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EXISTENCE OF DECAYING ENTIRE SOLUTIONS OF A CLASS OF SEMILINEAR ELLIPTIC EQUATIONS

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ABSTRACT. The main result establishes the existence of a nontrivial nonnegative radial solution $u \in C^2_{\text{loc}}(\mathbf{R}^N)$ of a semilinear elliptic eigenvalue problem in $\mathbf{R}^N$, $N \geq 3$, such that $u(|x|)$ has uniform limit zero as $|x| \to \infty$. Asymptotic decay estimates and necessary conditions are obtained. Since such solutions do not exist in the space $W_0^{1,2}(\mathbf{R}^N)$, a considerable departure from standard procedures is required.

The existence of a number $\lambda > 0$ and an associated nontrivial solution $u(x) \geq 0$ in $\mathbf{R}^N$, $N \geq 3$, will be proved for semilinear elliptic eigenvalue problems of the type[12]

$$-\Delta u + p_1(|x|)f_1(u) = \lambda p_2(|x|)f_2(u), \qquad x \in \mathbf{R}^N,\ N \geq 3, \tag{1}$$

$$\lim_{|x|\to\infty} u(x) = 0, \tag{2}$$

where Δ denotes the N-dimensional Laplacian and p_1, p_2, f_1, f_2 are locally Hölder continuous functions in $[0, \infty)$ satisfying growth conditions to be listed later. Such a solution $u \in C^2_{\text{loc}}(\mathbf{R}^N)$ is termed a *decaying entire* solution of equation (1). The prototype with $f_1(u) = u^\beta$, $f_2(u) = u^\gamma$, $1 < \gamma < \beta$, and constants p_1, p_2 (i.e., $a = 0$ in (H_1) below) has many well-known applications in quantum field theory, fluid mechanics, geometry, and other areas [**4**, **10**, **14**, **17**].

For some $\lambda > 0$, a nontrivial nonnegative entire solution $u(|x|)$ of (1) will be established satisfying the decay law

$$u(|x|) = O(|x|^{(2-N)/2}) \quad \text{as } |x| \to \infty. \tag{3}$$

For the equation considered by Strauss [**14**]:

$$-\Delta u + a_0 u + f_1(u) = \lambda f_2(u), \qquad a_0 > 0, \tag{4}$$

and for more general equations [**2**, **12**], the presence of a positive constant a_0 permits a variational approach to be used to construct a solution $u(|x|)$ in the Sobolev space $W_0^{1,2}(\mathbf{R}^N)$ which decays exponentially as $|x| \to \infty$. This of course cannot be true if $a_0 = 0$, the case under study here, as shown by easy examples. Generalizations and modifications of (4) considered by Berestycki and Lions [**1**, **2**],

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Berestycki, Lions, and Peletier [**3**], Berger [**4**], Ding and Ni [**5**], and others cited in these works, have character differing from (1) for the same reason.

The case $p_1(r) \equiv 0$ in (1), for which λ is superfluous and can be deleted, has been widely studied, in particular by Gidas and Spruck [**6**], Kawano [**7**], Kusano and Naito [**8**], Kusano and Oharu [**9**], Ni [**10**], and Toland [**15, 16**]. Neither the results nor the methods used in this case appear to be cogent to the present problem.

The method to be used here proceeds as follows: (i) A sequence of differential equations (5) below is considered for which radial solutions u_k in the space $W_0^{1,2}(\mathbf{R}^N)$ are obtained by the procedure of Strauss [**14**], $k = 1, 2, \ldots$.

(ii) Because of the direct variational method of constructing sequences $\{u_{kn} : n = 1, 2, \ldots\}$ which converge weakly in $W_0^{1,2}(\mathbf{R}^N)$ to u_k, a uniform bound on the $L^2(\mathbf{R}^N)$ norm of $|\nabla u_k|$ can be established.

(iii) Use of a uniform bound on $\{u_k\}$ establishes a subsequence of $\{u_k\}$ which converges in $C^2_{\mathrm{loc}}(\mathbf{R}^N)$ to a nonnegative solution of (1) satisfying the decay law (3). The hypotheses for (1) are listed below, where

$$g_i(s) = \int_0^s f_i(t)\,dt, \qquad s \geq 0,\ i = 1, 2.$$

(H_1) p_1 and p_2 are positive, bounded, locally Hölder continuous functions such that $p_2(r) \leq C_0 p_1(r)$ and $p_2(r) \leq C/(1 + r^a)$ in $0 \leq r < \infty$ for some constants $a \geq 0$, $C_0 > 0$, and $C > 0$.

(H_2) f_1 and f_2 are locally Hölder continuous in $[0, \infty)$, $f_1(s) \geq 0$, and $f_2(s) > 0$ for all $s > 0$.

(H_3) $f_1(s) = O(s)$, $f_2(s) = O(s^\gamma)$ as $s \to 0+$, where $\gamma > 1$ and

$$\gamma > (N + 2 - 2a)/(N - 2) \quad \text{if } 0 \leq a \leq 2.$$

(H_4) $f_2(s) = o(f_1(s))$ and $f_2(s) = O(g_1(s)/s)$ as $s \to \infty$.

(H_5) There exist positive constants C_1 and C_2 such that $sf_1(s) \leq C_1 g_1(s)$ and $sf_2(s) \geq C_2 g_2(s)$ for all $s \geq 0$.

REMARK. The case $a > 2$ also can be handled by a different procedure based on the existence of decaying entire positive solutions of linear elliptic equations and a comparison technique [**13**, Theorem 6]. This is possible in this case because $\int^\infty r p_2(r)\,dr < \infty$ by (H_1). We are unaware of any earlier methods which are applicable to the case $0 < a \leq 2$.

THEOREM. *Under these assumptions, there exists $\lambda > 0$ and a corresponding nontrivial solution $u(|x|) \geq 0$ of* (1) *in $\mathbf{R}^N$ satisfying the decay law* (3).

PROOF. Let E denote the subspace of all radial functions in $W_0^{1,2}(\mathbf{R}^N)$. Consider the sequence of problems

$$(5) \qquad -\Delta u_k + (1/k)u_k + p_1(r) f_1(u_k) = \lambda_k p_2(r) f_2(u_k), \qquad u_k \in E,\ r = |x| \geq 0,\ k = 1, 2, \ldots.$$

Let $\{f_{in}\}$ be the sequence of truncated functions defined by

$$f_{in}(s) = \begin{cases} f_i(s) & \text{if } s \leq n, \\ f_i(n) & \text{if } s > n, \end{cases}$$

for $n = 1, 2, \ldots$, $i = 1, 2$, and define

$$g_{in}(s) = \int_0^s f_{in}(t)\,dt, \qquad s \geq 0,\ i = 1, 2.$$

A slight modification of Lemma 3 of Strauss [**14**, p. 157] shows, for fixed k and n, that there exists a radial function u_{kn} which minimizes the functional

$$I_{kn}(u) = \frac{1}{2}\int_{\mathbf{R}^N}\left[|\nabla u|^2 + \frac{u^2}{k} + 2p_1 g_{1n}(u)\right] dx \tag{6}$$

subject to the constraints $u \in E$ and

$$J_n(u) = \int_{\mathbf{R}^N} p_2 g_{2n}(u)\,dx = 1. \tag{7}$$

By the Euler-Lagrange principle, there exists $\lambda_{kn} \in \mathbf{R}$ such that $I'_{kn}(u_{kn}) = \lambda_{kn} J'_n(u_{kn})$ in the dual space E^*, where the Fréchet derivatives are given by

$$I'_{kn}(u)v = \int_{\mathbf{R}^N}\left[\nabla u \cdot \nabla v + \frac{uv}{k} + p_1 f_{1n}(u)v\right] dx,$$

$$J'_n(u)v = \int_{\mathbf{R}^N} p_2 f_{2n}(u)v\,dx, \qquad u, v \in E.$$

(Clearly, $\lambda_{kn} \geq 0$ since the case $v = u$ is included.) This means that u_{kn} is a weak solution of the problem

$$-\Delta u_{kn} + \frac{1}{k}u_{kn} + p_1 f_{1n}(u_{kn}) = \lambda_{kn} p_2 f_{2n}(u_{kn}) \tag{8}$$

$$u_{kn} \in E,\ k, n = 1, 2, \ldots.$$

A standard bootstrap argument implies in view of the regularity hypotheses (H_1), (H_2) that $u_{kn} \in C^{2+\alpha}_{\mathrm{loc}}(\mathbf{R}^N)$ for some $\alpha \in (0, 1)$.

In analogy with (6), (7), the functionals associated with (5) are

$$I_k(u) = \frac{1}{2}\int_{\mathbf{R}^N}\left[|\nabla u|^2 + \frac{u^2}{k} + 2p_1 g_1(u)\right] dx, \tag{9}$$

$$J(u) = \int_{\mathbf{R}^N} p_2 g_2(u)\,dx, \qquad u \in E. \tag{10}$$

Since $g_2(s) > 0$ for all $s > 0$ by (H_2), there exists a radial function $\phi \in C_0^\infty(\mathbf{R}^N)$ such that $J(\phi) = 1$. If $n \geq \sup|\phi(x)|$, then $g_{in}(\phi) = g_i(\phi)$, $i = 1, 2$, and it follows from (6), (9), and the variational definition of u_{kn} that

$$I_{kn}(u_{kn}) \leq I_{1n}(\phi) = I_1(\phi) < \infty. \tag{11}$$

This implies that there exists a constant M, independent of k and n, such that

$$\int_{\mathbf{R}^N} |\nabla u_{kn}|^2\,dx \leq M \tag{12}$$

and

$$\|u_{kn}\|_{1,2}^2 \leq 2kM, \qquad k, n = 1, 2, \ldots, \tag{13}$$

where $\|\ \|_{1,2}$ denotes the norm in E. For fixed k, we conclude from (13) that $\{u_{kn}\}$ has a subsequence which converges weakly in E to a limit u_k. The proof given by

Strauss [**14**, pp. 159–160] shows that u_k is a nontrivial nonnegative exponentially decaying entire solution of (5), where λ_k in (5) is the limit of some subsequence of the (bounded) sequence $\{\lambda_{kn}\colon n = 1, 2, \ldots\}$. Closely related results appear in [**1**, **12**]. The weak convergence of u_{kn} to u_k in $L^2(\mathbf{R}^N)$ implies that $\partial u_{kn}/\partial x_i$ converges weakly in $L^2(\mathbf{R}^N)$ to $\partial u_k/\partial x_i$ for each $i = 1, \ldots, N$, and consequently

$$\int_{\mathbf{R}^N} |\nabla u_k|^2\,dx \le \liminf_{n\to\infty} \int_{\mathbf{R}^N} |\nabla u_{kn}|^2\,dx. \tag{14}$$

It follows from (12) and (14) that the sequence of norms $\|\nabla u_k\|_{L^2(\mathbf{R}^N)}$ is uniformly bounded. We now apply an estimate of Berestycki and Lions [**2**, Lemma AIII] for radial functions in $W_0^{1,2}(\mathbf{R}^N)$:

$$u_k(r) \le A r^{(2-N)/2} \|\nabla u_k\|_{L^2(\mathbf{R}^N)}, \qquad r \ge 1,$$

to conclude that there exists a constant A', independent of k, such that

$$0 \le u_k(r) \le A' r^{(2-N)/2}, \qquad r \ge 1. \tag{15}$$

To prove that $\{\lambda_k\}$ is bounded, we multiply (5) by u_k and integrate by parts to obtain

$$\int_{\mathbf{R}^N} \left[|\nabla u_k|^2 + \frac{1}{k} u_k^2 + p_1 u_k f_1(u_k) \right] dx = \lambda_k \int_{\mathbf{R}^N} p_2 u_k f_2(u_k)\,dx. \tag{16}$$

In the limit $n \to \infty$, the constraint (7) on u_{kn} becomes $J(u_k) = 1$ (see (10)), and hence (H_5) shows that

$$\int_{\mathbf{R}^N} p_2 u_k f_2(u_k)\,dx \ge C_2 \int_{\mathbf{R}^N} p_2 g_2(u_k)\,dx = C_2 J(u_k) = C_2 > 0.$$

Let $\tilde{C}_1 = \max(C_1, 2)$, where C_1 is as in (H_5). Then the left side of (16) is bounded above by

$$\frac{1}{2}\tilde{C}_1 \int_{\mathbf{R}^N} \left[|\nabla u_k|^2 + \frac{1}{k} u_k^2 + 2 p_1 g_1(u_k) \right] dx = \tilde{C}_1 I_k(u_k) = \tilde{C}_1 I_1(\phi),$$

where ϕ is as in (11), and therefore (16) shows that $\{\lambda_k\}$ is bounded.

In view of the uniform estimate (15), there exists a subsequence of $\{u_k(r)\}$ which converges locally uniformly in $C^2(\mathbf{R}^N)$ to a function $u \in C^2_{\mathrm{loc}}(\mathbf{R}^N)$. This is proved in the usual way from interior L^p-estimates and Schauder estimates on bounded domains in $\mathbf{R}^N$, permitting the selection of a convergent diagonal subsequence in $C^2_{\mathrm{loc}}(\mathbf{R}^N)$. We choose a subsequence $\lambda_k \to \lambda \ge 0$ in (5) and conclude in the limit $k \to \infty$ that $u = \lim_{k\to\infty} u_k$ is a nonnegative entire solution of (1).

To prove that u is nontrivial, first note from (H_3) and (H_5) that there exists a constant $C_3 > 0$ such that $0 \le g_2(s) \le C_3 s^{\gamma+1}$ for $0 \le s \le 1$. Let $h(r) = r^{(2-N)/2}$, so $h(r) \le 1$ for $r \ge 1$. Then by (H_1) there is a constant $C_4 > 0$ such that

$$p_2(r) g_2(h(r)) r^{N-1} \le C_4 r^{\delta} \quad \text{for } r \ge 1,$$

where

$$\delta = N - a - 1 + \left(\frac{2-N}{2}\right)(\gamma + 1) = -\frac{(N-2)\gamma}{2} + \frac{N}{2} - a.$$

It follows from the hypothesis on γ in (H_3) that $\delta < -1$, and therefore $J(h)$ converges, i.e., by (10),

$$J(h) = \int_{\mathbf{R}^N} p_2(|x|)g_2(h(|x|))\,dx < \infty.$$

Since $J(u_k) = 1$ and u_k satisfies (15), the dominated convergence theorem shows that

$$J(u) = \lim_{k\to\infty} J(u_k) = 1,$$

and hence u is not identically zero.

If $\lambda = 0$, (1) would imply that u is a nontrivial nonnegative decaying entire solution of $-\Delta u \le 0$, contradicting the maximum principle. Since obviously $\lambda \ge 0$, it follows that $\lambda > 0$. The decay law (3) is immediate from (15). This completes the proof.

A lower bound for the solution $u(|x|)$ in the theorem is obtained under the additional conditions:

(H_6) $p_2(r) = Bp_1(r)$ for some constant $B > 0$, $r \ge 0$;
(H_7) $f_1(s) = o(f_2(s))$ as $s \to 0+$.

COROLLARY 1. *If* (H_1)–(H_7) *hold, the solution u in the theorem satisfies*

$$K_1|x|^{2-N} \le u(|x|) \le K_2|x|^{(2-N)/2}, \qquad |x| \ge 1, \tag{17}$$

for some positive constants K_1 and K_2.

PROOF. It remains to prove the left inequality in (17). Let $r = |x|$. Since $u(r)$ satisfies (1) for some $\lambda > 0$, for arbitrary $\varepsilon > 0$, (H_6) and (H_7) show that there exists a corresponding $\delta > 0$ such that

$$-(\Delta u)(r) > (\lambda B - \varepsilon)p_1(r)f_2(u(r))$$

whenever $u(r) < \delta$. Choose $0 < \varepsilon < \lambda B$ and R large enough that $u(r) < \delta$ for all $r > R$, possible by (3). Then $-(\Delta u)(r) > 0$ for $r > R$. Since $0 \not\equiv u(r) \ge 0$, the strong maximum principle implies that $u(r) > 0$ for $r > R$. The left inequality (17) follows as a well-known consequence of the maximum principle.

The upper estimate in (17) can be sharpened i.e., (3) can be improved under the hypotheses (H_1)–(H_5).

COROLLARY 2. *The solution $u(|x|)$ in the theorem satisfies $u(r) = O(r^{-b})$ as $r \to \infty$ for any constant b in the interval*

$$\frac{N-2}{2} < b < \max\left\{\frac{(N-2)\gamma + 2a - 4}{2}, N-2\right\}. \tag{18}$$

PROOF. Since u satisfies (1), (H_1), (H_3), and (3) imply that there exists a constant $K > 0$ such that

$$-\Delta u \le Kr^{-\delta}, \qquad r = |x| \ge 1, \tag{19}$$

where

$$\delta = a + \frac{(N-2)\gamma}{2} = \frac{(N-2)\gamma + 2a}{2}.$$

Consider the function $v_A(r) = A(1+r^2)^{-b/2}$ for b satisfying (18) and a constant A to be chosen later. Define $D = \frac{1}{2}\min\{bN, b(N-b-2)\}$; $D > 0$, by (18). Differentiation yields

$$-\Delta v_A = \frac{Ab[N+(N-b-2)r^2]}{(1+r^2)^{(b+4)/2}} \geq \frac{2AD}{(1+r^2)^{(b+2)/2}}. \tag{20}$$

The choice of b in (18) means that $b+2 < \delta$. Then (19) and (20) imply the existence of a number $R \geq 1$ such that

$$-\Delta v_A \geq \frac{AD}{r^{b+2}} \geq \frac{K}{r^\delta} \geq -\Delta u, \qquad r \geq R.$$

Now choose A large enough that $v_A(R) \geq u(R)$. For such an A, it follows that

$$\begin{aligned} -\Delta(v_A - u) &\geq 0 \quad \text{for } |x| \geq R, \\ v_A - u &\geq 0 \quad \text{on } |x| = R, \\ v_A - u &\to 0 \quad \text{as } |x| \to \infty, \end{aligned}$$

and the maximum principle implies that $v_A - u \geq 0$ throughout $\{x \in \mathbf{R}^N : |x| \geq R\}$. This means that $u(r) \leq Cr^{-b}$ for some constant $C > 0, r \geq R$.

NECESSARY CONDITIONS. The sharpness of our results can be tested from the known necessary condition [**11**, pp. 76–77]

$$\int^\infty p(r) r^{N-1-\gamma(N-2)}\, dr < \infty \tag{21}$$

for the existence of a positive solution $u(x)$ of the differential inequality $\Delta u + p(r)u^\gamma \leq 0$ in an exterior domain $\Omega_R = \{x \in \mathbf{R}^N : |x| = r > R\}$, $N \geq 3$. As in the proof of Corollary 1, if (H_6) and (H_7) hold, a nontrivial nonnegative solution u of (1), (2) satisfies

$$-(\Delta u)(x) \geq \lambda_1 p_2(r) f_2(u(x)) \tag{22}$$

for all sufficiently large $r = |x|$, say $r > R$, for some positive constant λ_1. Since $-\Delta u \geq 0$ in Ω_R by (22), the strong maximum principle shows that $u > 0$ in Ω_R. If there exist positive constants C and K such that

$$p_2(r) f_2(s) \geq Cr^{-a} s^\gamma, \qquad r > R,\ 0 < s \leq K,$$

then (21) implies the necessary condition $-a + N - 1 - \gamma(N-2) < -1$, equivalent to

$$\gamma > \frac{N-a}{N-2}, \qquad N \geq 3. \tag{23}$$

EXAMPLE. Application of the existence theorem and the necessary condition (23) to the prototype

$$\begin{gathered} -\Delta u = \frac{\lambda u^\gamma - u^\beta}{1+r^a}, \qquad \gamma < \beta,\ a \geq 0, \\ \lim_{|x|\to\infty} u(x) = 0 \end{gathered} \tag{24}$$

yields the following conclusions:

(i) If $a = 2$, the condition $\gamma > 1$ is necessary and sufficient for the existence of a nontrivial nonnegative solution of (24) in $\mathbf{R}^N$ for some $\lambda > 0$.

(ii) If $a > 2$, the condition $\gamma > 1$ is sufficient for the existence of such a solution.

(iii) If $0 \leq a < 2$, no such solution exists if $\gamma \leq (N-a)/(N-2)$, but it does exist if $\gamma > (N+2-2a)/(N-2)$.

The case

$$\frac{N-a}{N-2} < \gamma \leq \frac{N+2-2a}{N-2}$$

remains open. Our results can be extended to generalizations of (1) of the type

$$-\Delta u + f_1(|x|, u) = \lambda f_2(|x|, u), \qquad x \in \mathbf{R}^N, \tag{25}$$

under suitable technical hypotheses on f_1 and f_2, by the same procedure. Also, Δ could be replaced by a selfadjoint linear uniformly elliptic operator of second order. The method also applies to boundary value problems for (1) or (25) in exterior domains Ω_R in $\mathbf{R}^N$.

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