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ON TOTALLY REAL 3-DIMENSIONAL SUBMANIFOLDS OF THE NEARLY KAEHLER 6-SPHERE

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ABSTRACT. Let M be a compact 3-dimensional totally real submanifold of the nearly Kaehler 6-dimensional unit sphere. Let K be the sectional curvature function of M. Then, if $K > 1/16$, M is a totally geodesic submanifold (and $K \equiv 1$).

1. Introduction. On a 6-dimensional unit sphere $S^6(1)$, one can construct a *nearly Kaehler structure* J making use of the *Cayley number system*. We recall this construction in §3.

In this paper we study 3-dimensional *totally real submanifolds* of $S^6(1)$. The definition and the basic formulas for such manifolds are given in §4. N. Ejiri **[E]** proved the following: If a 3-dimensional totally real submanifold of $S^6(1)$ has constant curvature K, then $K = 1$ or $K = \frac{1}{16}$. The main purpose of this article is the following theorem, which will be proved in §5.

THEOREM. *Let M be a compact 3-dimensional totally real submanifold of $S^6(1)$. If all sectional curvatures K of M satisfy $\frac{1}{16} < K \leqslant 1$, then $K = 1$ on M.*

The proof is based on integral formulas for Riemannian manifolds of A. Ros **[R]**, which are given in §2.

2. Integral formulas. Let M be a compact Riemannian manifold, UM its unit tangent bundle, and UM_p the fiber of UM over a point p of M. We denote by dp, du, and du_p respectively the canonical measures on M, UM, and UM_p.

For any continuous function $f\colon UM \to \mathbf{R}$ one has

$$\int_{UM} f\,du = \int_M \left(\int_{UM_p} f\,du_p \right) dp.$$

Let T be any k-covariant tensor field on M. Then the integral formulas state that

$$\int_{UM} (\nabla T)(u, u, u, \ldots, u)\,du = 0$$

and

$$\int_{UM} \sum_i (\nabla T)(e_i, e_i, u, u, \ldots, u)\,du = 0,$$

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where $\{e_i\}_{i=1}^n$ is an orthonormal basis of TM, the tangent bundle over M, and ∇ denotes the Levi Civita connection of M.

3. The nearly Kaehler $S^6(1)$. Let $e_0, e_1, \ldots, e_7$ be the standard basis of $\mathbf{R}^8$. Then each point α of $\mathbf{R}^8$ can be written in a unique way as $\alpha = Ae_0 + x$, where $A \in \mathbf{R}$ and x is a linear combination of $e_1, \ldots, e_7$. α can be viewed as a Cayley number, and is called purely imaginary when $A = 0$. For any pair of purely imaginary x and y, we consider the multiplication $\cdot$ given by

$$x \cdot y = -\langle x, y\rangle e_0 + x \times y,$$

where $\langle\ ,\ \rangle$ is the standard scalar product on $\mathbf{R}^8$ and $x \times y$ is defined by the following multiplication table for $e_j \times e_k$:

j/k	1	2	3	4	5	6	7
1	0	e_3	$-e_2$	e_5	$-e_4$	e_7	$-e_6$
2	$-e_3$	0	e_1	e_6	$-e_7$	$-e_4$	e_5
3	e_2	$-e_1$	0	$-e_7$	$-e_6$	e_5	e_4
4	$-e_5$	$-e_6$	e_7	0	e_1	e_2	$-e_3$
5	e_4	e_7	e_6	$-e_1$	0	$-e_3$	$-e_2$
6	$-e_7$	e_4	$-e_5$	$-e_2$	e_3	0	e_1
7	e_6	$-e_5$	$-e_4$	e_3	e_2	$-e_1$	0

For two Cayley numbers $\alpha = Ae_0 + x$ and $\beta = Be_0 + y$, the Cayley multiplication $\cdot$, which makes $\mathbf{R}^8$ the Cayley algebra $\mathscr{C}$, is defined by

$$\alpha \cdot \beta = ABe_0 + Ay + Bx + x \cdot y.$$

We recall that the multiplication $\cdot$ of $\mathscr{C}$ is neither commutative nor associative. The set $\mathscr{C}_+$ of all purely imaginary Cayley numbers clearly can be viewed as a 7-dimensional linear subspace $\mathbf{R}^7$ of $\mathbf{R}^8$. In $\mathscr{C}_+$ we consider the unit hypersphere which is centered at the origin:

$$S^6(1) = \{x \in \mathscr{C}_+ \mid \langle x, x\rangle = 1\}.$$

Then the tangent space T_xS^6 of $S^6(1)$ at a point x may be identified with the affine subspace of $\mathscr{C}_+$ which is orthogonal to x.

On $S^6(1)$ we now define a $(1,1)$-tensor field J by putting

$$J_xU = x \times U,$$

where $x \in S^6(1)$ and $U \in T_xS^6$. This tensor field is well defined (i.e., $J_xU \in T_xS^6$) and determines an almost complex structure on $S^6(1)$, i.e. $J^2 = -\mathrm{Id}$, where Id is the identity transformation [F]. The compact simple Lie group G_2 is the group of automorphisms of $\mathscr{C}$ and acts transitively on $S^6(1)$ and preserves both J and the standard metric on $S^6(1)$ [**FI**].

Further, let G be the $(2,1)$-tensor field on $S^6(1)$ defined by

$$G(X, Y) = (\tilde{\nabla}_X J)Y, \tag{3.1}$$

where X, $Y \in \mathscr{X}(S^6)$ and where $\tilde{\nabla}$ is the Levi Civita connection on $S^6(1)$. This tensor field has the following properties:

$$G(X, X) = 0, \tag{3.2}$$

$$G(X,Y)+G(Y,X)=0, \tag{3.3}$$

$$G(X,JY)+JG(X,Y)=0, \tag{3.4}$$

$$(\tilde{\nabla}_X G)(Y,Z)=\langle Y,JZ\rangle X+\langle X,Z\rangle JY-\langle X,Y\rangle JZ, \tag{3.5}$$

$$\langle G(X,Y),Z\rangle+\langle G(X,Z),Y\rangle=0, \tag{3.6}$$

$$\begin{aligned}\langle G(X,Y),G(Z,W)\rangle &= \langle X,Z\rangle\langle Y,W\rangle-\langle X,W\rangle\langle Z,Y\rangle\\ &\quad+\langle JX,Z\rangle\langle Y,JW\rangle-\langle JX,W\rangle\langle Y,JZ\rangle,\end{aligned} \tag{3.7}$$

where $X, Y, Z, W \in \mathscr{X}(S^6)$ [S, G]. We recall that (3.2) means that the structure J is *nearly Kaehler*, i.e. $\forall X \in \mathscr{X}(S^6)$: $(\tilde{\nabla}_X J)X = 0$.

4. Totally real submanifolds of S^6. A Riemannian manifold M, isometrically immersed in S^6, is called a totally real submanifold of S^6 if $J(TM) \subseteq T^{\perp}M$, where $T^{\perp}M$ is the normal bundle of M in S^6. Then, we have $\dim M \leqslant 3$. In this paper we consider the case $\dim M = 3$. In [E] Ejiri proved that a 3-dimensional totally real submanifold of S^6 is orientable and minimal, and that $G(X,Y)$ is orthogonal to M for $X, Y \in \mathscr{X}(M)$. We denote the Levi Civita connection of M by ∇. The formulas of Gauss and Weingarten are then given by

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{4.1}$$

and

$$\tilde{\nabla}_X \xi = -A_\xi X + D_X \xi, \tag{4.2}$$

where X and Y are vector fields on M and ξ is a normal vector field on M. The second fundamental form h is related to A_ξ by

$$\langle h(X,Y),\xi\rangle = \langle A_\xi X, Y\rangle. \tag{4.3}$$

From (4.1) and (4.2) we find

$$D_X(JY) = G(X,Y) + J\nabla_X Y \tag{4.4}$$

and

$$A_{JX}Y = -Jh(X,Y). \tag{4.5}$$

If we denote the curvature tensors of ∇ and D by R and R^D, respectively, then the equations of Gauss, Codazzi, and Ricci are given by

$$\begin{aligned}R(X,Y,Z,W) &= \langle X,Z\rangle\langle Y,W\rangle-\langle X,W\rangle\langle Y,Z\rangle\\ &\quad+\langle h(X,Z),h(Y,W)\rangle-\langle h(X,W),h(Y,Z)\rangle,\end{aligned} \tag{4.6}$$

$$(\nabla h)(X,Y,Z) = (\nabla h)(Y,X,Z), \tag{4.7}$$

$$\langle R^D(X,Y)\xi,\mu\rangle = \langle [A_\xi, A_\mu]X, Y\rangle, \tag{4.8}$$

where $X, Y, Z, W \in \mathscr{X}(M)$, ξ and μ are normal vector fields, and ∇h is defined by $\nabla h(X,Y,Z) = D_X h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z)$.

From (4.5), (4.6), and (4.8) we obtain

$$\langle R^D(X,Y)JZ,JW\rangle = \langle R(X,Y)Z,W\rangle + \langle Z,X\rangle\langle Y,W\rangle - \langle Z,Y\rangle\langle X,W\rangle. \tag{4.9}$$

We also define $\nabla^2 h$ by

$$(\nabla^2 h)(X, Y, Z, W) = D_X(\nabla h)(Y, Z, W) - \nabla h(\nabla_X Y, Z, W) \\ -\nabla h(Y, \nabla_X Z, W) - h(Y, Z, \nabla_X W).$$

Then $\nabla^2 h$ satisfies the following equation:

$$(4.10)\quad (\nabla^2 h)(X, Y, Z, W) = (\nabla^2 h)(Y, X, Z, W) + R^D(X, Y)h(Z, W) \\ -h(R(X, Y)Z, W) - h(Z, R(X, Y)W).$$

5. Proof of the theorem. In this section, X, Y, $Z, \ldots$ denote tangent vector fields on a 3-dimensional totally real submanifold M of S^6, with second fundamental tensor h. Then by straightforward computations one may prove the following.

LEMMA 1.

(a) $\langle h(X, Y), JZ\rangle = \langle h(X, Z), JY\rangle$,

(b) $\langle(\nabla h)(X, Y, Z), JW\rangle = \langle(\nabla h)(X, Y, W), JZ\rangle - \langle h(Y, Z), G(X, W)\rangle - \langle h(Y, W), G(X, Z)\rangle$,

(c) $\langle(\nabla^2 h)(X, Y, Z, W), JU\rangle = \langle(\nabla^2 h)(X, Y, Z, U), JW\rangle - \langle(\nabla h)(X, Z, W), G(Y, U)\rangle + \langle(\nabla h)(X, Z, U), G(Y, W)\rangle - \langle h(Z, W), (\tilde\nabla G)(X, Y, U)\rangle + \langle h(Z, U), (\tilde\nabla G)(X, Y, W)\rangle + \langle(\nabla h)(Y, Z, U), G(X, W)\rangle - \langle \nabla h(Y, Z, W), G(X, U)\rangle$.

Now let $v \in UM_p$, $p \in M$. If e_2 and e_3 are orthonormal vectors in UM_p, orthogonal to v, then we can consider $\{e_2, e_3\}$ as an orthonormal basis of $T_v(UM_p)$. We remark that $\{v, e_2, e_3\}$ is an orthonormal basis of T_pM. We can choose e_3 such that $G(v, e_2) = Je_3$, $G(e_2, e_3) = Jv$, and $G(e_3, v) = e_2$ (cf. [E]). If we denote the Laplacian of $UM_p \cong S^2$ by Δ, then $\Delta f = e_2e_2f + e_3e_3f$, where f is a differentiable function on UM_p.

LEMMA 2.

$$3\int_{UM_p} \|h(v, v)\|^2 = 7\int_{UM_p} \langle h(v, v), Jv\rangle^2.$$

PROOF. Define a function f on UM_p, $p \in M$, by $f(v) = \langle h(v, v), Jv\rangle^2$. Using Lemma 1(a) and the minimality of M we can prove that

$$(\Delta f)(v) = -42f(v) + 18\|h(v, v)\|^2.$$

Integrating this completes the proof. □

LEMMA 3.

$$\int_{UM} \sum_{i=1}^{3} \langle(\nabla h)(e_i, v, v), Jv\rangle\langle G(v, e_i), h(v, v)\rangle = \frac{1}{3}\int_{UM} \langle h(v, v), Jv\rangle^2,$$

where $\{e_1, e_2, e_3\}$ is an arbitrary orthonormal basis of TM.

PROOF. Define the covariant tensor field T_1 by

$$T_1(X_1, X_2, \ldots, X_7) = \langle G(X_1, X_2), h(X_3, X_4)\rangle \langle h(X_5, X_6), JX_7\rangle.$$

Then

$$\int_{UM} \sum_{i=1}^{3} (\nabla T_1)(e_i, e_i, v, v, v, v, v, v) = 0. \tag{5.1}$$

To compute $\sum_{i=1}^{3} (\nabla T_1)(e_i, e_i, v, v, v, v, v, v)$, we can choose an orthonormal basis $\{e_1, e_2, e_3\}$ such that $e_1 = v$, $G(e_1, e_2) = Je_3$, $G(e_2, e_3) = Je_1$, and $G(e_3, e_1) = Je_2$. This is allowed because $\sum_{i=1}^{3}(\nabla T_1)(e_i, e_i, v, v, v, v, v, v)$ does not depend on the choice of the basis. (Similar argumentations will be used through this paper, without mentioning it.) Since

$$\sum_{i=1}^{3} \langle G(e_i, v), (\nabla h)(e_i, v, v)\rangle = \langle h(v, v), Jv\rangle, \tag{5.2}$$

we find that

$$\begin{aligned}\sum_{i=1}^{3} (\nabla T_1)(e_i, e_i, v, v, v, v, v, v) &= \|h(v, v)\|^2 - 2\langle h(v, v), Jv\rangle^2 \\ &\quad + \sum_{i=3}^{3} \langle (\nabla h)(e_i, v, v), Jv\rangle \langle G(e_i, v), h(v, v)\rangle.\end{aligned}$$

Lemma 3 then follows from (5.1) and Lemma 2. □

Because of Lemma 1(b), we have

$$\begin{aligned}\|(\nabla h)(v, v, v)\|^2 = &\sum_{i=1}^{3} \langle (\nabla h)(e_i, v, v), Jv\rangle^2 \\ &-2\sum_{i=1}^{3} \langle (\nabla h)(v, v, e_i), Jv\rangle \langle h(v, v), G(v, e_i)\rangle \\ &+\|h(v, v)\|^2 - \langle h(v, v), Jv\rangle^2.\end{aligned}$$

Integrating this and using Lemmas 2 and 3, we also obtain the following.

LEMMA 4.

$$\begin{aligned}&\int_{UM} \|(\nabla h)(v, v, v)\|^2 \\ &\quad = \int_{UM} \sum_{i=1}^{3} \langle (\nabla h)(e_i, v, v), Jv\rangle^2 + \frac{2}{3}\int_{UM} \langle h(v, v), Jv\rangle^2.\end{aligned}$$ □

LEMMA 5.

$$\begin{aligned}\int_{UM} \sum_{i=1}^{3} \langle (\nabla h)(e_i, v, v), Jv\rangle^2 = &\frac{9}{4}\int_{UM} \langle (\nabla h)(v, v, v), Jv\rangle^2 \\ &+ \frac{1}{12}\int_{UM} \langle h(v, v), Jv\rangle^2.\end{aligned}$$

PROOF. Define the function h on UM_p, $p \in M$, by $h(v) = \langle (\nabla h)(v, v, v), Jv \rangle^2$. Then we obtain

$$(\Delta h)(v) = -72h(v) + 2\|(\nabla h)(v, v, v)\|^2 + 30 \sum_{i=1}^{3} \langle (\nabla h)(e_i, v, v), Jv \rangle^2$$
$$-12 \sum_{i=1}^{3} \langle \nabla h(e_i, v, v), Jv \rangle \langle G(v, e_i), h(v, v) \rangle.$$

Integrating this over UM, using Lemmas 3 and 4, we obtain Lemma 5. □

Combining Lemmas 4 and 5 we obtain

LEMMA 6.

$$\int_{UM} \|(\nabla h)(v, v, v)\|^2$$
$$= \frac{9}{4} \int_{UM} \langle (\nabla h)(v, v, v), Jv \rangle^2 + \frac{3}{4} \int_{UM} \langle h(v, v), Jv \rangle^2. \quad □$$

LEMMA 7.

$$\int_{UM} \langle (\nabla h)(v, v, v), Jv \rangle^2 + \int_{UM} \langle (\nabla^2 h)(v, v, v, v), Jv \rangle \langle h(v, v), Jv \rangle = 0.$$

PROOF. Define T_2 by

(5.3) $$T_2(X_1, X_2, \ldots, X_6) = \langle h(X_1, X_2), JX_3 \rangle \langle h(X_4, X_5), JX_6 \rangle.$$

We know that

$$\int_{UM} (\nabla^2 T_2)(v, v, v, v, v, v, v, v) = 0.$$

Since

$$(\nabla^2 T_2)(v, v, v, v, v, v, v, v) = 2\langle (\nabla^2 h)(v, v, v, v), Jv \rangle \langle h(v, v), Jv \rangle$$
$$+2\langle (\nabla h)(v, v, v), Jv \rangle^2,$$

Lemma 7 is proved. □

LEMMA 8.

$$\int_{UM} \|(\nabla h)(v, v, v)\|^2 + \int_{UM} \sum_{i=1}^{3} \langle (\nabla^2 h)(e_i, e_i, v, v), Jv \rangle \langle h(v, v), Jv \rangle = 0.$$

PROOF. Define T_2 by (5.3). We know that

$$\int_{UM} \sum_{i=1}^{3} (\nabla^2 T_2)(e_i, e_i, v, v, v, v, v, v) = 0.$$

By a straightforward computation, we can prove that

$$\begin{aligned}(\nabla^2 T_2)(e_i, e_i, v, v, v, v, v, v) &= 2\langle(\nabla h)(v, v, v), Je_i\rangle^2 + 2\langle(\nabla^2 h)(e_i, e_i, v, v), Jv\rangle\langle h(v, v), Jv\rangle \\ &\quad + 4\langle(\nabla h)(e_i, v, v), G(e_i, v)\rangle\langle h(v, v), Jv\rangle \\ &\quad + 2\langle h(v, v), (\tilde{\nabla} G)(e_i, e_i, v)\rangle\langle h(v, v), Jv\rangle.\end{aligned}$$

Because of (3.1) we have

$$\sum_{i=1}^{3} \langle h(v, v), (\tilde{\nabla} G)(e_i, e_i, v)\rangle\langle h(v, v), Jv\rangle = -2\langle h(v, v), Jv\rangle^2. \tag{5.4}$$

Using (5.4) we thus obtain

$$\sum_{i=1}^{3} (\nabla^2 T_2)(e_i, e_i, v, v, v, v, v, v) = 2\langle(\nabla^2 h)(e_i, e_i, v, v), Jv\rangle\langle h(v, v), Jv\rangle + 2\|(\nabla h)(v, v, v)\|^2.$$

Integrating this completes the proof. □

LEMMA 9.

$$\int_{UM} \|(\nabla h)(v, v, v)\|^2 + \int_{UM} \langle(\nabla^2 h)(v, v, v, v), h(v, v)\rangle = 0.$$

PROOF. Define the covariant tensor field T_3 on M by

$$T_3(X_1, X_2, X_3, X_4) = \langle h(X_1, X_2), h(X_3, X_4)\rangle.$$

We know that

$$\int_{UM} (\nabla^2 T_3)(v, v, v, v, v, v) = 0.$$

Because

$$(\nabla^2 T_3)(v, v, v, v, v, v) = 2\langle(\nabla^2 h)(v, v, v, v), h(v, v)\rangle + 2\|\nabla h(v, v, v)\|^2,$$

Lemma 9 follows. □

LEMMA 10.

$$\frac{3}{4}\int_{UM} \langle(\nabla h)(v, v, v), Jv\rangle^2 - \frac{1}{12}\int_{UM} \langle h(v, v), Jv\rangle^2 + \int_{UM} R(v, A_{Jv}v, A_{Jv}v, v) = 0.$$

PROOF. Define the function g on UM_p, $p \in M$, by

$$g(v) = \langle h(v, v), Jv\rangle\langle(\nabla^2 h)(v, v, v, v), Jv\rangle.$$

By a computation using (4.3), (4.4), the minimality of M, (5.2), and several times Lemma 1, we can prove that

$$\begin{aligned}(\Delta g)(v) = &-72g(v) + 30\langle h(v,v), (\nabla^2 h)(v,v,v,v)\rangle \\ &-24\langle h(v,v), Jv\rangle^2 + 30\|h(v,v)\|^2 \\ &-48\sum_{i=1}^{3}\langle(\nabla h)(v,v,e_i), Jv\rangle\langle h(v,v), G(v,e_i)\rangle \\ &-18R(v, A_{Jv}v, A_{Jv}v, v) \\ &+8\sum_{i=1}^{3}\langle h(v,v), Jv\rangle\langle(\nabla^2 h)(e_i,e_i,v,v), Jv\rangle.\end{aligned}$$

Integrating this over UM, using Lemmas 7, 9, 2, 3, and 8, we obtain

$$\begin{aligned}&72\int_{UM}\langle(\nabla h)(v,v,v), Jv\rangle + 30\int_{UM}\langle h(v,v), Jv\rangle^2 \\ &\quad -38\int_{UM}\|(\nabla h)(v,v,v)\|^2 - 18\int_{UM}R(v, A_{Jv}v, A_{Jv}v, v) = 0.\end{aligned}$$

Lemma 6 then completes the proof. □

Making use of Lemmas 2 and 4 we can rewrite Lemma 10 as follows.

LEMMA 11.

$$\begin{aligned}&\frac{3}{4}\int_{UM}\langle(\nabla h)(v,v,v), Jv\rangle^2 \\ &\quad + \int_{UM}\left[R(v, A_{Jv}v, A_{Jv}v, v) - \frac{1}{16}\left(\|A_{Jv}v\|^2 - \langle A_{Jv}v, v\rangle^2\right)\right] = 0.\end{aligned}$$ □

PROPOSITION. *If M is a 3-dimensional compact totally real submanifold of S^6 and if all sectional curvatures K of M satisfy $K \geqslant \frac{1}{16}$, then*

(1) $\langle(\nabla h)(v,v,v), Jv\rangle = 0$, and

(2) $R(v, A_{Jv}v, A_{Jv}v, v) = \frac{1}{16}(\|A_{Jv}v\|^2 - \langle A_{Jv}v, v\rangle^2)$, for all $p \in M$ and $v \in UM_p$.

PROOF. Under the assumptions of the proposition,

$$R(v, A_{Jv}v, A_{Jv}v, v) - \tfrac{1}{16}\left(\|A_{Jv}v\|^2 - \langle A_{Jv}v, v\rangle^2\right) \geqslant 0,$$

and Lemma 11 then implies the proposition. □

Now we can prove the theorem. Suppose that all sectional curvatures K of M satisfy $K > \frac{1}{16}$. By the proposition, we have

$$R(v, A_{Jv}v, A_{Jv}v) = \tfrac{1}{16}\left(\|A_{Jv}v\|^2 - \langle A_{Jv}v, v\rangle^2\right). \tag{5.5}$$

If there exist a unit vector $v \in UM_p$, $p \in M$, such that $A_{Jv}v$ is not parallel to v, then v and $A_{Jv}v$ determine a plane of which the sectional curvature equals $\frac{1}{16}$ according to (5.5). This is a contradiction. Therefore all $A_{Jv}v$ are parallel to v. This implies because of (4.5) that

$$\|h(v,v)\|^2 = \langle h(v,v), Jv\rangle^2 \tag{5.6}$$

for all $v \in UM$. (5.6) together with Lemma 2 implies that $h = 0$, i.e., that M is totally geodesic. This completes the proof.

6. Examples. 1. Let $M = \{x \in S^6 \mid x = x_1e_1 + x_1e_3 + x_5e_5 + x_7e_7\}$, and let i be the inclusion map from M into S^6. Then (M, i) is a 3-dimensional totally real and totally geodesic submanifold of S^6.

2. In **[E]** Ejiri announced that he can construct a totally real immersion of $S^3(1/16)$ into S^6.

REFERENCES

[E] N. Ejiri, *Totally real submanifolds in a 6-sphere*, Proc. Amer. Math. Soc. **83** (1981), 759–763.

[F] A. Fröhlicher, *Zur Differentialgeometrie der komplexen Strukturen*, Math. Ann. **129** (1955), 50–95.

[FI] F. Fukami and S. Ishihara, *Almost Hermitian structure on S^6*, Tôhoku Math. J. **7** (1955), 151–156.

[G] A. Gray, *Minimal varieties and almost Hermitian submanifolds*, Michigan Math. J. **12** (1965), 273–287.

[R] A. Ros, *A characterization of seven compact Kaehler submanifolds by holomorphic pinching*, Ann. of Math. (2) **121** (1985), 377–382.

[S] K. Sekigawa, *Almost complex submanifolds of a 6-dimensional sphere*, Kōdai Math. J. **6** (1983), 174–185.

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