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PRABIR ROY'S SPACE Δ AS A COUNTEREXAMPLE IN SHAPE THEORY

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ABSTRACT. In this note we use the space Δ in order to prove that a result concerning movability and mutational retractions cannot be transferred from the compact to the arbitrary metrizable case.

The space Δ was introduced by P. Roy **[6]** in order to prove that the equality $\operatorname{ind}(X) = \dim(X)$ does not hold in general for metrizable spaces. In particular he proved that $\operatorname{ind}(X) = 0$ but $\dim(X) = 1$. On the other hand, P. Nyikos **[5]** proved that Δ is not N-compact and then he showed that the relation "0-dimensional + realcompact $\Rightarrow$ N-compact" does not hold (0-dimensional here means "having a base of clopen sets").

The aim of this short note is to show that Δ can be used as a counterexample in shape theory too.

The following result is established in shape theory literature.

THEOREM 1. *Let X be a compact metrizable space, then:*

(a) *If all components of X are movable, X is movable (see* **[1**, *p.* 165]).

(b) *If $Y \subset X$ is a movable subcompactum of X such that Y is an intersection of open-closed sets of X, we have that Y is a fundamental retract (and consequently a mutational retract* **[4]**) *of X (see* **[8]**.)

(c) *If $Y \subset X$ is a mutational retract of X such that Y is an intersection of open-closed sets of X, we have that $\square(Y)$ is a retract of $\square(X)$, where $\square(Z)$ is the space of components of Z with the quotient topology.*

Part (c) is, in particular, an immediate consequence of Theorem (5.1) in **[1]** due to Borsuk (on p. 214).

Since Fox **[3]** extended the Borsuk shape theory to arbitrary metrizable spaces, some authors (see for example **[7, 4]**) transferred concepts and results from the Borsuk theory to the more general situation created by Fox. In this note we prove that Theorem 1 cannot be transferred to the case of arbitrary metrizable spaces, even if we restrict ourselves to the class U_0 of all spaces X such that $\operatorname{ind}(\square(X)) = 0$ and

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the decomposition of X into components is upper semicontinuous. In particular we have

Theorem 2. (*The movability used here is that introduced in* [7].)

At least one of the three following statements is true:

(a) *There exists a nonmovable metrizable space in the class U_0 such that all of its components are movable.*

(b) *There exists a metrizable space $X \in U_0$ and $Y \subset X$ a closed movable subset such that Y is an intersection of open-closed sets of X and Y is not a mutational retract of X.*

(c) *There exist a metrizable space $X \in U_0$ and $Y \subset X$, a mutational retract of X, such that Y is an intersection of open-closed sets of X and $\square(Y)$ is not a retract of $\square(X)$.*

Proof. Let us suppose that none of the three statements is true. Let us consider Roy's space Δ. As $\operatorname{ind}(\Delta) = 0$, we have that $\Delta = \square(\Delta)$; then all components of Δ are movable and the decomposition of Δ into components is upper semicontinuous. On the other hand, every closed subset of Δ is a movable intersection of open-closed subsets of Δ so it follows that every closed subset of Δ is a retract of Δ. Then every continuous map from a closed subset of Δ to the 0-dimensional sphere S^0 has a continuous extension to Δ and hence, see [**2**, p. 516], $\dim(\Delta) = 0$ which contradicts the fact, proved by Roy, that $\dim(\Delta) = 1$.

Remark. If there exist a metrizable N-compact space X with $\dim(X) \neq 0$, then we are able to prove that one of the statements (a) or (b) in Theorem 2 is true.

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