PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 102, Number 4, April 1988

SOME PROPERTIES OF MEASURES OF NONCOMPACTNESS IN PARANORMED SPACES

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(Communicated by John B. Conway)

ABSTRACT. This paper presents new properties of important measures of noncompactness in paranormed spaces. Using these properties some fixed point theorems for multivalued mappings in general topological vector spaces are obtained in a straightforward way.

1. Introduction. It is well known that in the fixed point theory in locally convex spaces the relation $\gamma(\operatorname{co} A) = \gamma(A)$ (γ is a measure of noncompactness and $\operatorname{co} A$ is the convex hull of A) is of great importance. But, if the considered topological vector space is not a locally convex space we cannot prove, in general, that $\gamma(\operatorname{co} A) = \gamma(A)$. We shall prove in this paper that under some additional conditions we can prove the inequality $\gamma(\operatorname{co} A) \leq \varphi[\gamma(A)]$, where $\varphi\colon [0,\infty) \to [0,\infty)$ and $\gamma \in \{\alpha, \beta\}$, where α is the Kuratowski measure of noncompactness and β the inner measure of noncompactness.

Using these results, fixed point theorems for multivalued mappings in general topological vector spaces will be obtained in a straightforward way. Fixed point theorems for single-valued and multivalued mappings in not necessarily locally convex topological vector spaces are proved by many authors [**3**–**8**, **10**–**12**, **16**]. More details about fixed point theorems in not necessarily locally convex topological vector spaces can be found in [**6**].

2. Notations and definitions. Let E be a vector space over the real or complex number field and $\| \ \|^*\colon E \to [0,\infty)$ so that the following conditions are satisfied: (1) $\|x\|^* = 0 \Leftrightarrow x = 0$. (2) $\|-x\|^* = \|x\|^*$ for every $x \in E$. (3) $\|x+y\|^* \leq \|x\|^* + \|y\|^*$ for every $x, y \in E$. (4) If $\|x_n - x_0\|^* \to 0$ and $t_n \to t_0$, then $\|t_n x_n - t_0 x_0\|^* \to 0$. Then the pair $(E, \| \ \|^*)$ is said to be a paranormed space. It is well known that E is a metrizable topological vector space in which the fundamental system of neighborhoods of zero is given by the family $\mathscr{V} = \{V_r \mid r > 0\}$, where

$$V_r = \{x \mid x \in E,\ \|x\|^* < r\}.$$

DEFINITION 1. Let $(E, \| \ \|^*)$ be a paranormed space, M a nonempty subset of E and $\varphi\colon (0,\infty) \to (0,\infty)$. The set M is said to be of Z_φ-type if, for every $r > 0$,

$$\operatorname{co}(V_r \cap (M - M)) \subset V_{\varphi(r)}. \tag{1}$$

In [**4**] we introduced the following

DEFINITION 2. Let E be a topological vector space, M a nonempty subset of E, and V be the fundamental system of neighborhoods of zero in E. The set

Received by the editors December 19, 1986.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 47H10; Secondary 54H25.
Key words and phrases. Fixed points, measure of noncompactness, paranormed spaces.

M is said to be of Zima type if for every $V \in \mathscr{V}$ there exists $U \in \mathscr{V}$ so that $\operatorname{co}(U \cap (M - M)) \subset V$.

It is obvious that a subset M of a paranormed space, which is of Z_φ-type, is of Zima type also, if the mapping φ is such that $\inf_{r>0} \varphi(r) = 0$.

We shall give an example of the space E, the subset M, and the mapping φ so that the set M is of Z_φ-type.

EXAMPLE. Let $E = S(0,1)$, where $S(0,1)$ is the space of all equivalence classes of finite, real measurable functions on $[0,1]$ (m is the Lebesgue measure) with paranorm

$$\|x\|^* = \int_0^1 \frac{|x(t)|}{1+|x(t)|} m(dt) \quad \text{for every } x \in S(0,1), \{x(t)\} \in x.$$

It is easy to see that $\| \ \|^*$ is a paranorm and let $s > 0$. Further, let $K_s \subset S(0,1)$ be defined in the following way: $K_s = \{x \mid x \in S(0,1),\ |x(t)| \le s, \text{ for every } t \in [0,1]\}$. It can be proved [**4**] that

$$\|r(x_1 - x_2)\|^* \le (1+2s)r\|x_1 - x_2\| \quad \text{for every } r > 0 \tag{2}$$

and every $x_1, x_2 \in K_s$. From (2) it follows that K_s is of Z_φ-type, where the mapping φ is defined by $\varphi(u) = (1+2s)u$ for every $u > 0$. V. Klee introduced the following [**11**].

DEFINITION 3. Let E be a topological vector space and $\mathscr{V}$ be the fundamental system of neighborhoods of zero in E. A subset $K \subset E$ ($K \neq \emptyset$) is said to be admissible if for every compact subset A of K and every $U \in \mathscr{V}$ there exists a continuous mapping $h\colon A \to K$ so that the following conditions are satisfied:

(1) $\dim \operatorname{Lin} h(A) < \infty$ (Lin = the linear hull);

(2) for every $x \in A$, $x - h(x) \in U$.

If $K = E$ we say that the space E is admissible.

An open question is the following: Does there exist a convex, nonadmissible subset of a topological vector space? An example of a nonconvex, compact nonadmissible subset of l^2 is given in [**12**].

Every closed and convex subset of a locally convex topological vector space is admissible. The spaces L^p $(0 < p < 1)$ and $S(0,1)$ are admissible nonlocally convex topological vector spaces.

It can be proved [**4**] that every subset of Zima type is an admissible subset. Hence, every subset which is of Z_φ-type is admissible if the mapping φ is such that $\inf_{r>0} \varphi(r) = 0$.

THEOREM. *Let E be a topological vector space and K a nonempty closed, convex and admissible subset of E, $b \in K$, $Y \subset K$ an, in K, closed neighborhood of b and let $F\colon Y \to \mathscr{R}(K)$ be a compact mapping, where $\mathscr{R}(K)$ is the family of all nonempty, closed and convex subsets of K. If for every $x \in \partial_K Y$ and every $t \in (0,1)$ $x \notin tF(x) + (1-t)b$, then there exists a fixed point of the mapping F.*

This result is obtained in [**10**].

Let (E,d) be a metric space and A be a bounded subset of E. The inner measure of noncompactness $\beta(A)$ of the set A is defined by

$$\beta(A) = \inf\{\varepsilon \mid \varepsilon > 0,\ A \text{ has a finite } \varepsilon\text{-net } \{x_1, x_2, \ldots, x_n\} \subset A\}.$$

The Kuratowski measure of noncompactness $\alpha(A)$ of the set A is defined by

$$\alpha(A) = \inf\{\varepsilon \mid \varepsilon > 0, \text{ there exists a finite cover } \{B_j\}_{j\in J} \text{ of } A \text{ such that } \operatorname{diam} B_j < \varepsilon, \text{ for every } j \in J\}.$$

3. Measures of noncompactness in paranormed spaces. First, we shall prove two lemmas.

LEMMA 1. *Let $(E, \|\ \|^*)$ be a paranormed space, K a nonempty bounded and convex subset of E which is of Z_φ-type where φ is a right continuous mapping from $[0,\infty)$ into $[0,\infty)$. Then, for every $A \subset K$, $\beta(\operatorname{co} A) \le \varphi(\beta(A))$.*

PROOF. We shall prove that for every $t > 0$ there exists a finite $\varphi(\beta(A))+t$-net of the set $\operatorname{co} A$. Since φ is right continuous there exists $u > 0$ so that $\varphi(\beta(A)+u)+u \le \varphi(\beta(A)) + t$. For such u there exists $\{x_1, x_2, \dots, x_n\} \subset A$ so that

$$A \subset \bigcup_{i=1}^{n} L(x_i, \beta(A) + u) \qquad (L(a,s) = \{x \mid x \in E,\ \|x - a\|^* < s\}). \tag{3}$$

From the precompactness of the set $\operatorname{co}\{x_1, x_2, \dots, x_n\}$ it follows that there exists $B = \{u_1, u_2, \dots, u_r\} \subset \operatorname{co}\{x_1, x_2, \dots, x_n\}$ so that

$$\operatorname{co}\{x_1, x_2, \dots, x_n\} \subset \bigcup_{j=1}^{r} L(u_j, u). \tag{4}$$

We shall prove that

$$\operatorname{co} A \subset \bigcup_{k=1}^{r} L(u_k, \varphi(\beta(A)) + t). \tag{5}$$

Let $y \in \operatorname{co} A$. Then there exist $y_i \in A$, $s_i \ge 0$ $(i \in \{1, 2, \dots, m\})$, $\sum_{i=1}^m s_i = 1$ so that $y = \sum_{i=1}^m s_i y_i$. Since $y_i \in A$ $(i \in \{1, 2, \dots, m\})$ from (3) it follows that there exists $x_{n(i)}$ $(n(i) \in \{1, 2, \dots, n\})$ so that $\|y_i - x_{n(i)}\|^* < \beta(A) + u$.

If $x = \sum_{i=1}^m s_i x_{n(i)}$, then $x \in \operatorname{co}\{x_1, x_2, \dots, x_n\}$, and from (4) we obtain that there exist $u_k \in B$ such that $\|x - u_k\|^* < u$. We have that

$$\begin{aligned} y - x &= \sum_{i=1}^{m} s_i y_i - s_i x_{n(i)} \\ &= \sum_{i=1}^{m} s_i (y_i - x_{n(i)}) \subset \operatorname{co}(U_{\beta(A)+u} \cap (A - A)) \subset U_{\varphi(\beta(A)+u)}. \end{aligned}$$

Hence, we have

$$y - u_k = y - x + x - u_k \in U_{\varphi(\beta(A)+u)} + U_u \subset U_{\varphi(\beta(A)+u)+u} \subset U_{\varphi(\beta(A))+t}.$$

This means that (5) holds.

LEMMA 2. *Let $(E, \|\ \|^*)$ be a paranormed space, K a nonempty, bounded and convex subset of E which is of Z_φ-type, where $\varphi\colon [0,\infty) \to [0,\infty)$ is a continuous mapping. Then for every $A \subset K$:*

$$\alpha(\operatorname{co} A) \le \varphi^2(\alpha(A)). \tag{6}$$

PROOF. Let $t > 0$. We prove that there exist $C_i \subset \operatorname{co} A$ ($i \in \{1, 2, \dots, m\}$) so that $\operatorname{co} A \subset \bigcup_{i=1}^m C_i$ and for every $j \in \{1, 2, \dots, m\}$ $C_j - C_j \subset U_{\varphi(\varphi(\alpha(A)))+t}$.

Since φ is a continuous mapping, it follows that there exists $u > 0$ such that $\varphi(\varphi(\alpha(A) + u)) + u \le \varphi(\varphi(\alpha(A))) + t$.

Let $\{D_1, D_2, \dots, D_n\}$ be a cover of A such that $D_i - D_i \subset U_{\alpha(A)+u}$, for every $i \in \{1, 2, \dots, n\}$. We can suppose that $D_i \subset A$, for every $i \in \{1, 2, \dots, n\}$ and so

$$D_i - D_i \subset U_{\alpha(A)+u} \cap (A - A) \subset U_{\alpha(A)+u} \cap (K - K).$$

From this we have that

$$\operatorname{co} D_i - \operatorname{co} D_i \subset \operatorname{co}(U_{\alpha(A)+u} \cap (K - K)) \subset U_{\varphi(\alpha(A)+u)} \cap (K - K).$$

Let

$$S = \left\{ v = (v_1, v_2, \dots, v_n),\ \sum_{i=1}^n v_i = 1,\ v_i \ge 0,\ i \in \{1, 2, \dots, n\} \right\}$$

and

$$B_v = v_1 \operatorname{co} D_1 + v_2 \operatorname{co} D_2 + \cdots + v_n \operatorname{co} D_n, \qquad v \in S.$$

Then

$$\begin{aligned} \operatorname{co} A &= \operatorname{co}(\operatorname{co} D_1 \cup \operatorname{co} D_2 \cup \cdots \cup \operatorname{co} D_n) \\ &= \bigcup_{v \in S} (v_1 \operatorname{co} D_1 + \cdots + v_n \operatorname{co} D_n) = \bigcup_{v \in S} B_v. \end{aligned}$$

Further, for every $v \in S$,

$$\begin{aligned} B_v - B_v &= v_1(\operatorname{co} D_1 - \operatorname{co} D_1) + v_2(\operatorname{co} D_2 - \operatorname{co} D_2) + \cdots + v_n(\operatorname{co} D_n - \operatorname{co} D_n) \\ &\subset v_1(U_{\varphi(\alpha(A)+u)} \cap (K - K)) + v_2(U_{\varphi(\alpha(A)+u)} \cap (K - K)) \\ &\quad + \cdots + v_n(U_{\varphi(\alpha(A)+u)} \cap (K - K)) \end{aligned}$$

and so

$$\begin{aligned} B_v - B_v &\subset \operatorname{co}(U_{\varphi(\alpha(A)+u)} \cap (K - K)) \subset U_{\varphi(\varphi(\alpha(A)+u))} \\ &\subset U_{\varphi(\varphi(\alpha(A)))+t-u}. \end{aligned}$$

Since $\operatorname{co} A$ is bounded, there is $M > 0$ such that

$$|v_i| < M,\ i \in \{1, 2, \dots, n\} \Rightarrow v_1 \operatorname{co} D_1 + v_2 \operatorname{co} D_2 + \cdots + v_n \operatorname{co} D_n \subset U_u.$$

Let for $v \in R^n$: $\|v\| = \max_{i \in \{1, 2, \dots, n\}} |v_i|$. We shall prove that

(7) $$v, w \in S,\ \|v - w\| < M \Rightarrow B_v - B_w \subset U_{\varphi(\varphi(\alpha(A)))+t}.$$

Let $x = \sum_{i=1}^n v_i x_i \in B_v$, $x_i \in \operatorname{co} D_i$, $i \in \{1, 2, \dots, n\}$, $v = (v_i) \in S$ and $y = \sum_{i=1}^n w_i y_i \in B_w$, $y_i \in B_w$, $y_i \in \operatorname{co} D_i$, $i \in \{1, 2, \dots, n\}$, $w = (w_i) \in S$. Then

$$\begin{aligned} x - y &= \sum_{i=1}^n (v_i - w_i) x_i + \sum_{i=1}^n w_i (x_i - y_i) \\ &\in \sum_{i=1}^n (v_i - w_i) \operatorname{co} D_i + U_{\varphi(\varphi(\alpha(A)))+t-u} \\ &\subset U_u + U_{\varphi(\varphi(\alpha(A)))+t-u} \subset U_{\varphi(\varphi(\alpha(A)))+t} \end{aligned}$$

and hence (7) is proved.

Let $S = \bigcup_{j=1}^m S_j$ so that $\operatorname{diam} S_j < M$, for every $j \in \{1, 2, \dots, m\}$. If C_j, $j \in \{1, 2, \dots, m\}$ are defined by $C_j = \bigcup_{v \in S_j} B_v$ then $\operatorname{co} A = \bigcup_{j=1}^m C_j$ and $C_j - C_j \subset U_{\varphi(\varphi(\alpha(A)))+t}$.

4. Fixed point theorems. Let E be a topological vector space, K a nonempty subset of E, A a partially ordered set with the partial ordering $\leq$, and ψ a system of subsets of $\overline{\text{co}}\, K$ with the following properties:

$$M \in \psi \Rightarrow (\overline{M} \in \psi,\ \text{co}\, M \in \psi,\ M \cup \{u\} \in \psi\ (u \in K),\ N \in \psi\ (N \subset M)).$$

DEFINITION 4. Let γ be a mapping of ψ into A. The mapping γ is said to be a φ-measure of noncompactness on K, where φ is a mapping of A into A if the following conditions are satisfied:
(1) $\gamma(\overline{M}) = \gamma(M) = \gamma(M \cup \{u\})$ $(M \in \psi,\ u \in K)$.
(2) $\gamma(\text{co}\, M) \leq \varphi(\gamma(M))$ $(M \in \psi)$.

THEOREM 1. *Let E be a topological vector space, K a closed and convex subset of E with the property that every closed and convex subset of K is admissible, $u \in K$, $U \subset K$ an, in K, closed neighborhood of u, $F\colon U \to \mathscr{R}(K)$ an upper semicontinuous mapping and γ a φ-measure of noncompactness on K so that the following conditions hold*:
(1) $x \notin tF(x) + (1-t)u$ $(x \in \partial_K U$ *and* $t \in (0,1))$.
(2) *For every* $A \subset U$: $A, F(A) \in \psi$ *and* $\gamma(A) \leq \varphi(\gamma(F(A))) \Rightarrow \overline{F(A)}$ *is compact.*
(3) *For every* N_1, $N_2 \in \psi$, $N_1 \subset N_2$: $\gamma(N_1) \leq \gamma(N_2)$.
Then there exists $x \in U$ *so that* $x \in F(x)$.

PROOF. Let $\mathscr{S} = \{S \mid S \in E,\ S = \overline{\text{co}}\ S,\ u \in S,\ F(U \cap S) \subset S\}$. Then $\mathscr{S} \neq \emptyset$ and let $S_0 = \bigcap_{S \in \mathscr{S}} S$, $S_1 = \overline{\text{co}}(\{u\} \cup F(U \cap S_0))$. It is easy to see that $S_1 \in \mathscr{S}$ and so $S_0 \subset S_1$. Since $S_1 \subset S_0$ it follows that

$$S_0 = \overline{\text{co}}(\{u\} \cup F(U \cap S_0)).$$

From this we obtain that

$$U \cap S_0 \subset \overline{\text{co}}(\{u\} \cup F(U \cap S_0))$$

and hence using the properties of the measure γ we obtain

$$\gamma(U \cap S_0) \leq \gamma[\overline{\text{co}}(\{u\} \cup F(U \cap S_0))] \leq \varphi(\gamma(F(U \cap S_0))).$$

This implies the compactness of the set $\overline{F(U \cap S_0)}$.

Let $\widetilde{K} = K \cap S_0$, $Y = U \cap S_0$, and $\widetilde{F} = F \mid Y$. We have that $\partial_{\widetilde{K}} Y \subset \partial_K U$ and so $x \notin t\widetilde{F}(x) + (1-t)u$, for all $t \in (0,1)$ and $x \in \partial_{\widetilde{K}} Y$. Since $\widetilde{F}$ is a compact mapping and $\widetilde{K}$ is admissible from the theorem of Jerofsky it follows that there exists $x \in Y$ so that $x \in \widetilde{F}(x)$.

Using Lemma 2 we shall prove the following Corollary to Theorem 1.

COROLLARY. *Let $(E, \|\ \|^*)$ be a complete paranormed space, φ a continuous mapping from the interval $[0,\infty)$ into $[0,\infty)$ so that $\varphi(0) = 0$, K a bounded, closed and convex subset of E which is of Z_φ-type, M the family of all nonempty subsets of K, $u \in K$, $U \subset K$ an, in K, closed neighborhood of u, and $F\colon U \to \mathscr{R}(K)$ an upper semicontinuous mapping so that the conditions* (1) *and* (2) *from Theorem* 1 *are satisfied for $\gamma = \alpha$, $\psi = M$. Then there exists $x \in U$ so that $x \in F(x)$.*

PROOF. From Definition 1, it follows that every subset of the set K is of Z_φ-type and since φ is continuous and $\varphi(0) = 0$ it follows $\inf_{r>0} \varphi(r) = 0$. This implies that every closed and convex subset of K (in fact, every subset of K) is admissible. Further, the Kuratowski measure of noncompactness satisfies condition (3) of Theorem 1 and so all the conditions of Theorem 1 are satisfied which implies the existence of an element $x \in U$ such that $x \in F(x)$.

REMARK. Since the inner measure of noncompactness β is not a monotone measure of noncompactness, we cannot take in the above Corollary that $\gamma = \beta$.

From the proof of Theorem 1 it is obvious that the following result holds.

THEOREM 2. *Let E be a topological vector space, K a closed and convex subset of E with the property that every closed and convex subset of K is admissible, $u \in K$, $U \subset K$ an, in K, closed neighborhood of u, and $F\colon U \to \mathscr{R}(K)$ an upper semicontinuous mapping with the following properties*:

(1) *$x \notin tF(x) + (1-t)u$ $(x \in \partial_K U$ and $t \in (0,1))$.*

(2) *For every $S \subset E$,*

$$S = \overline{\mathrm{co}}(\{u\} \cup F(U \cap S)) \Rightarrow \overline{F(U \cap S)} \text{ is compact.}$$

Then there exists $x \in U$ so that $x \in Fx$.

Let us give an example of a not locally convex topological vector space E and $K \subset E$ so that every subset of the set K is admissible.

EXAMPLE [**12**]. Let (X, A, m) be a complete measure space such that A does not contain any atom of measure ∞. Further, let F be the real or complex number field and

$S(X, \mathsf{A}, m) = \{f \mid f\colon X \to \mathsf{F},\ f \text{ is } \mathsf{A}\text{-measurable}\}/\{f \mid f\colon X \to \mathsf{F},\ f = 0 \text{ a.e.}\}$.

$O = \{\varphi \mid \varphi\colon R^+ \to R^+,\ \varphi \text{ is continuous and monotone increasing, } \varphi(t) = 0 \Leftrightarrow t = 0\}$.

$L_\varphi = \{f \mid f \in S(X, \mathsf{A}, m), \omega_\varphi(f) = \int_X \varphi(|f|)\, dm < \infty\}$.

$SL_\varphi = \bigcup_{a>0} a \cdot L_\varphi$, $KL_\varphi = \bigcap_{a>0} a \cdot L_\varphi$ $(\varphi \in O)$, $|\cdot|_\varphi\colon SL_\varphi \to R^+$ is the mapping defined by $|f|_\varphi = \inf\{k \mid k > 0,\ \omega_\varphi(f/k) \le k\}$.

Then $(SL_\varphi, |\cdot|_\varphi)$ is an (F)-space and KL_φ is a closed subspace of SL_φ. If $g \in KL_\varphi$ then every subset of the set K is admissible where

$$K = \{f \mid f \in KL_\varphi, |f(t)| \le g(t), t \in X\}.$$

Let E denote a locally convex topological vector space, $\mathscr{U}$ a base of absolutely convex neighborhoods of zero in E, and for any $A \subset E$ [**9**],

$$Q(A) = \{U \mid U \in \mathscr{U},\ A \subset B + U, \text{ for some totally bounded subset } B \text{ of } E\}.$$

Let $S \subset E$. A mapping $F\colon S \to 2^E$ is condensing iff [**9**]: $A \subset S$, A is bounded and not totally bounded $\Rightarrow Q(A) \subsetneqq Q(F(A))$.

From Theorem 2 we obtain a generalization of Theorem 2 from [**17**].

COROLLARY. *Let E be a locally convex topological vector space, G a nonempty subset of E such that $\overline{G}$ is convex and quasi-complete. If $F\colon \overline{G} \to \mathscr{R}(E)$ is an upper semicontinuous mapping, which is condensing and such that*

(a) *$F(G)$ is bounded.*

(b) *There is an $\omega \in G$ such that for all $y \in \partial(\overline{G})$ and $z \in F(y)$: $z - \omega \ne m(y - \omega)$ for $m > 1$.*

Then there exists $x \in \overline{G}$ so that $x \in Fx$.

If γ is a not necessarily monotone φ-measure of noncompactness we can prove the following fixed point theorem.

THEOREM 3. *Let E be a topological vector space, G a nonempty closed and convex subset of E so that every compact and convex subset of K is admissible, γ a φ-measure of noncompactness on G and $F\colon G \to \mathscr{R}(G)$ an upper semicontinuous mapping so that, for every $A \subset G$, $\gamma(A) \leq \varphi(\gamma(F(A))) \Rightarrow \overline{F(A)}$ is compact. Then there exists $x \in G$ so that $x \in Fx$.*

PROOF. For every $z \in G$ there exists $Z \subset G$ so that $Z = \overline{\text{co}}(F(Z) \cup \{z\})$, which can be easily proved. The rest of the proof is as the proof of Theorem 1.

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