\cdot, \varsigma_j) \to K(\cdot, \varsigma)$ pointwise on B, it will suffice to prove that weak* convergence implies pointwise convergence. But if f_j is a sequence in BMOA which is weak* convergent to f, then by Cauchy's formula, for any $z \in B$,

$$f_j(z) = \int \frac{f_j(\varsigma)\, d\sigma(\varsigma)}{(1 - \langle z, \varsigma\rangle)^n} \to \int \frac{f(\varsigma)\, d\sigma(\varsigma)}{(1 - \langle z, \varsigma\rangle)^n} = f(z)$$

and the proof is complete.

(2.5) THEOREM. *Every function in* $A^1_{0,n}$ *has a continuous extension to* $\overline{B}$.

PROOF. In view of Corollary 2.4, the mapping $z \to \int g\overline{K(\cdot, z)}\, d\sigma = \int gK(z, \cdot)\, d\sigma$ is continuous on $\overline{B}$ for any $g \in A^1_0$. In particular, for $f \in A^1_{0,n}$, the mapping $z \to \int \mathcal{D}^n f K(z, \cdot)\, d\sigma$ is continuous on $\overline{B}$. In light of Theorem 2,1, the proof is complete.

We now turn to the study of the behavior of functions in $A^1_{0,n}$ along nontangential, real analytic curves in ∂B. We will use the elementary fact that such a curve can be realized locally as the transverse intersection of a (nonsingular) complex curve with ∂B. It will be necessary to know the behavior of the restrictions to such complex curves of functions in $A^1_{0,n}$, which is given by the following special case of Theorem 1.6 of [**1**]. Following [**1**], when D is a smoothly bounded domain in $\mathbf{C}^n$ and s is a nonnegative integer, we will denote by $A^p_{0,s}(D)$ the space of all holomorphic functions on D with partial derivatives up to order s in the Hardy class $H^p(D)$.

(2.6) THEOREM. *Let D be a bounded strictly pseudoconvex domain in $\mathbf{C}^n$ with smooth boundary, and let M be a one-dimensional complex submanifold of a neighborhood of $\overline{D}$ (so that $M \cap \overline{D}$ is a bordered Riemann surface). Then for any $f \in A^1_{0,n}(D)$ the restriction to M of the partial derivatives $\partial_j f$, $1 \leq j \leq n$, are in the Hardy class $H^1(M \cap D)$.*

We can now easily deduce the boundary behavior of functions in $A^1_{0,n}$ along nontangential, real analytic curves by using the preceding result to reduce the general case to the (known) one-dimensional case.

(2.7) THEOREM. *The restriction of any function in $A^1_{0,n}$ to any nontangential, real analytic curve in ∂B (which is well defined by Theorem* 2.5) *is absolutely continuous.*

PROOF. Let Γ be a nontangential, real analytic curve in ∂B and let $f \in A^1_{0,n}$. Since absolute continuity is a local property, it suffices to show that the restriction of f to a small neighborhood of any point in Γ is absolutely continuous. Let $p \in \Gamma$ be arbitrary. Choose a one-dimensional complex submanifold M of a neighborhood U of p such that $\Gamma \cap U = M \cap \partial B$, and a strictly convex domain D with smooth boundary such that $\overline{D} \subset U \cap \overline{B}$, and such that ∂D contains a neighborhood of p in Γ. We may also assume by choosing D sufficiently small that M meets ∂D transversally and that $M \cap D$ is simply connected. Thus, letting Δ denote the unit disk, there is a C^∞ diffeomorphism $\Phi\colon \overline{\Delta} \to M \cap \overline{D}$ which is holomorphic on Δ, and we may assume without loss of generality that $\Phi(1) = p$. It follows from Theorem 2.6 and the chain rule that $(f \circ \Phi)'$ is in the Hardy class $H^1(\Delta)$, so by the classical one variable result of Privalov (see [**4**, p. 42]), the restriction to the unit circle of $f \circ \Phi$ is absolutely continuous. But since Φ is a diffeomorphism of a neighborhood of 1 in the unit circle to a neighborhood of p in Γ, it follows that $f|_\Gamma$ is absolutely continuous in a neighborhood of p, and the theorem is proved.

The final link in the proof of Theorem 1.4 is provided by the next result which generalizes to the ball a pair of well-known inequalities in the disk due to Hardy and Littlewood (see [**4**, p. 87]) and to Littlewood and Paley [**10**]. The result is a special case of Theorems 5.12 and 5.13 of [**2**], and the proof will not be included here.

(2.8) THEOREM. *Assume that $0 < p \leq 1$, $q \geq 0$ and that $s = (n+q)/p$. Then $A^p_{q,s} \subset A^1_{0,n}$.*

Theorem 1.4 now follows immediately from Theorems 2.5, 2.7 and 2.8.

3. Some final remarks. In the preceding two sections we have restricted our attention to the unit ball for the sake of clarity. However, all of the results have direct analogues for smoothly bounded, strictly pseudoconvex domains. (In this case we obtain absolute continuity along any curve in the boundary which can be realized locally as the transverse intersection with the boundary of a complex curve.) For the proof, one must begin with a suitable analogue of Theorem 2.1. This can be obtained by starting with the Henkin-Ramirez integral formula [**7**, **11**] and integrating by parts n times to obtain an integral formula which reproduces a holomorphic function as a boundary integral of an nth derivative against a kernel with a logarithmic singularity.

The real analyticity condition on the curves in Theorem 1.4 can clearly be relaxed a bit: it suffices to assume that the curve in question is nontangential and is locally the boundary in an appropriate sense of a complex curve in B. However, it is certainly not the case that an arbitrary smooth, nontangential curve can be realized as the boundary of such a complex curve. We do not know whether the restriction to such a curve of a function in $A^1_{0,n}$ is necessarily absolutely continuous.

ADDED IN PROOF. After this paper was accepted, I discovered that the real analyticity hypothesis in Theorem 1.4 can be reduced to C^2. The proof of this fact will be included in a later paper.

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