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CONVERGENCE AND DIVERGENCE ALMOST EVERYWHERE OF SPHERICAL MEANS FOR RADIAL FUNCTIONS

YUICHI KANJIN

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ABSTRACT. Let $d > 1$. It will be shown that the maximal operator S^* of spherical means S_R, $R > 0$, is bounded on $L^p(\mathbf{R}^d)$ radial functions when $2d/(d+1) < p < 2d/(d-1)$, and it implies that, for every $L^p(\mathbf{R}^d)$ radial function $f(t)$, $S_R f(t)$ converges to $f(t)$ for a.e. $t \in \mathbf{R}^d$ when $2d/(d+1) < p \leq 2$. Also, it will be proved that there is an $L^{2d/(d+1)}(R^d)$ radial function $f(t)$ with compact support such that $S_R f(t)$ diverges for a.e. $t \in R^d$.

Introduction. Define linear operators S_R, $R > 0$, and S^* on $L^2(R^d)$ by

$$S_R f(t) = (2\pi)^{-d/2} \int_{|\xi| \leq R} \hat{f}(\xi) e^{it\xi} \, d\xi,$$

and

$$S^* f(t) = \sup_{R>0} |S_R f(t)|, \qquad t \in \mathbf{R}^d,$$

respectively, where $\hat{f}(\xi)$ is the Fourier transform of a function $f(t)$, that is, $\hat{f}(\xi) = (2\pi)^{-d/2} \int_{\mathbf{R}^d} f(t) e^{-i\xi t} \, dt$, $\xi \in \mathbf{R}^d$.

We shall assume for the rest of the paper that $d > 1$. We put $S = S_1$. Fefferman [**F**] showed that S is bounded on $L^p(\mathbf{R}^d)$ if and only if $p = 2$. On the other hand, Herz [**H**] showed that, when restricted to $L^p(\mathbf{R}^d)$ radial functions, S is bounded if and only if $2d/(d+1) < p < 2d/(d-1)$. In this connection, Kenig and Tomas [**KT1**] showed that, for $p = 2d/(d+1)$ and $p = 2d/(d-1)$, S is not weak type on $L^p(\mathbf{R}^d)$ radial functions, and Chanillo [**C**] showed that S is restricted weak type on $L^p(\mathbf{R}^d)$ radial functions for $p = 2d/(d+1)$.

This paper deals with boundedness of S^* on $L^p(\mathbf{R}^d)$ radial functions. We shall prove a theorem which transplants a norm inequality for the maximal operators defined by Fourier Jacobi multipliers to a corresponding inequality for the maximal operators defined by Fourier Hankel multipliers. It follows from the theorem that S^* is bounded on $L^p(\mathbf{R}^d)$ radial functions if $2d/(d+1) < p < 2d/(d-1)$, which implies that $S_R f(t)$ converges to $f(t)$ almost everywhere for every $L^p(\mathbf{R}^d)$ radial function $f(t)$ if $2d/(d+1) < p \leq 2$. We shall also show that there is an $L^{2d/(d+1)}(\mathbf{R}^d)$ radial function $f(t)$ with compact support such that $S_R f(t)$ diverges almost everywhere.

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1. Statements of results. Let $\alpha \geq -1/2$. For a function $g(x)$ on the interval $(0,\infty)$, the Hankel transform $\tau_\alpha g(y)$, $y \geq 0$, of order α is given by

$$\tau_\alpha g(y) = \int_0^\infty g(x) \frac{J_\alpha(yx)}{(yx)^\alpha} x^{2\alpha+1}\, dx,$$

where $J_\alpha(x)$ is the Bessel function of the first kind of order α. We define

$$L^p_\alpha = \left\{ g(x) \text{ on } (0,\infty); \|g\|_{\alpha,p} = \left(\int_0^\infty |g(x)|^p x^{2\alpha+1}\, dx \right)^{1/p} < \infty \right\}.$$

Then it is known that $\|\tau_\alpha g\|_{\alpha,q} \leq C_{\alpha,p} \|g\|_{\alpha,p}$ for $g(x)$ in L^p_α if $1 \leq p \leq 2$, where $1/p + 1/q = 1$ and $C_{\alpha,p}$ is a constant depending only on α and p (cf. [**H**]).

Let $\lambda(y) \in L^\infty_\alpha$. Define operators Λ_R, $R > 0$, and Λ^* on L^2_α by

$$\tau_\alpha(\Lambda_R g)(y) = \lambda(y/R)\tau_\alpha g(y) \quad \text{and} \quad \Lambda^* g(x) = \sup_{R>0} |\Lambda_R g(x)|,$$

respectively.

Let $P_n^{(\alpha,\beta)}(u)$ denote the Jacobi polynomial of degree n and order (α,β), $\alpha, \beta > -1$ defined by

$$(1-u)^\alpha (1+u)^\beta P_n^{(\alpha,\beta)}(u) = \frac{(-1)^n}{2^n n!} \left(\frac{d}{du} \right)^n \{(1-u)^{n+\alpha}(1+u)^{n+\beta}\}.$$

The functions $P_n^{(\alpha,\beta)}(\cos\theta)$ are orthogonal on $(0,\pi)$ with respect to

$$d\mu(\theta) = (\sin\theta/2)^{2\alpha+1} (\cos\theta/2)^{2\beta+1}\, d\theta.$$

We define

$$L^p_{(\alpha,\beta)} = \Big\{ h(\theta) \text{ on } (0,\pi); \quad \|h\|_{(\alpha,\beta),p} = \left(\int_0^\pi |h(\theta)|^p \, d\mu(\theta) \right)^{1/p} < \infty \Big\}.$$

For a function $h(\theta)$ in $L^1_{(\alpha,\beta)}$, we have the Fourier Jacobi series

$$h(\theta) = \sum_{n=0}^\infty \tilde{h}(n) \rho_n P_n^{(\alpha,\beta)}(\cos\theta),$$

where

$$\tilde{h}(n) = \int_0^\pi h(\theta) P_n^{(\alpha,\beta)}(\cos\theta)\, d\mu(\theta),$$

and

$$\rho_n = \left[\int_0^\pi \{P_n^{(\alpha,\beta)}(\cos\theta)\}^2 \, d\mu(\theta) \right]^{-1}.$$

Define operators $\tilde{\Lambda}_R$, $R > 0$, and $\tilde{\Lambda}^*$ on $L^2_{(\alpha,\beta)}$ by

$$(\tilde{\Lambda}_R h)\tilde{}(n) = \lambda(n/R)\tilde{h}(n) \quad \text{and} \quad \tilde{\Lambda}^* h(\theta) = \sup_{R>0} |\tilde{\Lambda}_R h(\theta)|,$$

respectively.

We are now ready to state the first theorem.

THEOREM 1. *Let $\alpha, \beta \geq -1/2$. Let $1 < p < \infty$ and let $\lambda(y)$ be a function in L^∞_α continuous for almost every y. If $\tilde{\Lambda}^*$ is bounded on $L^p_{(\alpha,\beta)}$, then Λ^* is bounded on L^p_α.*

Define linear operators T_R, $R > 0$, and T^* on L^2_α by

$$T_R g(x) = \int_0^R \tau_\alpha g(y) \frac{J_\alpha(xy)}{(xy)^\alpha} y^{2\alpha+1}\, dy,$$

and

$$T^* g(x) = \sup_{R>0} |T_R g(x)|, \qquad x \geq 0,$$

respectively. Let $\chi_{(0,1)}(y)$ be the characteristic function of the interval $(0,1)$. For $\lambda(y) = \chi_{(0,1)}(y)$, we notice that $\Lambda^* = T^*$ and $\tilde{\Lambda}^*$ is the maximal operator of partial sum operators of the Fourier Jacobi series. From Badkov's theorem [**B**, Theorem 1.7], it follows that $\tilde{\Lambda}^*$ is bounded on $L^p_{(\alpha,\beta)}$ if

$$\max\{4(\alpha+1)/(2\alpha+3), 4(\beta+1)/(2\beta+3)\} < p$$
$$< \min\{4(\alpha+1)/(2\alpha+1), 4(\beta+1)/(2\beta+1)\}$$

and $\alpha, \beta \geq -1/2$. Thus, Theorem 1 yields Corollary 1 from which Corollary 2 is obtained by using routine method.

COROLLARY 1. *Let $\alpha \geq -1/2$ and $4(\alpha+1)/(2\alpha+3) < p < 4(\alpha+1)/(2\alpha+1)$. Then, $\|T^* g\|_{\alpha,p} \leq C\|g\|_{\alpha,p}$ for $g(x)$ in L^p_α, where C is a constant depending only on α and p.*

COROLLARY 2. *Let $\alpha \geq -1/2$ and $4(\alpha+1)/(2\alpha+3) < p \leq 2$. Then, for $g(x)$ in L^p_α, $T_R g(x)$ converges to $g(x)$ for almost every x in $(0,\infty)$ as $R \to \infty$.*

By familiar relationship between Fourier transforms of radial functions and Hankel transforms, these corollaries are restated as follows.

COROLLARY 3. *Let $2d/(d+1) < p < 2d/(d-1)$. Then, $\|S^* f\|_p \leq C\|f\|_p$ for $L^p(\mathbf{R}^d)$ radial functions $f(t)$, where C is a constant depending only on d and p.*

COROLLARY 4. *Let $2d/(d+1) < p \leq 2$. Then, for every $L^p(\mathbf{R}^d)$ radial function $f(t)$, $S_R f(t)$ converges to $f(t)$ for almost every t in $\mathbf{R}^d$ as $R \to \infty$.*

On the divergence, the following theorem holds.

THEOREM 2. *Let $\alpha > -1/2$. If $p = 4(\alpha+1)/(2\alpha+3)$, then there is a function $g(x)$ in L^p_α with compact support such that $T_R g(x)$ diverges almost everywhere in $(0,\infty)$ as $R \to \infty$.*

COROLLARY 5. *If $p = 2d/(d+1)$, then there is an $L^p(\mathbf{R}^d)$ radial function $f(t)$ with compact support such that $S_R f(t)$ diverges almost everywhere in $\mathbf{R}^d$ as $R \to \infty$.*

2. Proofs. A formulation (Lemma 1) of linearization of Kenig and Tomas [**KT2**] in our situation will enable us to prove Theorem 1 by following the proof of the theorem of Igari [**I**] which transplants a norm inequality for Fourier Jacobi multipliers to a corresponding inequality for Fourier Hankel multipliers. Theorem 2 will be proved by an argument similar to that used in Stanton and Tomas [**ST**] to prove divergence of central Fourier series on compact Lie groups, or in Meaney [**M**] to prove divergence of Jacobi polynomial series. See also Sogge [**S**].

LEMMA 1 [**KT2**, §1, LEMMA]. *Let $1 < p < \infty$ and $1/p + 1/q = 1$. Let $\lambda(y) \in L^\infty_\alpha$.*

(i) *The operator $\tilde{\Lambda}^*$ is bounded on $L^p_{(\alpha,\beta)}$ if and only if*

$$\left\| \sum_{k=1}^{\infty} \tilde{\Lambda}_{R_k} h_k \right\|_{(\alpha,\beta),q} \le C \left\| \sum_{k=1}^{\infty} |h_k| \right\|_{(\alpha,\beta),q}, \tag{1}$$

where C is a constant not depending on sequences $\{h_k(\theta)\}_{k=1}^{\infty} \subset L^q_{(\alpha,\beta)}$ and $\{R_k\}_{k=1}^{\infty}$, $R_k > 0$.

(ii) *The operator Λ^* is bounded on L^p_α if and only if*

$$\left\| \sum_{k=1}^{\infty} \Lambda_{R_k} g_k \right\|_{\alpha,q} \le C' \left\| \sum_{k=1}^{\infty} |g_k| \right\|_{\alpha,q}, \tag{2}$$

where C' is a constant not depending on sequences $\{g_k(x)\}_{k=1}^{\infty} \subset L^q_\alpha$ and $\{R_k\}_{k=1}^{\infty}$, $R_k > 0$.

Now we prove Theorem 1. Assume that (1) is valid. It suffices to show that (2) is valid for finite sequences $\{g_k(x)\}_{k=1}^{m}$ such that every $g_k(x)$ is an infinitely differentiable function with compact support in $(0, \infty)$. For $j > 0$, define $g_{k,j}(\theta) = g_k(j\theta)$. Then $\operatorname{supp} g_{k,j}(\theta) \subset (0, \pi)$ for large $j > 0$ and $k = 1, 2, 3, \ldots, m$. We consider $g_{k,j}(\theta)$ as a function belonging to $L^q_{(\alpha,\beta)}$. By the assumption, we have

$$\left\| \sum_{k\in S} \tilde{\Lambda}_{jR_k} g_{k,j} \right\|_{(\alpha,\beta),q} \le C \left\| \sum_{k\in S} |g_{k,j}| \right\|_{(\alpha,\beta),q}, \tag{3}$$

where S is any subset of $\{1, 2, 3, \ldots, m\}$. Multiplying $\{j(2j)^{2\alpha+1}\}^{1/q}$ by both sides of (3) and changing variable, we have

$$\begin{aligned}
&\left\{ \int_0^{j\pi} \left| \sum_{k\in S} G_k(x,j) \right|^q x^{2\alpha+1} \left(\frac{2j}{x} \sin \frac{x}{2j} \right)^{2\alpha+1} \left(\cos \frac{x}{2j} \right)^{2\beta+1} dx \right\}^{1/q} \\
&\quad \le C \left\{ \int_0^{j\pi} \left(\sum_{k\in S} |g_k(x)| \right)^q x^{2\alpha+1} \left(\frac{2j}{x} \sin \frac{x}{2j} \right)^{2\alpha+1} \left(\cos \frac{x}{2j} \right)^{2\beta+1} dx \right\}^{1/q} \\
&\quad \le C \left\| \sum_{k\in S} |g_k| \right\|_{\alpha,q},
\end{aligned}$$

where

$$\begin{aligned}
G_k(x,j) &= \tilde{\Lambda}_{jR_k} g_{k,j}(x/j) \\
&= \sum_{n=0}^{\infty} \lambda\left(\frac{n}{jR_k} \right) \tilde{g}_{k,j}(n) \rho_n P_n^{(\alpha,\beta)} \left(\cos \frac{x}{j} \right).
\end{aligned}$$

Fix an arbitrary positive number K. For $j > 3K/2\pi$, we have

$$\left(\frac{2j}{x} \sin \frac{x}{2j} \right)^{2\alpha+1} \left(\cos \frac{x}{2j} \right)^{2\beta+1} \ge \left(\frac{\sqrt{3}}{2} / \frac{\pi}{3} \right)^{2\alpha+1} \left(\frac{1}{2} \right)^{2\beta+1}, \qquad x \in (0, K).$$

Thus we have

$$(4)\qquad \left\{\int_0^K \left|\sum_{k\in S} G_k(x,j)\right|^q x^{2\alpha+1}\,dx\right\}^{1/q} \le C'' \left\|\sum_{k\in S} |g_k|\right\|_{\alpha,q}$$

for $j > 3K/2\pi$, where C'' is a constant depending only on α, β and q. Notice that (4) is valid for $q = 2$ since $\tilde{\Lambda}^*$ and Λ^* are bounded on $L^2_{(\alpha,\beta)}$ and L^2_α, respectively. Considering the case of $S = \{k\}$, $k = 1,2,3,\dots,m$, and taking a subsequence if necessary, we have that the sequence $\{G_k(x,j)\}_j$ converges weakly to a function $G_k(x)$ in $L^q_\alpha(0,K)$ and, simultaneously, in $L^2_\alpha(0,K)$ for every k, where $L^q_\alpha(0,K) = \{g(x) \in L^q_\alpha; \operatorname{supp} g(x) \subset (0,K)\}$. From the weak convergence of $\{G_k(x,j)\}_j$ in $L^q_\alpha(0,K)$ and the inequality (4) with $S = \{1,2,3,\dots,m\}$, it follows that

$$\left\{\int_0^K \left|\sum_{k=1}^m G_k(x)\right|^q x^{2\alpha+1}\,dx\right\}^{1/q} \le C'' \left\|\sum_{k=1}^m |g_k|\right\|_{\alpha,q}.$$

To complete the proof, it suffices to obtain $G_k(x) = \Lambda_{R_k} g_k(x)$ a.e. $x \in (0,K)$ from the fact that $G_k(x)$ is the weak limit of $\{G_k(x,j)\}_j$ in $L^2_\alpha(0,K)$. But, it is a strict imitation of the method of [**I**, p. 203, l.10~], and so we omit it.

Next we turn to the proof of Theorem 2. We divide the proof into two lemmas.

LEMMA 2. *Let $\alpha > -1/2$ and $p = 4(\alpha+1)/(2\alpha+3)$. Define a sequence $\{\varphi_k\}_{k=1}^\infty$ of bounded linear functionals of the space $L^p_\alpha(0,1)$ by $\varphi_k(g) = \int_k^{k+1} \tau_\alpha g(y) y^{\alpha+1/2}\,dy$. Then the norms $\|\varphi_k\|$ of the functionals φ_k satisfy that $\|\varphi_k\| \ge C(\log k)^{1/q}$, where $q = 4(\alpha+1)/(2\alpha+1)$ and C is a constant not depending on k.*

LEMMA 3. *Let $\alpha > -1/2$ and $(4\alpha+2)/(2\alpha+3) \le p \le 2$. If a function $g(x)$ in L^p_α satisfies the condition that $T_R g(x)$ converges on a set E of positive measure, then $\lim_{R\to\infty} \int_R^{R+h} \tau_\alpha g(y) y^{\alpha+1/2}\,dy = 0$ uniformly in $0 \le h \le 1$.*

By Lemma 2, there is a function $g(x)$ in $L^p_\alpha(0,1)$, $p = 4(\alpha+1)/(2\alpha+3)$ such that

$$\limsup_{k\to\infty} \left|\int_k^{k+1} \tau_\alpha g(y) y^{\alpha+1/2}\,dy\right| = \infty.$$

It follows from Lemma 3 that $T_R g(x)$ diverges almost everywhere in $(0,\infty)$ as $R \to \infty$, which completes the proof of Theorem 2.

PROOF OF LEMMA 2. By Fubini's theorem, we have

$$\varphi_k(g) = \int_0^1 g(x) \left\{\int_k^{k+1} \frac{J_\alpha(yx)}{(yx)^\alpha} y^{\alpha+1/2}\,dy\right\} x^{2\alpha+1}\,dx,$$

and thus

$$\|\varphi_k\| = \left(\int_0^1 \left|\int_k^{k+1} \frac{J_\alpha(yx)}{(yx)^\alpha} y^{\alpha+1/2}\,dy\right|^q x^{2\alpha+1}\,dx\right)^{1/q}.$$

By the asymptotic formula

$$(5)\qquad J_\alpha(z) = \left(\frac{2}{\pi z}\right)^{1/2} \cos(z-\delta) + O(z^{-3/2}) \qquad z\to\infty,\ \delta = (2\alpha+1)\pi/4,$$

we have

$$\|\varphi_k\| \geq \left(\int_{1/k}^{1}\left|\int_{k}^{k+1}\frac{J_\alpha(yx)}{(yx)^\alpha}y^{\alpha+1/2}\,dy\right|^q x^{2\alpha+1}\,dx\right)^{1/q}$$

$$\geq C_1\left(\int_{1/k}^{1}\left|\int_{k}^{k+1}\cos(yx-\delta)\,dy\right|^q x^{2\alpha+1-q(\alpha+1/2)}\,dx\right)^{1/q}$$

$$-C_2\left(\int_{1/k}^{1}\left|\int_{k}^{k+1}y^{-1}\,dy\right|^q x^{2\alpha+1-q(\alpha+3/2)}\,dx\right)^{1/q}$$

$$= C_1A_k - C_2B_k, \quad \text{say,}$$

where C_1 and C_2 are positive constants not depending on k. The terms B_k satisfy that

$$B_k = \left\{\log\left(1+\frac{1}{k}\right)\right\}\left(\frac{k^q-1}{q}\right)^{1/q} = O(1) \qquad (k\to\infty).$$

By a simple calculation, we have

$$A_k^q = 2^q\int_{1/k}^{1}\left|\cos((k+\tfrac{1}{2})x-\delta)\sin\frac{x}{2}\right|^q x^{-(q+1)}\,dx$$

$$\geq \left(\frac{2}{\pi}\right)^q\int_{1/k}^{1}|\cos((k+\tfrac{1}{2})x-\delta)|^q x^{-1}\,dx$$

$$\geq C_3\log k,$$

where C_3 is a constant not depending on k. This completes the proof of Lemma 2.

PROOF OF LEMMA 3. By Egorov's theorem, we have a closed set E of positive measure such that $T_Rg(x)$ converges uniformly on E. Then the integral

$$U_R = \int_R^{R+h}\tau_\alpha g(y)\frac{J_\alpha(xy)}{(xy)^\alpha}y^{2\alpha+1}\,dy$$

converges to 0 as $R\to\infty$, uniformly in $x\in E$ and $0\leq h\leq 1$. Without loss of generality, we may assume that $E\subset(a,b)$ with $0<a<b<\infty$ and $R\geq a^{-1}$. By the asymptotic formula (5), we have

$$U_R = \frac{(2/\pi)^{1/2}}{x^{\alpha+1/2}}\int_R^{R+h}\tau_\alpha g(y)\cos(xy-\delta)y^{\alpha+1/2}\,dy$$

$$+\int_R^{R+h}\tau_\alpha g(y)O((xy)^{-3/2})(xy)^{-\alpha}y^{2\alpha+1}\,dy$$

$$= \frac{(2/\pi)^{1/2}}{x^{\alpha+1/2}}V_R + W_R, \quad \text{say,}$$

and thus $|V_R|\leq C_4(|U_R|+|W_R|)$ for $x\in E$ and $0\leq h\leq 1$, where C_4 is a constant depending only on a, b and α. We have

$$|W_R| \leq C_5\left|\int_R^{R+h}\tau_\alpha g(y)y^{-(\alpha+3/2)}y^{2\alpha+1}\,dy\right|$$

$$\leq C_5\left(\int_R^{R+h}|\tau_\alpha g(y)|^q y^{2\alpha+1}\,dy\right)^{1/q}\cdot\left(\int_R^{R+h}y^{-p(\alpha+3/2)}y^{2\alpha+1}\,dy\right)^{1/p},$$

where C_5 is a constant depending only on a, b and α. Since $\|\tau_\alpha g\|_{\alpha,q} < \infty$ and $(4\alpha+2)/(2\alpha+3) \le p \le 2$, it follows that $W_R = o(1)$ $(R \to \infty)$ uniformly in $x \in E$ and $0 \le h \le 1$ which implies that $V_R = o(1)$ $(R \to \infty)$ uniformly in $x \in E$ and $0 \le h \le 1$. We write V_R in the form

$$\int_R^{R+h} \cos xy \, d\chi_1(y) + \sin xy \, d\chi_2(y),$$

where

$$d\chi_1(y) = (\cos\delta)\tau_\alpha g(y) y^{\alpha+1/2}\, dy,$$
$$d\chi_2(y) = (\sin\delta)\tau_\alpha g(y) y^{\alpha+1/2}\, dy.$$

By the proof of [**Z**, Chapter XVI Theorem (8.4)], which is a trigonometric integral analogue of the Cantor-Lebesgue theorem, we have that $\int_R^{R+h} d\chi_j(y) = o(1)$ $(R \to \infty)$ uniformly in $0 \le h \le 1$ for $j = 1, 2$ and thus

$$\int_R^{R+h} \tau_\alpha g(y) y^{\alpha+1/2}\, dy = \int_R^{R+h} \cos\delta \, d\chi_1(y) + \sin\delta\, d\chi_2(y)$$
$$= o(1) \qquad (R \to \infty),$$

uniformly in $0 \le h \le 1$. This completes the proof of Lemma 3.

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DEPARTMENT OF MATHEMATICS, COLLEGE OF LIBERAL ARTS, KANAZAWA UNIVERSITY, KANAZAWA 920, JAPAN