

CONVERGENCE AND DIVERGENCE ALMOST EVERYWHERE OF SPHERICAL MEANS FOR RADIAL FUNCTIONS

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(Communicated by Richard R. Goldberg)

ABSTRACT. Let $d > 1$. It will be shown that the maximal operator S^* of spherical means S_R , $R > 0$, is bounded on $L^p(\mathbf{R}^d)$ radial functions when $2d/(d+1) < p < 2d/(d-1)$, and it implies that, for every $L^p(\mathbf{R}^d)$ radial function $f(t)$, $S_R f(t)$ converges to $f(t)$ for a.e. $t \in \mathbf{R}^d$ when $2d/(d+1) < p \leq 2$. Also, it will be proved that there is an $L^{2d/(d+1)}(\mathbf{R}^d)$ radial function $f(t)$ with compact support such that $S_R f(t)$ diverges for a.e. $t \in \mathbf{R}^d$.

Introduction. Define linear operators S_R , $R > 0$, and S^* on $L^2(\mathbf{R}^d)$ by

$$S_R f(t) = (2\pi)^{-d/2} \int_{|\xi| \leq R} \hat{f}(\xi) e^{it\xi} d\xi,$$

and

$$S^* f(t) = \sup_{R > 0} |S_R f(t)|, \quad t \in \mathbf{R}^d,$$

respectively, where $\hat{f}(\xi)$ is the Fourier transform of a function $f(t)$, that is, $\hat{f}(\xi) = (2\pi)^{-d/2} \int_{\mathbf{R}^d} f(t) e^{-i\xi t} dt$, $\xi \in \mathbf{R}^d$.

We shall assume for the rest of the paper that $d > 1$. We put $S = S_1$. Fefferman [F] showed that S is bounded on $L^p(\mathbf{R}^d)$ if and only if $p = 2$. On the other hand, Herz [H] showed that, when restricted to $L^p(\mathbf{R}^d)$ radial functions, S is bounded if and only if $2d/(d+1) < p < 2d/(d-1)$. In this connection, Kenig and Tomas [KT1] showed that, for $p = 2d/(d+1)$ and $p = 2d/(d-1)$, S is not weak type on $L^p(\mathbf{R}^d)$ radial functions, and Chanillo [C] showed that S is restricted weak type on $L^p(\mathbf{R}^d)$ radial functions for $p = 2d/(d+1)$.

This paper deals with boundedness of S^* on $L^p(\mathbf{R}^d)$ radial functions. We shall prove a theorem which transplants a norm inequality for the maximal operators defined by Fourier Jacobi multipliers to a corresponding inequality for the maximal operators defined by Fourier Hankel multipliers. It follows from the theorem that S^* is bounded on $L^p(\mathbf{R}^d)$ radial functions if $2d/(d+1) < p < 2d/(d-1)$, which implies that $S_R f(t)$ converges to $f(t)$ almost everywhere for every $L^p(\mathbf{R}^d)$ radial function $f(t)$ if $2d/(d+1) < p \leq 2$. We shall also show that there is an $L^{2d/(d+1)}(\mathbf{R}^d)$ radial function $f(t)$ with compact support such that $S_R f(t)$ diverges almost everywhere.

Received by the editors April 17, 1987.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 42B15; Secondary 42C10.

Key words and phrases. Maximal operator of spherical means for radial functions, a.e. convergence, a.e. divergence, transplantation theorem, Hankel transform.

1. Statements of results. Let $\alpha \geq -1/2$. For a function $g(x)$ on the interval $(0, \infty)$, the Hankel transform $\tau_\alpha g(y)$, $y \geq 0$, of order α is given by

$$\tau_\alpha g(y) = \int_0^\infty g(x) \frac{J_\alpha(yx)}{(yx)^\alpha} x^{2\alpha+1} dx,$$

where $J_\alpha(x)$ is the Bessel function of the first kind of order α . We define

$$L_\alpha^p = \left\{ g(x) \text{ on } (0, \infty); \|g\|_{\alpha,p} = \left(\int_0^\infty |g(x)|^p x^{2\alpha+1} dx \right)^{1/p} < \infty \right\}.$$

Then it is known that $\|\tau_\alpha g\|_{\alpha,q} \leq C_{\alpha,p} \|g\|_{\alpha,p}$ for $g(x)$ in L_α^p if $1 \leq p \leq 2$, where $1/p + 1/q = 1$ and $C_{\alpha,p}$ is a constant depending only on α and p (cf. [H]).

Let $\lambda(y) \in L_\alpha^\infty$. Define operators Λ_R , $R > 0$, and Λ^* on L_α^2 by

$$\tau_\alpha(\Lambda_R g)(y) = \lambda(y/R) \tau_\alpha g(y) \quad \text{and} \quad \Lambda^* g(x) = \sup_{R>0} |\Lambda_R g(x)|,$$

respectively.

Let $P_n^{(\alpha,\beta)}(u)$ denote the Jacobi polynomial of degree n and order (α, β) , $\alpha, \beta > -1$ defined by

$$(1-u)^\alpha (1+u)^\beta P_n^{(\alpha,\beta)}(u) = \frac{(-1)^n}{2^n n!} \left(\frac{d}{du} \right)^n \{ (1-u)^{n+\alpha} (1+u)^{n+\beta} \}.$$

The functions $P_n^{(\alpha,\beta)}(\cos \theta)$ are orthogonal on $(0, \pi)$ with respect to

$$d\mu(\theta) = (\sin \theta/2)^{2\alpha+1} (\cos \theta/2)^{2\beta+1} d\theta.$$

We define

$$L_{(\alpha,\beta)}^p = \left\{ h(\theta) \text{ on } (0, \pi); \|h\|_{(\alpha,\beta),p} = \left(\int_0^\pi |h(\theta)|^p d\mu(\theta) \right)^{1/p} < \infty \right\}.$$

For a function $h(\theta)$ in $L_{(\alpha,\beta)}^1$, we have the Fourier Jacobi series

$$h(\theta) = \sum_{n=0}^\infty \tilde{h}(n) \rho_n P_n^{(\alpha,\beta)}(\cos \theta),$$

where

$$\tilde{h}(n) = \int_0^\pi h(\theta) P_n^{(\alpha,\beta)}(\cos \theta) d\mu(\theta),$$

and

$$\rho_n = \left[\int_0^\pi \{ P_n^{(\alpha,\beta)}(\cos \theta) \}^2 d\mu(\theta) \right]^{-1}.$$

Define operators $\tilde{\Lambda}_R$, $R > 0$, and $\tilde{\Lambda}^*$ on $L_{(\alpha,\beta)}^2$ by

$$(\tilde{\Lambda}_R h)\tilde{h}(n) = \lambda(n/R) \tilde{h}(n) \quad \text{and} \quad \tilde{\Lambda}^* h(\theta) = \sup_{R>0} |\tilde{\Lambda}_R h(\theta)|,$$

respectively.

We are now ready to state the first theorem.

THEOREM 1. *Let $\alpha, \beta \geq -1/2$. Let $1 < p < \infty$ and let $\lambda(y)$ be a function in L^∞_α continuous for almost every y . If Λ^* is bounded on $L^p_{(\alpha,\beta)}$, then Λ^* is bounded on L^p_α .*

Define linear operators T_R , $R > 0$, and T^* on L^2_α by

$$T_R g(x) = \int_0^R \tau_\alpha g(y) \frac{J_\alpha(xy)}{(xy)^\alpha} y^{2\alpha+1} dy,$$

and

$$T^* g(x) = \sup_{R>0} |T_R g(x)|, \quad x \geq 0,$$

respectively. Let $\chi_{(0,1)}(y)$ be the characteristic function of the interval $(0, 1)$. For $\lambda(y) = \chi_{(0,1)}(y)$, we notice that $\Lambda^* = T^*$ and $\tilde{\Lambda}^*$ is the maximal operator of partial sum operators of the Fourier Jacobi series. From Badkov's theorem [B, Theorem 1.7], it follows that $\tilde{\Lambda}^*$ is bounded on $L^p_{(\alpha,\beta)}$ if

$$\begin{aligned} \max\{4(\alpha + 1)/(2\alpha + 3), 4(\beta + 1)/(2\beta + 3)\} < p \\ < \min\{4(\alpha + 1)/(2\alpha + 1), 4(\beta + 1)/(2\beta + 1)\} \end{aligned}$$

and $\alpha, \beta \geq -1/2$. Thus, Theorem 1 yields Corollary 1 from which Corollary 2 is obtained by using routine method.

COROLLARY 1. *Let $\alpha \geq -1/2$ and $4(\alpha + 1)/(2\alpha + 3) < p < 4(\alpha + 1)/(2\alpha + 1)$. Then, $\|T^* g\|_{\alpha,p} \leq C \|g\|_{\alpha,p}$ for $g(x)$ in L^p_α , where C is a constant depending only on α and p .*

COROLLARY 2. *Let $\alpha \geq -1/2$ and $4(\alpha + 1)/(2\alpha + 3) < p \leq 2$. Then, for $g(x)$ in L^p_α , $T_R g(x)$ converges to $g(x)$ for almost every x in $(0, \infty)$ as $R \rightarrow \infty$.*

By familiar relationship between Fourier transforms of radial functions and Hankel transforms, these corollaries are restated as follows.

COROLLARY 3. *Let $2d/(d + 1) < p < 2d/(d - 1)$. Then, $\|S^* f\|_p \leq C \|f\|_p$ for $L^p(\mathbf{R}^d)$ radial functions $f(t)$, where C is a constant depending only on d and p .*

COROLLARY 4. *Let $2d/(d + 1) < p \leq 2$. Then, for every $L^p(\mathbf{R}^d)$ radial function $f(t)$, $S_R f(t)$ converges to $f(t)$ for almost every t in \mathbf{R}^d as $R \rightarrow \infty$.*

On the divergence, the following theorem holds.

THEOREM 2. *Let $\alpha > -1/2$. If $p = 4(\alpha + 1)/(2\alpha + 3)$, then there is a function $g(x)$ in L^p_α with compact support such that $T_R g(x)$ diverges almost everywhere in $(0, \infty)$ as $R \rightarrow \infty$.*

COROLLARY 5. *If $p = 2d/(d + 1)$, then there is an $L^p(\mathbf{R}^d)$ radial function $f(t)$ with compact support such that $S_R f(t)$ diverges almost everywhere in \mathbf{R}^d as $R \rightarrow \infty$.*

2. Proofs. A formulation (Lemma 1) of linearization of Kenig and Tomas [KT2] in our situation will enable us to prove Theorem 1 by following the proof of the theorem of Igari [I] which transplants a norm inequality for Fourier Jacobi multipliers to a corresponding inequality for Fourier Hankel multipliers. Theorem 2 will be proved by an argument similar to that used in Stanton and Tomas [ST] to prove divergence of central Fourier series on compact Lie groups, or in Meaney [M] to prove divergence of Jacobi polynomial series. See also Sogge [S].

LEMMA 1 [KT2, §1, LEMMA]. Let $1 < p < \infty$ and $1/p + 1/q = 1$. Let $\lambda(y) \in L^\infty_\alpha$.

(i) The operator $\tilde{\Lambda}^*$ is bounded on $L^p_{(\alpha,\beta)}$ if and only if

$$(1) \quad \left\| \sum_{k=1}^\infty \tilde{\Lambda}_{R_k} h_k \right\|_{(\alpha,\beta),q} \leq C \left\| \sum_{k=1}^\infty |h_k| \right\|_{(\alpha,\beta),q},$$

where C is a constant not depending on sequences $\{h_k(\theta)\}_{k=1}^\infty \subset L^q_{(\alpha,\beta)}$ and $\{R_k\}_{k=1}^\infty, R_k > 0$.

(ii) The operator Λ^* is bounded on L^p_α if and only if

$$(2) \quad \left\| \sum_{k=1}^\infty \Lambda_{R_k} g_k \right\|_{\alpha,q} \leq C' \left\| \sum_{k=1}^\infty |g_k| \right\|_{\alpha,q},$$

where C' is a constant not depending on sequences $\{g_k(x)\}_{k=1}^\infty \subset L^q_\alpha$ and $\{R_k\}_{k=1}^\infty, R_k > 0$.

Now we prove Theorem 1. Assume that (1) is valid. It suffices to show that (2) is valid for finite sequences $\{g_k(x)\}_{k=1}^m$ such that every $g_k(x)$ is an infinitely differentiable function with compact support in $(0, \infty)$. For $j > 0$, define $g_{k,j}(\theta) = g_k(j\theta)$. Then $\text{supp } g_{k,j}(\theta) \subset (0, \pi)$ for large $j > 0$ and $k = 1, 2, 3, \dots, m$. We consider $g_{k,j}(\theta)$ as a function belonging to $L^q_{(\alpha,\beta)}$. By the assumption, we have

$$(3) \quad \left\| \sum_{k \in S} \tilde{\Lambda}_{jR_k} g_{k,j} \right\|_{(\alpha,\beta),q} \leq C \left\| \sum_{k \in S} |g_{k,j}| \right\|_{(\alpha,\beta),q},$$

where S is any subset of $\{1, 2, 3, \dots, m\}$. Multiplying $\{j(2j)^{2\alpha+1}\}^{1/q}$ by both sides of (3) and changing variable, we have

$$\begin{aligned} & \left\{ \int_0^{j\pi} \left| \sum_{k \in S} G_k(x, j) \right|^q x^{2\alpha+1} \left(\frac{2j}{x} \sin \frac{x}{2j} \right)^{2\alpha+1} \left(\cos \frac{x}{2j} \right)^{2\beta+1} dx \right\}^{1/q} \\ & \leq C \left\{ \int_0^{j\pi} \left(\sum_{k \in S} |g_k(x)| \right)^q x^{2\alpha+1} \left(\frac{2j}{x} \sin \frac{x}{2j} \right)^{2\alpha+1} \left(\cos \frac{x}{2j} \right)^{2\beta+1} dx \right\}^{1/q} \\ & \leq C \left\| \sum_{k \in S} |g_k| \right\|_{\alpha,q}, \end{aligned}$$

where

$$\begin{aligned} G_k(x, j) &= \tilde{\Lambda}_{jR_k} g_{k,j}(x/j) \\ &= \sum_{n=0}^\infty \lambda \left(\frac{n}{jR_k} \right) \tilde{g}_{k,j}(n) \rho_n P_n^{(\alpha,\beta)} \left(\cos \frac{x}{j} \right). \end{aligned}$$

Fix an arbitrary positive number K . For $j > 3K/2\pi$, we have

$$\left(\frac{2j}{x} \sin \frac{x}{2j} \right)^{2\alpha+1} \left(\cos \frac{x}{2j} \right)^{2\beta+1} \geq \left(\frac{\sqrt{3}}{2} / \frac{\pi}{3} \right)^{2\alpha+1} \left(\frac{1}{2} \right)^{2\beta+1}, \quad x \in (0, K).$$

Thus we have

$$(4) \quad \left\{ \int_0^K \left| \sum_{k \in S} G_k(x, j) \right|^q x^{2\alpha+1} dx \right\}^{1/q} \leq C'' \left\| \sum_{k \in S} |g_k| \right\|_{\alpha, q}$$

for $j > 3K/2\pi$, where C'' is a constant depending only on α, β and q . Notice that (4) is valid for $q = 2$ since $\tilde{\Lambda}^*$ and Λ^* are bounded on $L^2_{(\alpha, \beta)}$ and L^2_α , respectively. Considering the case of $S = \{k\}$, $k = 1, 2, 3, \dots, m$, and taking a subsequence if necessary, we have that the sequence $\{G_k(x, j)\}_j$ converges weakly to a function $G_k(x)$ in $L^q_\alpha(0, K)$ and, simultaneously, in $L^2_\alpha(0, K)$ for every k , where $L^q_\alpha(0, K) = \{g(x) \in L^q_\alpha; \text{supp } g(x) \subset (0, K)\}$. From the weak convergence of $\{G_k(x, j)\}_j$ in $L^q_\alpha(0, K)$ and the inequality (4) with $S = \{1, 2, 3, \dots, m\}$, it follows that

$$\left\{ \int_0^K \left| \sum_{k=1}^m G_k(x) \right|^q x^{2\alpha+1} dx \right\}^{1/q} \leq C'' \left\| \sum_{k=1}^m |g_k| \right\|_{\alpha, q}.$$

To complete the proof, it suffices to obtain $G_k(x) = \Lambda_{R_k} g_k(x)$ a.e. $x \in (0, K)$ from the fact that $G_k(x)$ is the weak limit of $\{G_k(x, j)\}_j$ in $L^2_\alpha(0, K)$. But, it is a strict imitation of the method of [I, p. 203, l.10~], and so we omit it.

Next we turn to the proof of Theorem 2. We divide the proof into two lemmas.

LEMMA 2. Let $\alpha > -1/2$ and $p = 4(\alpha+1)/(2\alpha+3)$. Define a sequence $\{\varphi_k\}_{k=1}^\infty$ of bounded linear functionals of the space $L^p_\alpha(0, 1)$ by $\varphi_k(g) = \int_k^{k+1} \tau_\alpha g(y) y^{\alpha+1/2} dy$. Then the norms $\|\varphi_k\|$ of the functionals φ_k satisfy that $\|\varphi_k\| \geq C(\log k)^{1/q}$, where $q = 4(\alpha+1)/(2\alpha+1)$ and C is a constant not depending on k .

LEMMA 3. Let $\alpha > -1/2$ and $(4\alpha+2)/(2\alpha+3) \leq p \leq 2$. If a function $g(x)$ in L^p_α satisfies the condition that $T_R g(x)$ converges on a set E of positive measure, then $\lim_{R \rightarrow \infty} \int_R^{R+h} \tau_\alpha g(y) y^{\alpha+1/2} dy = 0$ uniformly in $0 \leq h \leq 1$.

By Lemma 2, there is a function $g(x)$ in $L^p_\alpha(0, 1)$, $p = 4(\alpha+1)/(2\alpha+3)$ such that

$$\limsup_{k \rightarrow \infty} \left| \int_k^{k+1} \tau_\alpha g(y) y^{\alpha+1/2} dy \right| = \infty.$$

It follows from Lemma 3 that $T_R g(x)$ diverges almost everywhere in $(0, \infty)$ as $R \rightarrow \infty$, which completes the proof of Theorem 2.

PROOF OF LEMMA 2. By Fubini's theorem, we have

$$\varphi_k(g) = \int_0^1 g(x) \left\{ \int_k^{k+1} \frac{J_\alpha(yx)}{(yx)^\alpha} y^{\alpha+1/2} dy \right\} x^{2\alpha+1} dx,$$

and thus

$$\|\varphi_k\| = \left(\int_0^1 \left| \int_k^{k+1} \frac{J_\alpha(yx)}{(yx)^\alpha} y^{\alpha+1/2} dy \right|^q x^{2\alpha+1} dx \right)^{1/q}.$$

By the asymptotic formula

$$(5) \quad J_\alpha(z) = \left(\frac{2}{\pi z} \right)^{1/2} \cos(z - \delta) + O(z^{-3/2}) \quad z \rightarrow \infty, \delta = (2\alpha+1)\pi/4,$$

we have

$$\begin{aligned} \|\varphi_k\| &\geq \left(\int_{1/k}^1 \left| \int_k^{k+1} \frac{J_\alpha(yx)}{(yx)^\alpha} y^{\alpha+1/2} dy \right|^q x^{2\alpha+1} dx \right)^{1/q} \\ &\geq C_1 \left(\int_{1/k}^1 \left| \int_k^{k+1} \cos(yx - \delta) dy \right|^q x^{2\alpha+1-q(\alpha+1/2)} dx \right)^{1/q} \\ &\quad - C_2 \left(\int_{1/k}^1 \left| \int_k^{k+1} y^{-1} dy \right|^q x^{2\alpha+1-q(\alpha+3/2)} dx \right)^{1/q} \\ &= C_1 A_k - C_2 B_k, \quad \text{say,} \end{aligned}$$

where C_1 and C_2 are positive constants not depending on k . The terms B_k satisfy that

$$B_k = \left\{ \log \left(1 + \frac{1}{k} \right) \right\} \left(\frac{k^q - 1}{q} \right)^{1/q} = O(1) \quad (k \rightarrow \infty).$$

By a simple calculation, we have

$$\begin{aligned} A_k^q &= 2^q \int_{1/k}^1 \left| \cos\left(\left(k + \frac{1}{2}\right)x - \delta\right) \sin \frac{x}{2} \right|^q x^{-(q+1)} dx \\ &\geq \left(\frac{2}{\pi}\right)^q \int_{1/k}^1 |\cos\left(\left(k + \frac{1}{2}\right)x - \delta\right)|^q x^{-1} dx \\ &\geq C_3 \log k, \end{aligned}$$

where C_3 is a constant not depending on k . This completes the proof of Lemma 2.

PROOF OF LEMMA 3. By Egorov's theorem, we have a closed set E of positive measure such that $T_R g(x)$ converges uniformly on E . Then the integral

$$U_R = \int_R^{R+h} \tau_\alpha g(y) \frac{J_\alpha(xy)}{(xy)^\alpha} y^{2\alpha+1} dy$$

converges to 0 as $R \rightarrow \infty$, uniformly in $x \in E$ and $0 \leq h \leq 1$. Without loss of generality, we may assume that $E \subset (a, b)$ with $0 < a < b < \infty$ and $R \geq a^{-1}$. By the asymptotic formula (5), we have

$$\begin{aligned} U_R &= \frac{(2/\pi)^{1/2}}{x^{\alpha+1/2}} \int_R^{R+h} \tau_\alpha g(y) \cos(xy - \delta) y^{\alpha+1/2} dy \\ &\quad + \int_R^{R+h} \tau_\alpha g(y) O((xy)^{-3/2}) (xy)^{-\alpha} y^{2\alpha+1} dy \\ &= \frac{(2/\pi)^{1/2}}{x^{\alpha+1/2}} V_R + W_R, \quad \text{say,} \end{aligned}$$

and thus $|V_R| \leq C_4(|U_R| + |W_R|)$ for $x \in E$ and $0 \leq h \leq 1$, where C_4 is a constant depending only on a, b and α . We have

$$\begin{aligned} |W_R| &\leq C_5 \left| \int_R^{R+h} \tau_\alpha g(y) y^{-(\alpha+3/2)} y^{2\alpha+1} dy \right| \\ &\leq C_5 \left(\int_R^{R+h} |\tau_\alpha g(y)|^q y^{2\alpha+1} dy \right)^{1/q} \cdot \left(\int_R^{R+h} y^{-p(\alpha+3/2)} y^{2\alpha+1} dy \right)^{1/p}, \end{aligned}$$

where C_5 is a constant depending only on a , b and α . Since $\|\tau_\alpha g\|_{\alpha,q} < \infty$ and $(4\alpha + 2)/(2\alpha + 3) \leq p \leq 2$, it follows that $W_R = o(1)$ ($R \rightarrow \infty$) uniformly in $x \in E$ and $0 \leq h \leq 1$ which implies that $V_R = o(1)$ ($R \rightarrow \infty$) uniformly in $x \in E$ and $0 \leq h \leq 1$. We write V_R in the form

$$\int_R^{R+h} \cos xy \, d\chi_1(y) + \sin xy \, d\chi_2(y),$$

where

$$\begin{aligned} d\chi_1(y) &= (\cos \delta) \tau_\alpha g(y) y^{\alpha+1/2} dy, \\ d\chi_2(y) &= (\sin \delta) \tau_\alpha g(y) y^{\alpha+1/2} dy. \end{aligned}$$

By the proof of [Z, Chapter XVI Theorem (8.4)], which is a trigonometric integral analogue of the Cantor-Lebesgue theorem, we have that $\int_R^{R+h} d\chi_j(y) = o(1)$ ($R \rightarrow \infty$) uniformly in $0 \leq h \leq 1$ for $j = 1, 2$ and thus

$$\begin{aligned} \int_R^{R+h} \tau_\alpha g(y) y^{\alpha+1/2} dy &= \int_R^{R+h} \cos \delta \, d\chi_1(y) + \sin \delta \, d\chi_2(y) \\ &= o(1) \quad (R \rightarrow \infty), \end{aligned}$$

uniformly in $0 \leq h \leq 1$. This completes the proof of Lemma 3.

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