PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 102, Number 4, April 1988

A NONMOVABLE SPACE WITH MOVABLE COMPONENTS

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(Communicated by Dennis Burke)

ABSTRACT. In this paper we construct a nonmovable complete metric space of which every component is movable. This construction is based on P. Roy's famous example of a complete metric space X which has inductive dimension $\operatorname{ind} X = 0$ and covering dimension $\dim X = 1$.

Introduction. K. Borsuk [**1**, Theorem 7.1] proved that if every component of a compactum X is movable, then X is movable. In this paper we construct a nonmovable complete metric space every component of which is movable. The construction is based on P. Roy's famous example [**5**] of a complete metric space X which has inductive dimension $\operatorname{ind} X = 0$ and covering dimension $\dim X = 1$.

Actually for any metric space X with $\dim X = 1$ we construct in §I–IX a related solenoidal-type space Z and a morphism $\mathbf{p} = \{p_n\}$ from Z into inverse system of the dyadic solenoid. We show that $p_1 \colon Z \to S^1$ induces a nontrivial homomorphism on the first Čech cohomology $H^1(p) \colon H^1(S^1) \to H^1(Z)$ and that this implies Z is nonmovable. So in the case X is Roy's example we get that Z is nonmovable but each component is movable (each component is a point). The reader is referred to Spanier [**6**] as a general reference for the Hopf theorem.

We also present a second example in §XI which is a totally disconnected, nonmovable space. This example is easier to construct but fails to have inductive dimension 0 and to be completely metrizable like the solenoidal Roy example mentioned above.

We were led to this question by a paper of M. A. Moron [**4**]. He showed that at least one of three results in shape theory for compacta did not carry over to the noncompact case. However, he could not ascertain which one failed in the noncompact case.

I. Construction of a solenoidal space. Let X be a metric space with the covering dimension $\dim X = 1$. There exists a closed subset $A \subset X$ and a map $f \colon (X, A) \to (I, \dot{I})$ that induces a nontrivial homomorphism $H^1(f) \colon H^1(I, \dot{I}) \to H^1(X, A)$. Let $f_i \colon (X_i, A_i) \to (I, \dot{I})$, $i = 0, 1$, be two copies of f. We denote by Y the space obtained from the topological sum of X_0 and X_1 by the natural identification of $f_0^{-1}(1)$ and $f_1^{-1}(1)$. We assume that X_0 and X_1, and so also $B_0 = f_0^{-1}(0)$, $B_1 = f_1^{-1}(0)$ and $B = B_0 \cup B_1$ are subsets of Y. One can prove that the map $g \colon (Y, B) \to (I, \dot{I})$, given by

$$g(y) = \begin{cases} \frac{1}{2} f_0(y) & \text{for } y \in X_0, \\ 1 - \frac{1}{2} f_1(y) & \text{for } y \in X_1, \end{cases}$$

Received by the editors January 28, 1987.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 54F43; Secondary 54F45.
Key words and phrases. Movability, component, dimension, and shape.

induces a nontrivial homomorphism $H^1(g)\colon H^1(I,\dot{I}) \to H^1(Y,B)$. Let $s\colon Y \to Y$ be the symmetry function that assigns to a point $x \in X_0$ the corresponding point $s(x) \in X_1$. There is a homeomorphism h defined on the Cantor set C onto itself such that the quotient space $C \times I/(h(c),0) \sim (c,1)$, where $c \in C$, is the dyadic solenoid. We define a quotient space

$$Z = C \times Y/(h(c),b) \sim (c,s(b)),$$

where $c \in C$ and $b \in B_0$. Let $h\colon C \times Y \to C \times Y$ be defined by $h(c,y) = (h(c),y)$ for any $c \in C$ and $y \in Y$.

II. Now we will describe a morphism $\mathbf{p} = \{p_n\}$ from Z into the dyadic solenoid's inverse system $S^1 \overset{2}{\leftarrow} S^1 \overset{2}{\leftarrow} S^1 \leftarrow \cdots$ (see [**3**, p. 122]).

Let $\{\mathcal{C}_n\}_{n=0}^{\infty}$ be the standard system of open coverings of the Cantor set C, i.e., $\mathcal{C}_n = \{C_n^k \mid k = 1,\ldots,2^n\}$ consists of nonempty, open, pairwise disjoint subsets of C, each element of $\mathcal{C}_n$ (a copy of C) contains exactly two elements of $\mathcal{C}_{n+1}$ and $\{\mathcal{C}_n\}_{n=0}^{\infty}$ is cofinal in the family of all open coverings of C. W may assume that the homomorphism h induces a cyclic permutation on the set $\mathcal{C}_n$, $h(C_n^k) \in \mathcal{C}_n$ for each $C_n^k \in \mathcal{C}_n$, $n > 0$.

The set Z is the sum of 2^n copies of $C_n^k \times Y$. The (continuous) map $p_n\colon Z \to S^1$ restricted to $C_n^k \times Y$ is the composition of the projection $C_n^k \times Y \to Y$, the map $g\colon Y \to I$ and an inclusion $i_k\colon I \to S^1$ (2^n segments $i_k(I)$ have disjoint interiors). We may assume that for each n the following diagram commutes:

$$\begin{array}{ccccc} & & Z & & \\ & \overset{p_n}{\swarrow} & & \overset{p_{n+1}}{\searrow} & \\ S^1 & & \overset{2}{\leftarrow} & & S^1 \end{array}$$

III. We will prove that $p = p_1\colon Z \to S^1$ induces a nontrivial homomorphism $H^1(p)\colon H^1(S^1) \to H^1(Z)$. Let $Z = Z_1 \cup Z_2$, where Z_i is the image of $C_1^i \times Y$, where $C_1^i \in \mathcal{C}_1$, under the projection $C \times Y \to Z$. Let T, T_0, and T_1 be the images of $C \times B$, $C_1^1 \times B_0$, and $C_1^1 \times B_1$, respectively, under this projection. So $T = Z_1 \cap Z_2$. The map $p\colon Z \to S^1$ maps Z_i onto a segment $I_i \subset S^1$. The intersection $I_1 \cap I_2$ is S^0 (a two-point set). Let us consider the following diagram:

$$\begin{array}{ccccccc} H^0(T) & \xrightarrow{\partial^*} & H^1(Z,T) & \xrightarrow{j^*} & H^1(Z) & \longrightarrow & \cdots \\ \uparrow{\scriptstyle H^0(p)} & & \uparrow{\scriptstyle H^1(p)} & & \uparrow{\scriptstyle H^1(p)} & & \\ H^0(S^0) & \xrightarrow{\partial^*} & H^1(S^1,S^0) & \xrightarrow{j^*} & H^1(S^1) & \longrightarrow & \cdots \end{array}$$

induced by the map p. By the Mayer-Vietoris sequence we can identify $H^1(Z,T)$ with $H^1(Z_1,T) \oplus H^1(Z_2,T)$ and $H^1(S^1,S^0)$ with $H^1(I_1,S^0) \oplus H^1(I_2,S^0)$ (the isomorphisms are induced by inclusions). Then $H^1(p)\colon H^1(S^1,S^0) \to H^1(Z,T)$ we can identify with $H^1(p \mid Z_1) \oplus H^1(p \mid Z_2)$, where $H^1(p \mid Z_i)\colon H^1(I_i,S_0) \to H^1(Z_iT)$. Let α be the image of the generator of $H^1(I_1,S^0)$. To prove that $H^1(p)\colon H^1(S^1) \to H^1(Z)$ is nontrivial, it is enough to prove that $j^*(a)$ is nontrivial. Assume that $j^*(a) = 0$. Then $a = \partial^*(d)$ for some $d \in H^0(T)$.

IV.

REMARK. Let $a \in H^1(Z,T)$. Assume that $a = \partial^*(d)$ for some $d \in H^0(T)$, where $\partial^*\colon H^0(T) \to H^1(Z,T)$ is the coboundary operator. Then there exists a covering $\mathcal{V}$ of Z such that

(i) $d = j^*_{\mathcal{V}}(d_{\mathcal{V}})$ for some $d_{\mathcal{V}} \in H^0(N(\mathcal{V} \mid T))$,

(ii) $a = j^*_{\mathcal{V}}(a_{\mathcal{V}})$ for some $a_{\mathcal{V}} \in H^1(N(\mathcal{V}), N(\mathcal{V} \mid T))$,

(iii) $\partial^*(d_{\mathcal{V}}) = a_{\mathcal{V}}$,

where

$$\begin{array}{ccc} H^0(N(\mathcal{V} \mid T)) & \xrightarrow{\partial^*} & H^1(N(\mathcal{V}), N(\mathcal{V} \mid T)) \\ \Big\downarrow j^*_{\mathcal{V}} & & \Big\downarrow j^*_{\mathcal{V}} \\ H^0(T) & \xrightarrow{\partial^*} & H^1(Z,T) \end{array}$$

and the ∂^* are coboundary operators and the $j^*_{\mathcal{V}}$ are natural projections.

V. System of neighborhoods. By $\{\mathcal{U}_\alpha\}_{\alpha\in\Lambda}$ we denote the system of all open symmetric (i.e., $s(U) \in \mathcal{U}_\alpha$ if $U \in \mathcal{U}_\alpha$) coverings of Y of order 1. For any covering $\mathcal{U}_\alpha$ of Y we define a system $\{\mathcal{V}_\varphi\}_{\varphi\in\Phi_\alpha}$ by

$$\mathcal{V}_\varphi = \{C' \times U \mid U \in \mathcal{U}_\alpha \text{ and } C' \in \mathcal{C}_{\varphi(U)}\},$$

where Φ_α is the set of all symmetric functions φ (i.e., $\varphi(U) = \varphi(s(U))$) from $\mathcal{U}_\alpha$ into the set of all nonnegative integers. Let $\Phi = \bigcup\{\Phi_\alpha \mid \alpha \in \Lambda\}$. The system $\{\mathcal{U}_\alpha\}_{\alpha\in\Lambda}$ is cofinal in the system of all open coverings of Y and the system $\{\mathcal{V}_\varphi\}_{\varphi\in\Phi}$ is cofinal in the system of all open coverings of $C \times Y$.

Now, let A be any closed subset of Y. Suppose that the nerve $N(\mathcal{V})$ is a component of the nerve $N(\mathcal{V}_\varphi|C \times A)$, $\varphi \in \Phi_\alpha$. Then there is a k an $C^0 \in \mathcal{C}_k$ and a subfamily $\mathcal{U}^0 \subset \mathcal{U}_\alpha$ such that

$$\mathcal{V} = \{C' \times U \mid U \in \mathcal{U}^0, C' \in \mathcal{C}_{\varphi(U)}, \text{ and } C' \subset C^0\},$$

so $\bigcup \mathcal{V} = \bigcup \mathcal{U}^0 \times C^0$. Let us observe that

$$h(\mathcal{V}) = \{C' \times U \mid U \in \mathcal{U}^0, C' \in \mathcal{C}_{\varphi(U)}, \text{and } C' \subset h(C^0)\},$$

and $N(h(\mathcal{V}))$ is a component of $N(\mathcal{V}_\varphi|C \times A)$ which we denote by $hN(\mathcal{V})$.

VI. Lemma. Let $\mathcal{V}_\varphi$ be a covering of $C \times Y$, $\varphi \in \Phi_\alpha$, induced by $\mathcal{U}_\alpha$ (see §V). Symmetric components (with respect to s) of the simplicial complex

$$L = N(\mathcal{V}_\varphi) - N(\mathcal{V}_\varphi|C \times B)$$

"join" $C \times B_0$ and $C \times B_1$. By Σ_i, $i = 0, 1$, we denote the set of all 1-simplexes that have a vertex in $N(\mathcal{V}_\varphi|C \times B_i)$ and that are in symmetric components of L. We can consider the group G_i of all functions from Σ_i into the integer group as a subgroup (by natural embedding) of $H^1(N(\mathcal{V}_\varphi), N(\mathcal{V}_\varphi|C \times B))$.

Let us denote by Δ_i, $i = 0, 1$, the set of all components of $N(\mathcal{V}_\varphi|C \times B_i)$. We can consider the group $H^0(N(\mathcal{V}_\varphi|C \times B_i))$ as the group of all functions from Δ_i into the integer group. By Δ_i' we denote the set of components of $N(\mathcal{V}_\varphi|C \times B_i)$ that contains a vertex of some simplex of Σ_i. Let $\Delta_i'' = \Delta_i - \Delta_i'$. The group of all functions from Δ_i' (resp. Δ_i'') into the integer group we denote by H_i' (resp. H_i''). Let us observe that $\partial^*(H^0(N(\mathcal{V}_\alpha|C \times B_i))) = \partial^*(H_i') \subset G_i$ and $\partial\omega^*(H_i'') = 0$, where

$$\partial^*\colon H^0(N(\mathcal{V}_\varphi|C \times B)) \to H^1(N(\mathcal{V}_\varphi), N(\mathcal{V}_\varphi|C \times B))$$

is the coboundary operator. The group $H^0(N(\mathcal{V}_\varphi|C \times B)$ is the direct sum

$$H^0(N(\mathcal{V}_\varphi|C \times B_0)) \oplus H^0(N(\mathcal{V}_\varphi|C \times B_1)).$$

The map $s\colon C \times Y \to C \times Y$, given by $s(c,y) = (c, s(y))$, induces a symmetric isomorphism

$$H^0(s)\colon H^0(N(\mathcal{V}_\varphi|C \times B)) \to H^0(N(\mathcal{V}_\varphi|C \times B))$$

in the following way. Any element $d \in H^0(N(\mathcal{V}_\alpha|C \times B))$ is represented by a function d from $\Delta = \Delta_0 \cup \Delta_1$ into the integer group. The element $H^0(s)(d)$ can be represented as a map from Δ into the integer group such that $H^0(s)(d)(K) = d(s(K))$ for any component K of $N(\mathcal{V}_\varphi|C \times B)$. We have

$$H^0(s)(H^0(N(\mathcal{V}_\varphi|C \times B_0))) = H^0(N(\mathcal{V}_\varphi|C \times B_1)).$$

Now, let $d = d_0 + d_1$ be an element of $H^0(N(\mathcal{V}_\varphi|C \times B)$, where

$$d_i \in H^0(N(\mathcal{V}_\varphi|C \times B_i)).$$

Each d_i is represented by a map d_i from Δ_i into the integer group. Suppose that $\delta^*(d) = 0$. Let $\partial_0 \in \Delta_0'$ and $\partial_1 = s(\partial_0) \in \Delta_1'$. Then there is a (symmetric) simplicial path in $N(\mathcal{V}_\varphi)$ with end points $V_0 \in \partial_0$ and $V_1 = s(V_0) \in \partial_1$ such that all other simplexes of this path are in L. Since $\partial^*(d) = 0$ it follows that $\partial^*(d_0)$ and $\partial^*(d_1)$ take opposite values on the first and the last simplex of this path. So d_0 and d_1 take the same value on V_0 and V_1, respectively, and so on ∂_0 and ∂_1, respectively. So we obtain the following

LEMMA. *Let $d = d_0 + d_1$, where $d_i \in H^0(N(\mathcal{V}_\varphi|C \times B_i)$, $i = 0, 1$. If $\partial^*(d) = 0$, then $d_0(K) = d_1(s(K))$ for every $K \in \Delta_0'$ (and so for every $K \in \Delta_1'$).*

VII. Notation. We consider (Z,T) as the sum $(Z_1,T) \cup (Z_2,T)$ of two copies of $(C \times Y,\ C \times B)$, where $Z_1 \cap Z_2 = T$. Let $j_1\colon (C \times Y, C \times B) \to (Z,T)$ and $j_2\colon (C \times Y, C \times B) \to (Z,T)$ be corresponding embeddings, so

$$j_i(C \times Y, C \times B) = (Z_i, T) \quad \text{for } i = 1,2.$$

We may assume that

$$j_1|C \times B_1 = j_2|C \times B_1 \quad \text{and} \quad j_1|C \times B_0 = j_2 h|C \times B_0,$$

where $h\colon C \times Y \to C \times Y$ is the homeomorphism given by $h(c,y) = (h(c),y)$ for any $c \in C$ and $y \in Y$. Let $\mathcal{V}_\varphi$ be the covering of $C \times Y$ described in V. By $\widetilde{\mathcal{V}}_\varphi$ we denote the open covering of Z that satisfies two conditions:

$$j_1(\mathcal{V}_\varphi) = \widetilde{\mathcal{V}}_\varphi \cap Z_1 \quad \text{and} \quad j_2(\mathcal{V}_\varphi) = \widetilde{\mathcal{V}}_\varphi \cap Z_2.$$

Let us observe that the embeddings j_1 and j_2 induce isomorphisms of complexes

$$j_i\colon (N(\mathcal{V}_\varphi), N(\mathcal{V}_\varphi|C \times B) \to (N(\widetilde{\mathcal{V}}_\varphi|Z_i), N(\widetilde{\mathcal{V}}_\varphi|T)) \quad \text{for } i = 1,2.$$

The system of all such coverings $\{\widetilde{\mathcal{V}}_\varphi\}_{\alpha\in\Phi}$ of Z is cofinal in the system of all open coverings of Z.

VIII. We have assumed (§III) that $a \in \partial^*(H^0(T))$ i.e. $a = \partial^*(d)$ for some $d \in H^0(T)$. By the Remark (§IV) there exists a covering $\widetilde{\mathcal{V}}_\varphi$ of Z that satisfies the conditions (i), (ii), (iii). We have

$$\begin{aligned} H^0(N(\widetilde{\mathcal{V}}_\varphi|T)) &\xrightarrow{\partial^*} H^1(N(\widetilde{\mathcal{V}}_\varphi), N(\widetilde{\mathcal{V}}_\varphi|T)) \\ &= H^1(N(\widetilde{\mathcal{V}}_\varphi|Z_1), N(\widetilde{\mathcal{V}}_\varphi|T)) \oplus H^1(N(\widetilde{\mathcal{V}}_\varphi|Z_2), N(\widetilde{\mathcal{V}}_\varphi|T)). \end{aligned}$$

So there exists $a_\nu \in H^1(N(\widetilde{\mathcal{V}}_\varphi), N(\widetilde{\mathcal{V}}_\varphi|T)$ and $d_\nu \in H^0(N(\widetilde{\mathcal{V}}_\varphi|T))$ such that a is the image of a_ν under j_ν^* and $a_\nu = \partial^*(d_\nu)$. We may assume (see §III) that

$$a_\nu \in H^1(N(\widetilde{\mathcal{V}}_\varphi|Z_1), N(\widetilde{\mathcal{V}}_\varphi|T)) \subset H^1(N(\widetilde{\mathcal{V}}_\varphi), N(\widetilde{\mathcal{V}}_\varphi|T)).$$

The embeddings j_1 and j_2 induce isomorphisms

$$H^1(j_i)\colon H^1(N(\widetilde{\mathcal{V}}_\varphi|Z_i), N(\widetilde{\mathcal{V}}_\varphi|T)) \to H^1(N(\mathcal{V}_\varphi), N(\mathcal{V}_\varphi|C \times B)) \quad \text{for } i = 1, 2,$$

and isomorphisms

$$H^0(j_i)\colon H^0(N(\widetilde{\mathcal{V}}_\varphi|T)) \to H^0(N(\mathcal{V}_\varphi|C \times B)) \quad \text{for } i = 1, 2.$$

We know that

$$H^0(N(\widetilde{\mathcal{V}}_\varphi|T)) = H^0(N(\widetilde{\mathcal{V}}_\varphi|T_0)) \oplus H^0(N(\widetilde{\mathcal{V}}_\varphi|T_1))$$

and

$$H^0(N(\mathcal{V}_\varphi|C \times B)) = H^0(N(\mathcal{V}_\varphi|C \times B_0)) \oplus H^0(N(\mathcal{V}_\varphi|C \times B_1))$$

and that $H^0(j_i)$ maps $H^0(N(\widetilde{\mathcal{V}}_\varphi|T_0))$ onto $H^0(N(\mathcal{V}_\varphi|C \times B_0))$ and $H^0(N(\widetilde{\mathcal{V}}_\varphi|T_1))$ onto $H^0(N(\mathcal{V}_\varphi|C \times B_1))$.

Let us observe that

$$H^0(j_1)|H^0(N(\widetilde{\mathcal{V}}_\varphi|T_1)) = H^0(j_2)|H^0(N(\widetilde{\mathcal{V}}_\varphi|T_1))$$

and

$$H^0(j_1)|H^0(N(\widetilde{\mathcal{V}}_\varphi|T_0)) = H^0(h)H^0(j_2)|H^0(N(\widetilde{\mathcal{V}}_\varphi|T_0)),$$

where $H^0(h)$ is an automorphism on $H^0(N(\mathcal{V}_\varphi|C \times B_0))$ induced by $h\colon C \times Y \to C \times Y$.

Let $d_\nu = d_{\nu,0} + d_{\nu,1}$ where $d_{\nu,i} \in H^0(N(\widetilde{\mathcal{V}}_\varphi|T_i))$, for $i = 0, 1$. Then

$$H^0(j_1)(d_{\nu,0}) = H^0(h)H^0(j_2)(d_{\nu,0}) = H^0(h)(d_0),$$

where $d_0 = H^0(j_2)(d_{\nu,0})$ and

$$H^0(j_1)(d_{\nu,1}) = H^0(j_2)(d_{\nu,1}) = d_1.$$

We can assume (see §III) that

$$H^1(j_1)(a_\nu) \in G_0 \subset H^1(N(\mathcal{V}_\varphi), N(\mathcal{V}_\varphi|C \times B))$$

may be represented as a function that takes the value 1 on each 1-simplex of Σ_0 and that

$$H^1(j_2)(a_\nu) \in H^1(N(\mathcal{V}_\varphi), N(\mathcal{V}_\varphi|C \times B))$$

is trivial. Let us denote by d^1 an element of $H^0(N(\mathcal{V}_\varphi|C \times B_0))$ that takes the value 1 on each component K of $N(\mathcal{V}_\varphi|C \times B_0)$.

We have

$$\partial^*(H^0(j_1)(d_\nu)) = H^1(j_1)(a_\nu) = \partial^*(d^1)$$

and

$$\partial^*(H^0(j_2)(d_\nu)) = H^1(j_2)(a_\nu) = 0.$$

It follows that

$$\partial^*(H^0(h)(d_0) + d_1) = \partial^*(d^1) \quad \text{and} \quad \partial^*(d_0 + d_1) = 0.$$

by the Lemma in §VI,

$$(H^0(h)(d_0) - d^1)(K) = d_1(sK) \quad \text{and} \quad d_0(K) = d_1(sK)$$

for every $K \in \Delta_0'$. So

$$(*) \qquad d_0(K) = (H^0(h)(d_0) - d^1)(K) = d_0(K) - 1 \quad \text{for every } K \in \Delta_0'.$$

By §V there is a sequence in Δ_0':

$$(**) \qquad K_1, K_2, \ldots, K_{2^k}$$

such that $K_n = h(K_{n-1})$ for each $n = 2, \ldots, 2^{2k}$ and $K_1 = h(K_{2^k})$. The equality $(*)$ considered on the components $(**)$ gives a contradiction. Thus a_ν is not in $\partial^*(H^0(\widetilde{\mathcal{V}}_\varphi|T))$ and so a is not in $\partial^*(H^0(T))$. It follows that $H^1(p)\colon H^1(S^1) \to H^1(Z)$ is not trivial.

IX. Let D be the dyadic solenoid, i.e., $D = \operatorname{inv\,lim}(S^1 \overset{2}{\leftarrow} S^1 \overset{2}{\leftarrow} \cdots)$ and let $\pi\colon D \to S^1$ be the projection onto the first circle.

LEMMA. *Let g be a map from a topological space Z into D. If $f = \pi \cdot g\colon Z \to S^1$ is essential then, Z is not movable.*

PROOF. Since $\pi\colon D \to S^1$ is divisible by 2^n for all n in the group $[D, S^1]$, f is divisible by 2^n for all n in the group $[Z, S^1]$.

Now let $\{Z_\lambda, r_{\lambda,\mu}\}$ be the Čech system of Z and let $f_\lambda\colon Z_\lambda \to S^1$ be a map such that $f \approx f_\lambda \pi_\lambda$, $\pi_\lambda\colon Z \to Z_\lambda$ being the projection. Suppose Z is movable, so there is $\mu > \lambda$ such that for arbitrary $\gamma > \mu$ there is a map $h\colon Z_\mu \to Z_\gamma$ with $r_{\lambda\gamma} \cdot h \approx r_{\lambda\mu}$.

Observe that $[Z_\mu, S^1]$ does not contain nontrivial elements divisible by all the powers of 2. This is so because such a map induces the trivial homomorphism of fundamental groups. Let m be an integer such that $[f_\lambda \cdot r_{\lambda\mu}]$ is not divisible by 2^m. Since $[Z, S^1] = \operatorname{dir\,lim}[Z_\gamma, S^1]$, there is $\gamma > \mu$ with $[f_\lambda \cdot r_{\lambda\gamma}]$ divisible by 2^m. Then $[f_\lambda \cdot r_{\lambda\gamma} \cdot h] = [f_\lambda \cdot r_{\lambda\mu}]$ is divisible by 2^m, a contradiction.

REMARK. In the previous Lemma, D can be replaced by any nontrivial solenoid.

X. Solenoidal Roy's example. In [**5**], P. Roy has given an example of a metric space X with the covering dimension $\dim X = 1$ and with the small inductive dimension $\operatorname{ind} X = 0$. Our solenoidal space Z obtained from the Roy's examples is not movable and it is easy to see that $\operatorname{ind} Z = 0$. Thus we have the following

THEOREM. *There exists a nonmovable* 1*-dimensional complete metric space Z, with* $\operatorname{ind} Z = 0$.

COROLLARY. *There exists a nonmovable space Z with movable components (each components is a point).*

REMARK. If the covering dimension of a space is 0, then it is movable. (See [**2**].)

XI. Second example. Let D be the dyadic solenoid, i.e.

$$D = \operatorname{inv}\lim(S^1 \overset{2}{\leftarrow} S^1 \overset{2}{\leftarrow} \cdots).$$

Let $\pi\colon D \to S^1$ be the projection onto the first circle. The crucial property of π we are going to use is that π is essential (not homotopic to a constant map).

Consider the family $\mathbf{A}$ of all open subsets U of $D\times C$, C being the Cantor set, such that the composition $U \to D\times C \to D \to S^1$ is inessential, where $U \to D\times C$ is the inclusion map, $D \times C \to D$ is the projection and $D \to S^1$ is π. Choose a bijection $\alpha\colon C \to \mathbf{A}$ between $\mathbf{A}$ and the Cantor set C. Such a bijection exists because given a point (d, c) in $D\times C$ there is $\varepsilon > 0$ such that the ball $B((d, c), \varepsilon)$ centered at (d, c) and of radius ε belongs to $\mathbf{A}$. Hence $B((d, c), \delta) \in \mathbf{A}$ for all $\delta < \varepsilon$, and since D is connected $B((d, c), \delta) \neq B((d, c), \mu)$ for $\delta \neq \mu$ sufficiently small. Given $c \in C$, the composition $D\times\{c\} \to D\times C \to D \to S^1$ is essential. Therefore, $D\times\{c\}-\alpha(c) \neq \emptyset$ and we can choose $\beta(c) \in D\times\{c\}-\alpha(c)$. Let $Y = \{\beta(c)\colon c \in C\} \subset D\times C$. Observe that $Y \to D \times C \to C$ is a bijection ($D \times C \to C$ being the projection), so Y is totally disconnected. Suppose $\pi\cdot p|Y\colon Y \to S^1$ is inessential, where $p\colon D\times C \to D$ is the projection. Then there is a map $H\colon D\times C\times\{0,1\}\cup Y\times[0,1] \to S^1$ such that $H|D\times C\times\{0\}$ is $\pi\cdot p$ and $H|D\times C\times\{1\}$ is a constant map. By [**7**, p. 107], H is homotopic to a map G extendable over a neighborhood V of $D\times C\times\{0,1\}\cup Y\times[0,1]$ in $D\times C\times[0,1]$. Choose a neighborhood U of Y in $D\times C$ such that $U\times[0,1] \subset V$. Then $\pi\cdot p|U$ is inessential, so $U = \alpha(c)$ for some $c \in C$. This leads to a contradiction because $Y \cap (D \times \{c\} - \alpha(c)) \neq \emptyset$, so it is impossible for Y to be contained in U. Thus $f = \pi\cdot p|Y\colon Y \to S^1$ is essential. By §IX, Y is nonmovable.

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