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CORRIGENDUM TO "PARTITIONS AND DIAMOND"

PIERRE MATET

(Communicated by Thomas J. Jech)

We recall that given an uncountable cardinal κ, $\Diamond_\kappa$ asserts the existence of a family $s_\alpha \subseteq \alpha$, $\alpha < \kappa$, such that the set $\{\alpha < \kappa \colon A \cap \alpha = s_\alpha\}$ is stationary in κ for all $A \subseteq \kappa$.

It occurred to us that the implication (iv)$\to$(i) in Proposition 4 of [**4**] needs not hold for $\kappa > \omega_1$. In that case, a slight modification of the original argument yields that (i)–(iii) in the proposition are indeed equivalent, and that they are equivalent to this stronger form of (iv): There exists a family $Z_\alpha \in (\kappa)^2$, $\alpha < \kappa$, such that the diagonal intersection $\Delta\{Z_\alpha(h(\alpha)) \colon \alpha < \kappa\}$ is stationary for every $h \in 2^\kappa$.

The incorrect proof relied on the claim, p. 39 of [**1**], that assuming κ is regular, $\Diamond_\kappa$ follows from the existence of a sequence $s_\alpha \subseteq \alpha$, $\alpha < \kappa$, such that whenever $A \subseteq \kappa$, there is an infinite α with $A \cap \alpha = s_\alpha$. That this is indeed the case when $\kappa = \omega_1$ was shown by Devlin in [**2**]. However, this cannot be true in general, as it would imply that $\Diamond_\kappa$ holds whenever κ is regular and $\Diamond_\lambda$ holds for some uncountable cardinal $\lambda < \kappa$. It is nevertheless possible to generalize Devlin's result as follows:

PROPOSITION. *Let λ, κ be infinite cardinals with $2^\lambda \geq \kappa$. Assume there are P_α, $\alpha < \kappa$, such that each P_α is a collection of size $\leq |\alpha|$ of subsets of α, and that for every $A \subseteq \kappa$, there is an $\alpha \geq \lambda$ with $A \cap \alpha \in P_\alpha$. Then $\Diamond_\kappa$ holds.*

PROOF. Let P_α, $\alpha < \kappa$, be as in the statement of the proposition. By a well-known result of Kunen [**3**], it is enough to show that κ is regular and there exist Q_α, $\alpha < \kappa$, such that each Q_α is a collection of size $\leq |\alpha|$ of subsets of α, and the set $\{\alpha \colon B \cap \alpha \in Q_\alpha\}$ is stationary in κ for all $B \subseteq \kappa$. Define functions i, j from κ to κ by letting $i(\alpha) = \lambda\alpha$ and $j(\alpha) = 2\alpha$. Set $N = \kappa - j[\kappa]$. For each $\alpha \in [\lambda, \kappa)$, denote by Q_α the collection of all those subsets D of α such that there are $\beta < \lambda$ and $H \in P_{\alpha+\beta}$ with $D = j^{-1}[H \cap \alpha]$. Fix $B \subseteq \kappa$, and let C be a closed unbounded subset of κ. Let c_β, $0 < \beta < \rho$, be the increasing enumeration of the set $C \cap i[\kappa - 2]$, and put $c_0 = 0$. Choose $E_\beta \subset N$, $\beta < \rho$, with the following properties: $E_\beta \subseteq N \cap [c_\beta, c_\beta + \lambda)$, and $E_\beta \neq H \cap N \cap [c_\beta, c_\beta + \lambda)$ whenever $H \in P_\alpha$ with $\alpha \in [c_\beta + \lambda, c_{\beta+1})$. Then set $A = j[B] \cup \left(\bigcup_{\beta<\rho} E_\beta\right)$. Select $\alpha \geq \lambda$ with $A \cap \alpha \in P_\alpha$. It is easily verified that $B \cap c_\beta \in Q_{c_\beta}$, where β is such that $\alpha \in [c_\beta, c_\beta + \lambda)$. It only remains to show that κ is regular. First note that $2^\mu = \kappa$ holds for every cardinal $\mu \in [\lambda, \kappa)$, as the set $\bigcup_{\alpha \geq \lambda} Q_\alpha$ has size κ. Thus $\operatorname{cf} \kappa = \operatorname{cf}(2^\mu) > \mu$ for all $\mu \in [\lambda, \kappa)$, and consequently $\operatorname{cf} \kappa = \kappa$.

We remark that Theorem 4 (where (b) should read $\Diamond^-_{\lambda^+}(\lambda^+ - \lambda)$) of [**5**] is the special case of our result when $\kappa = \lambda^+$. Also, a straightforward modification of the proof of the proposition yields the implication c)$\to$a) of Theorem 3 of [**5**].

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Finally, we would like to point out that in [**4**], Proposition 3 easily follows from Proposition 1, by the following remark: given a cardinal κ and a family $A_\alpha \in [\kappa]^\kappa$, $\alpha < \kappa$, with the property that $A_\alpha \subseteq A_\beta$ whenever $\beta < \alpha$, there is a $B \in [\kappa]^\kappa$ such that $|B - A_\alpha| < \kappa$ for all α.

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FREIE UNIVERSITÄT BERLIN, INSTITUT FÜR MATHEMATIK II, ARNIMALLEE 3, 1000 BERLIN 33, WEST GERMANY