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CORRIGENDUM TO "A TOOL IN ESTABLISHING TOTAL VARIATION CONVERGENCE"

K. R. PARTHASARATHY AND TON STEERNEMAN

In Theorem 2.1 of [**1**] the measure μ on the Borel-sets of a complete and seperable metric space $\mathcal{X}$ was assumed to be σ-finite. The σ-finiteness condition has to be replaced by the condition that μ is a Radon-measure. In §4 we stated that the bounded continuous functions with support of finite diameter are dense in $\mathcal{L}_1$. This is not true in general, as can be seen by the following simple counterexample due to Professor Erik Thomas. Take $\mathcal{X} = [0,1]$ and $d(x,y) = |x-y|$. Define the σ-finite measure $\mu = \delta_0 + \sum_{n=1}^{\infty} \delta_{1/n}$, where δ_x denotes the Dirac-measure in the point x. The continuous, integrable functions are not dense in $\mathcal{L}_1$. For any $f \in C \cap \mathcal{L}_1$ we have $\int f\, d\mu = f(0) + \sum_{n=1}^{\infty} f(n^{-1}) < \infty$. Hence $f(n^{-1}) \to 0$ as $n \to \infty$. From the continuity of f it follows that $f(0) = 0$. Define $f^* \in \mathcal{L}_1$ by $f^* = I_{\{0\}}$. Note that $\int |f - f^*|\, d\mu \geq 1$ for any $f \in C \cap \mathcal{L}_1$. The problem arises because the point 0 does not have an open neighborhood of finite measure. The outer regularity is needed. So, we have to assume that μ is a Radon-measure, which is σ-finite. In this case the bounded, continuous, integrable functions with support of finite diameter are dense in $\mathcal{L}_1$.

A direct proof can easily be given, but it is also an easy consequence of a nice result (Proposition 2.1) of Thomas [**2**].

We wish to thank Professor E. Thomas for drawing our attention to his counterexample.

REFERENCES

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INDIAN STATISTICAL INSTITUTE, DELHI CENTRE, 7, S.J.S. SANSANWAL MARG, NEW DELHI 110016, INDIA

ECONOMETRICS INSTITUTE, UNIVERSITY OF GRONINGEN, P. O. BOX 80, 9700 AV GRONINGEN, THE NETHERLANDS

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