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TWO THEOREMS ON TRUTH TABLE DEGREES

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ABSTRACT. In this article we solve two questions of Odifreddi on the r.e. tt-degrees. First we construct an r.e. tt-degree with anticupping property. In fact, we construct r.e. tt-degrees $\mathbf{a}, \mathbf{b}$ with $\mathbf{0} < \mathbf{a} < \mathbf{b}$ and such that for all (not necessarily r.e.) tt-degrees $\mathbf{c}$ if $\mathbf{a} \cup \mathbf{c} \geq \mathbf{b}$ then $\mathbf{a} \leq \mathbf{c}$. This result also has ramifications in, for example, the r.e. wtt-degrees. Finally we solve another question of Odifreddi by constructing an r.e. tt-degree with no greatest r.e. m-degree.

1. Introduction. The goal of this paper is to answer two questions of Odifreddi concerning r.e. tt-degrees. For background we refer to Odifreddi [**9**, **10**] or Rogers [**12**]. The relevant questions are:

(1.1) [**9**, Problem 11] Does every r.e. tt-degree have greatest r.e. m-degree.

(1.2) Does $\mathbf{0}'_{\mathrm{tt}}$ have the *anticupping property*? Namely, is there an r.e. tt-degree $\mathbf{a} \neq \mathbf{0}$ such that for all r.e. tt-degrees $\mathbf{b}$, if $\mathbf{b} \cup \mathbf{a} = \mathbf{0}'_{\mathrm{tt}}$ then $\mathbf{b} = 0'_{\mathrm{tt}}$?

We solve (1.1) negatively. The method extends to construct on r.e. tt-degree $\mathbf{a}$ containing no n-r.e. m-degree exceeding all r.e. m-degrees in $\mathbf{a}$.

We solve (1.2) affirmatively. This question was particularly interesting in view of the fact that the analogous questions had been solved for all other (major) reducibilities. The method we use is very different from those used for other reducibilities. Although it is not difficult this method is quite powerful. In fact we are able to show that $\mathbf{0}'_{\mathrm{tt}}$ has the *global anticupping property*: let $\mathscr{D}_{\mathrm{tt}}$ denotes the collection of all tt-degrees. We say an r.e. tt-degree $\mathbf{d} \neq \mathbf{0}$ has the global anticupping property if there exists an r.e. tt-degree $\mathbf{a}$ with $\mathbf{0} < \mathbf{a} \leq \mathbf{d}$ such that

$$\forall \mathbf{b} \in \mathscr{D}_{\mathrm{tt}} (\mathbf{b} \cup \mathbf{a} \geq \mathbf{d} \rightarrow \mathbf{b} \geq \mathbf{a}). \tag{1.3}$$

The technique used to establish that $\mathbf{0}'_{\mathrm{tt}} = \mathbf{d}$ satisfies (1.3) is also applicable to various other situations and reducibilities. To demonstrate this, a minor variation of the construction establishes a result from [**1**]: if $\mathbf{d}$ is any nonzero r.e. wtt-degree, there exists an r.e. wtt-degree $\mathbf{a}$ with $\mathbf{0} < \mathbf{a} < \mathbf{d}$ such that for all wtt-degrees $\mathbf{b}$, if $\mathbf{a} \cup \mathbf{b} \geq \mathbf{d}$ then $\mathbf{a} \leq \mathbf{b}$. In particular, all nonzero r.e. wtt-degrees have the global anticupping property. This last result does not hold for the r.e. tt-degrees since, for example, there are minimal r.e. tt-degrees (Kobzev [**7**]). The reader should note that the analogue of (1.3) does not hold for $\mathbf{d} = \mathbf{0}'_{\mathrm{T}}$ in the T-degrees since the upper semilattice of T-degrees $\geq \mathbf{0}'_{\mathrm{T}}$ is complemented (Posner [**11**]).

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It seems convenient to adopt a variation of the notation of Fejer and Shore [**4**]. Thus if $\{e\}(x)\downarrow$ then $[e]$ is the truth table with index $\{e\}(x)$. For a set A we define

$$[e](A;x)=\begin{cases}1 & \text{if } \{e\}(x)\downarrow \text{ and } A\vDash [e](x),\\ 0 & \text{if } \{e\}(x)\downarrow \text{ and } \neg\,(A\vDash [e](x)),\\ \uparrow & \text{if } \{e\}(x)\uparrow .\end{cases}$$

Similarly, we define $[e]_s(A_s;x)$ according to whether or not $\{e\}_x(x)\downarrow$. When the context is clear, we shall write $[e](A_s;x)$ for $[e]_s(A_s;x)$ to simplify notation. We let $u(-)$ denote the use of $(-)$. We let $A[x]=\{z\in A\colon z\le x\}$, and use $\langle\ ,\ \rangle$ to denote a standard pairing function, we assume $\langle\ ,\ \rangle$ is monotone in both variables. We assume that all computations, etc. are bounded by s at stage s. All other notation and terminology is standard and we refer the reader to [**9**, **10**, or **12**].

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2. Anticupping.

(2.1) THEOREM. *There exists an r.e. tt-degree* **d** *with the global anticupping property.*

REMARK. As a corollary we see that $\mathbf{0}'_{\mathrm{tt}}$ has the global anticupping property since $\mathbf{0}'_{\mathrm{tt}}\ge\mathbf{d}$. However the reader should note that the construction below is more flexible and we can, for instance, make **d** low. In the notation of our construction, this involves the use of many "entourages of followers". We do not pursue such variations as they do not seem central to the issues of this paper.

PROOF. We shall construct r.e. sets $A=\bigcup_s A_s$ and $D=\bigcup_s D_s$ to satisfy the requirements below

$P_e\colon \overline{A}\ne W_e$,

N_e: If B is any set then $[e](A\oplus B)=D$ implies $A\le_{\mathrm{tt}} B$.

Additionally, we arrange that $A\le_{\mathrm{tt}} D$ (in fact the construction ensures $A\le_m D$). We shall use σ,τ,γ etc. to denote strings (i.e. $\sigma,\tau\in 2^{<\omega}$) and $\mathrm{lh}(\sigma)$ will denote the length of σ. Let

$$l(e,s)=\max\{x\colon \exists\sigma\forall y<x([e](A_s\oplus\sigma;y)=D_s(y))\}.$$

Define

$$u(e,y,s)=\begin{cases}u([e](A_s\oplus\sigma;y)) & \text{for any } \sigma \text{ with } [e](A_s\oplus\sigma;y)\downarrow \text{ and } \mathrm{lh}(\sigma)\le s,\\ \uparrow & \text{if } \forall\sigma(\mathrm{lh}(\sigma)\le s\to [e](A_s\oplus\sigma;y)\uparrow).\end{cases}$$

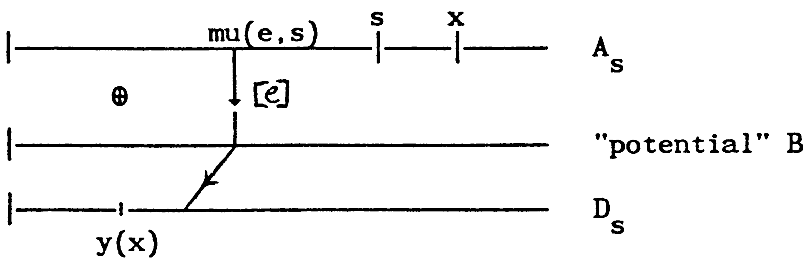
Note that the use of tt-reductions means that it is irrelevant which σ we use in the first case above. Now let

$$\begin{aligned} ml(e,s)&=\max\{l(e,t)\colon t<s\},\\ mu(e,s)&=\max\{u(e,y,s)\colon y<l(e,s)\}.\end{aligned}$$

We shall satisfy the P_e by *followers*. Each follower of P_e is targeted for A and is attached to a unique *prefollower* targeted for D. $E+3$ prefollowers are appointed to P_e at the beginning of the construction and are used to satisfy the N_j for $j<e$. The number $e+3$ comes from the quantity of N_j with which P_e must cooperate. (More on this later.)

We shall first briefly describe our basic strategy for satisfying N_e. It is important to note that B is unknown during the construction and we must play for all possible B. Our fundamental idea—for a single N_e—is to use the prefollower $y(x)$ of a follower x to force a B-predictable change in D via $[e]$. The implementation is roughly as follows.

We have, at each stage, a least unused prefollower $y \in E(j)$ (to be defined later) targeted for D. We shall wait until a stage s such that $l(e,s) > y$, declare P_j as e-confirmed and reset a new follower x of P_j targeted for A and attached to y. We appoint $x > s$ and so obtain the situation described in the diagram below.



We now promise that for all stages $t \geq s$

(2.2) $$x \in A_{t+1} - A_t \quad \text{iff } y(x) \in D_{t+1} - D_t.$$

Now suppose that we have built our reduction procedure so that B can tt-compute $A[x-1]$, which we write as "$A[x-1] \leq_{\mathrm{tt}} B$". It follows by (2.2) and the fact that $x > mu(e,s)$, that if $[e](A \oplus B) = D$ then

(2.3) $$x \in A \quad \text{iff } [e](A[x-1] \oplus B; y(x)) = 1.$$

But (2.3) and the assumption that $A[x-1] \leq_{tt} B$ mean that $A[x] \leq_{tt} B$. In this way we show that $A \leq_{tt} B$.

The reader should note that it is necessary to use more than one prefollower for a single P_e for the following reason. Suppose in the situation of the diagram we are concerned with two N_e say N_{e_1} and N_{e_2}. Now when we see $l(e_1, s) < y(x)$ we attached x to $y(x)$. Now this gives us a *permanent* commitment to enumerate x into A iff $y(x) \in D$. The trouble is that perhaps at some $\hat{s} > s$ for e_2 it may be that $u([e_2](A_{\hat{s}} \oplus \sigma; y(x))) > x$. This means that x is no longer a good follower for P_j from N_{e_2}'s point of view, since the driving force is to have followers beyond the use regions of the prefollowers. The solution is to pick a new follower $\hat{x}$ which we must attach to some prefollower $\hat{y} \neq y(x)$ (*since we must still respect the* e_1 *commitment*). Since we wish now to respect e_1 and e_2 commitments we would like $\hat{x}$ to exceed both $u([e_1](A \oplus \sigma; \hat{y}))$ and $u([e_2](A \oplus \sigma; \hat{y}))$. To do this we need to have already seen $l(e_1, t) > \hat{y}$ some $t \leq \hat{s}$. Our basic idea is to set aside as many potential prefollowers as we will need in advance and only act when $l(e,s)$ exceeds all of them. Then if we ever need to switch we will know that previous N_e commitments remain respected.

We now give the formal details of the argument. First to each P_e for $e \in \omega$ we assign an *entourage* of prefollowers

$$E(e) = \{\langle e, 1\rangle, \ldots, \langle e, e+3\rangle\}.$$

In the course of the construction numbers in $E(e)$ may be *used* or *unused* (or also cancelled). If $x \in E(e)$ and x is used and uncancelled then x is attached to some follower y of P_e. Initially all of $E(e)$ except $\langle e, 1\rangle$ are unused.

We shall say that P_e *requires attention* at stage $s+1$ if $A_s \cap W_{e,s} = \varnothing$ and $x \in W_{e,s}$ where x is the current follower of P_e.

Construction.

Stage 0. Declare each P_j for $j \in \omega$ as not e-confirmed for all $e \leq j$. Assign $\langle j, 0\rangle$ as a follower of P_j targeted for A and attached to $\langle j, 1\rangle$. Declare $\langle j, 1\rangle$ as used.

Stage $s+1$.

Step 1. For each unsatisfied P_j and each $e \leq j$ if $j \leq s$ and if

(i) $l(e,s) > \langle j, j+3\rangle$, and

(ii) P_j is not yet e-confirmed,

declare P_j as e-confirmed. Find the least unused member $\langle j, i\rangle$ of $E(j)$. (This will exist.) Declare $\langle j, i\rangle$ as used. Appoint $x = \langle j, s+1\rangle$ as a follower of P_j. Declare $y(x) = \langle j, i\rangle$ as attached to x. Cancel the previous follower of P_j together with its prefollower.

Step 2. For each $j \leq s$ if P_j requires attention find the appropriate follower x and enumerate x into $A_{s+1} - A_s$ and $y(x)$ (x's prefollower) into $D_{s+1} - D_s$.

End of Construction.

(2.4) LEMMA. *All the P_e are met and not all of the members of $E(e)$ are used.*

PROOF. It is clear that P_e always has a follower provided we don't run out of prefollowers. We need to reset P_e's follower at most once for each N_j with $j \leq e$, and so most $e+1$ times. Thus at most $e+2$ members of $E(e)$ are used. Once we reach a stage after which step 1 never again pertains to P_e, P_e will have a final follower x. For this follower, as usual, either P_e never receives attention (and so $\overline{W}_e \neq A$ by fiat) or step 2 pertains to x ensuring $W_{e,s} \cap A_s \neq \varnothing$. □

(2.5) LEMMA. *All the N_e are met.*

PROOF. Let B be any set and suppose $[e](A \oplus B) = D$. Then $l(e,s) \to \infty$ since the appropriate initial segments σ of B will satisfy the definition for $l(e,s)$. We must show that $A \leq_{\mathrm{tt}} B$. Let s_0 be a stage such that

$$\forall s \geq s_0\ \forall j < e\ (P_j \text{ does not receive attention at stage } s). \tag{2.6}$$

Our procedure is inductive. Let z be given. Suppose $A[z-1] \leq_{\mathrm{tt}} B$. Now numbers may enter A after stage s_0 only if they follow some P_j for some $j \geq e$. Let $s_1 = \max\{z, s_0\}$. If z does not follow some P_j for $j \geq e$ at stage s_1 then $z \in A$ iff $z \in A_{s_1}$. Assuming z follows P_j, say, find the least stage $s_2 > s_1$ such that one of the following options holds.

(i) P_j is e-confirmed at stage s_2,

(ii) $z \in A_{s_2}$,

(iii) $W_{e,s} \cap A_s \neq \varnothing$, or

(iv) z is cancelled.

If (ii), (iii) or (iv) hold then $z \in A$ iff $z \in A_{s_2}$. If (i) holds we proceed as follows. Find the stage $t \leq s_2$ at which z is appointed. By (i) above as z is uncancelled P_j is e-confirmed at stage t. This means z is given a prefollower $y(z) = \langle j, i\rangle$ for some i with $l(e,t) > \langle j, i\rangle$ and

$$z > u([e](A \oplus B; \langle j, i\rangle)). \tag{2.7}$$

Now $z \in A$ iff $\langle j, i\rangle \in D$ and by (2.7) (if case (i) holds) we see

$$z \in A \quad \text{iff } [e](A[z-1] \oplus B; \langle j, i\rangle) = 1. \tag{2.8}$$

Hence by induction B can tt-compute $A[z]$ and hence $A \leq_{\mathrm{tt}} B$. □

(2.6) LEMMA. $A \leq_m D$.

PROOF. To compute if $z \in A$, see if z is a follower by stage z. If not then $z \notin A$. If z is a follower then z has a prefollower $y(z)$. Then $z \in A$ iff $y(z) \in D$. □

There is nothing special here about tt-reductions. We remark that the same proof also shows:

(2.7) COROLLARY [1]. *There exist r.e. wtt-degrees with the global anticupping property.*

We remark that by using an infinite collection of $\{E(e)\}$ for each P_e in place of $E(e)$ a standard permitting argument (on D) shows.

(2.8) COROLLARY [1]. *Each nonzero r.e. wtt-degree has the global anticupping property.*

PROOF. Left to reader. □

Of course (2.8) fails in the r.e. tt-degrees since Kobzev [7] has constructed a minimal r.e. tt-degree. Jeanleah Mohrherr [8] has asked the related question of the extent which Friedberg's [3] completeness criterion holds in $\mathscr{D}_{\mathrm{tt}}$. In [8], Mohrherr showed (for $\mathscr{D}_{\mathrm{tt}}$)

$$\forall \mathbf{a} \geq \mathbf{0}'\ \exists \mathbf{b}(\mathbf{b}' = \mathbf{a}). \tag{2.9}$$

The question is whether there exists a tt-degree $\mathbf{d}$ $(= \mathbf{0}'?)$ such that

$$\forall \mathbf{a} \geq \mathbf{d}\ \exists \mathbf{b}(\mathbf{b} < \mathbf{a} \& \mathbf{b} \cup \mathbf{d} = \mathbf{a}). \tag{2.10}$$

In view of our results, I conjecture that (2.10) fails for $\mathscr{D}_{tt}$.

3. ***M*-tops**. Our result for this section is to solve Odifreddi's question [9, Problem 11].

(3.1) THEOREM. *There exists an r.e. tt-degree without greatest r.e. m-degree.*

PROOF. We shall build $A = \bigcup_s A_s$ with auxiliary r.e. sets $B_e = \bigcup_s B_{e,s}$ to satisfy the requirements (taken over 3-tuples $\langle [e], W_e, \gamma_i\rangle$)

$$R_{e,i}\colon [e](A) = W_e \quad \text{implies} \quad B_e \leq_{\mathrm{tt}} A \text{ and } \neg\ (B_e \leq_m W_e \text{ via } \gamma_i).$$

The reader should note that meeting all the $R_{e,i}$ gives (3.1). For suppose W_e is an r.e. set of greatest r.e. m-degree in tt-degree of A. Now as $B_e \leq_{\mathrm{tt}} A$, $B_e \oplus A \equiv_{\mathrm{tt}} A$. Since $B_e \not\leq_m W_e$ in particular $B_e \oplus A \not\leq_m W_e$. In fact, we ensure that $B_e \leq_{btt} A$ with norm 2. The reduction is

$$x \in B_e \quad \text{iff} \begin{cases} x \text{ is a follower target for } B_e \text{ be stage } x \text{ and} \\ 2x \in A \text{ and } 2x+1 \notin A. \end{cases}$$

This reduction is predicated, of course, on the assumption that $[e](A) = W_e$. Followers of $R_{e,i}$ may be *active* or *passive*. If a follower x of $R_{e,j}$ is active (and so targeted for B_e) then if x is cancelled at stage s, we automatically enumerate x into B_e. "Activity" therefore involves a "pending commitment" to B_e.

We shall say that $R_{e,i}$ *requires attention* at stage $s+1$ if $R_{e,i}$ is not (declared) satisfied and one of the following options holds.

(3.2) $R_{e,i}$ has no follower.

(3.3) $R_{e,i}$ has a follower x and

(i) $\gamma_{i,s}(x)\downarrow$,

(ii) $l(e,s) > x,\ \gamma_i(x)$; where

$$l(e,s) = \max\{x\colon \forall y < x([e](A_s;y) = W_{e,s}(y))\}$$

Construction.

Stage $s+1$. Find a least $\langle e,i\rangle$ such that $R_{e,i}$ requires attention. First cancel the satisfaction of all $R_{j,k}$ for $\langle e,i\rangle < \langle j,k\rangle$. If $R_{j,k}$ is currently active and x is its follower enumerate x into $B_{j,s+1} - B_{j,s}$. Now cancel all followers of $R_{j,k}$.

Now attack the $R_{e,i}$ by adopting the appropriate case below.

Case 1. (3.2) holds; Assign $x = s$ as a follower of $R_{e,i}$. Declare $R_{e,i}$ as passive.

Case 2. (3.3) holds and $R_{e,i}$ is passive. Define $C_s = A_s \cup \{2x\}$.

Subcase (a). $[e](C_s;\gamma_i(x)) = 0$. Set $A_{s+1} = A_s \cup \{2x\}$ and $B_{e,s+1} = B_{e,s}\cup\{x\}$. Declare $R_{e,i}$ as satisfied.

Subcase (b). $[e](C_s;\gamma_i(x)) = 1$. Set $A_{s+1} = A_s \cup \{2x\}$ and $B_{e,s+1} = B_{e,s}$. Declare $R_{e,i}$ as active.

Case 3. (3.3) holds and $R_{e,i}$ is active. Set $A_{s+1} = A_s\cup\{2x+1\}$ and $B_{e,s+1} = B_{e,s}$. Declare $R_{e,i}$ as satisfied.

End of construction.

Verification. The argument is finite injury. Let s_0 be a stage such that

$$\forall s > s_0 \forall m > \langle e,i\rangle\ (R_m \text{ does not receive attention at stage } s).$$

By our cancellation procedure we may suppose that $R_{e,i}$ has no follower at stage s_0. Now if $R_{e,i}$ is to fail then $l(e,s)\to\infty$.

At some stage $s_1 > s_0$, $R_{e,i}$ receives attention and gets a follower x. If $R_{e,i}$ is to fail, then (3.3) must hold at some stage $s_2 > s_1$. At stage s_2 we see

$$[e](A_{s_2};\gamma_i(x)) = W_{e,s}(\gamma_i(x)). \tag{3.4}$$

By our cancellation procedure, the only numbers (possibly) $< s$ which can ever enter $A - A_{s_2}$ are $2x$ and $2x+1$. Hence the only numbers (possibly) $< u([e](A;\gamma_i(x)))$ which can ever enter $A - A_{s_2}$ are $2x$ and $2x+1$. Now if subcase (a) holds then

$$[e](A;\gamma_i(x)) = 0 = W_e(\gamma_i(x))$$

and $\neg$ ($B_e \le_m W_e$ via $\gamma_i(x)$) since $B_e(x) = 1$ and $\gamma_i(x)\notin W_e$.

If subcase (b) holds we set $A_{s+1} = A_s\cup\{2x\}$ and activate x. Since $[e](C_s;\gamma_i(x))$ $= 1$, when (3.3) next (say at stage $s_2 > s_2$) pertains to $R_{e,i}$ we must have

$$[e](A;\gamma_i(x)) = 1 = W_{e,s_3}(\gamma_i(x)).$$

But now $\gamma_i(x)\in W_{e,s_3}$ and so $\gamma_i(x)\in W_e$ as W_e is r.e. But now we win since $x\notin B_e$ and $\gamma_i(x)\in W_e$. It is clear that $B_e \le_{tt} A$ if $l(e,s)\to\infty$ and so all the $R_{e,i}$ are met. □

(3.5) *Variations and comments.*

The reader should note that we win above for one of two reasons. In subcase (a) by enumerating $2x$ into A we cause $[e]$ (A) to believe $\gamma_1(x)\notin W_e$ yet enumerating x into B_e causes $\gamma_i(x)\in W_e$. In subcase (b) we first set A so that $[e]$ (A) believes

$\gamma_i(x) \in W_e$ but does not enumerate x into B_e. Thus once we see $\gamma_i(x) \in W_{e,s}$ at some s we then use $2x+1$ to allow us to keep x out of B_e while $\gamma_i(x) \in W_e$.

The "punch line" here is that once $\gamma_i(x)$ enters W_e it cannot be retracted. Let n be given. An easy modification (using a larger norm for the $B_e \leq_{\mathrm{btt}} W_e$) will construct an r.e. tt-degree with no n-r.e. m-degree exceeding all r.e. m-degrees. By dovetailing and using full tt-reductions, it is also possible to construct an r.e. tt-degree **a** with no k-r.e. m-degree exceeding all r.e. m-degrees in **a** for any k. I do not know if all nonzero r.e. T-degrees contain r.e. tt-degrees without r.e. m-tops. We should point out that Rogers and Jockusch (cf. [**12**]) have shown that all tt-degrees contain a greatest m-degree. Thus the restriction "*r.e.* m-degree" is necessary. Finally it seems conceivable that the methods of this section might be useful in solving the following question implicit in [**2**]: Is there tt-topped r.e. T-degree that is not m-topped?

ADDED IN PROOF. The result above can be extended to construct an r.e. T-degree **a** such that if **b** is any r.e. tt-degree within **a**, then **b** has no greatest r.e. m-degree. See [**13**].

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