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NOTE ON A PAPER OF J. L. PALACIOS

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ABSTRACT. J. L. Palacios claimed [2] that the author's paper [1] contained errors. This note refutes those claims by showing that Palacios misunderstood [1] and adopted assumptions different from those of [1].

J. L. Palacios [2] claimed to find errors in my article [1]. The purpose of this note is my assertion that Palacios' claims are false and based on a misunderstanding of [1].

Let $\{X_n, n \geq 1\}$ be a sequence of real-valued random variables on the probability space $(\mathcal{R}^\infty, \mathcal{B}^\infty, P)$ where $\mathcal{R}^\infty$ and $\mathcal{B}^\infty$ are the usual product space and product σ-field. Let $\mathcal{I}, \mathcal{T}$, and $\mathcal{E}$ be the strict sense versions of the invariant, tail, and exchangeable σ-fields. In [1] the class of sets was considered for which $\sigma^{-1}A = A$ (P) for all permutations σ that permute at most a finite number of positive integer coordinates. What this clearly means is that instead of the strict sense σ-field $\mathcal{E}$ the σ-field

$$\mathcal{E}_1 = \{A\colon P(A \Delta \sigma^{-1} A) = 0 \text{ for every } \sigma\}$$

is to be considered, where Δ is symmetric difference. The crucial question is: does $\mathcal{E}_1$ necessarily contain all null sets in B^∞? The answer is No. To see this, first note that any null set N in $\mathcal{E}_1$ must have $\sigma^{-1}N$ also null for each σ. This need not hold for all null sets in B^∞ as the following example shows: let $x = (1,0,1,0,\dots)$ and $y = (0,1,0,1,\dots)$ be alternating sequences each with probability $\frac{1}{2}$, and let $z = (0,1,1,0,\dots)$ where $z = \sigma x$ for σ the permutation interchanging the first two coordinates of x. Then $N = \{z\}$ is null, but $\sigma^{-1}N$ has probability $\frac{1}{2}$.

Palacios' error consists in implicitly assuming that $\mathcal{E}_1$ contains all null sets in B^∞. This is a strong assumption, indeed, it is my nonsingularity condition [1, top of p. 314]. Thus, when Palacios says [2, p. 139, §4] that he has not used this condition, he is unaware that he has implicitly adopted it. Each of Palacios' criticisms of [1] can be traced to this misunderstanding.

REFERENCES

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