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MONOMIAL SPACE CURVES IN P_k^3 AS BINOMIAL SET THEORETIC COMPLETE INTERSECTIONS

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ABSTRACT. We give a necessary and sufficient condition for monomial curves in P_k^3 to be set theoretic complete intersections on a binomial surface. Using this condition, we prove that the twisted cubic curve is the only smooth monomial curve which is a set theoretic complete intersection on a binomial surface, in characteristic zero.

1. INTRODUCTION

In this paper we are interested in the following problem: Whether C, a monomial space curve in P_k^3 (k an algebraically closed field), is a binomial set theoretic complete intersection.

By a monomial space curve we mean a curve embedded in P_k^3 with generic zero $(t^d, t^{a_1}u^{b_1}, t^{a_2}u^{b_2}, u^d)$ where d, a_1, a_2, b_1, b_2 are positive integers such that $a_1 \neq a_2$, $a_1 + b_1 = d$, $a_2 + b_2 = d$ and g.c.d $(d, a_1, a_2) = 1$.

By a binomial set theoretic complete intersection we mean that there exist homogeneous polynomials F and G in $I(C)$, the associated ideal of C, with F binomial such that $\sqrt{(F, G)} = I(C)$, i.e. C is the intersection of two surfaces and the equation of one of them is binomial.

There is a large number of results in the literature that emphasizes the strong connection between monomial curves and binomial surfaces. Among them R. Hartshorne in [H] proves that all smooth monomial curves in P_k^3 are binomial set theoretic complete intersections if the characteristic of k is positive. In [M], T. T. Moh generalizes Hartshorne's results by proving that every monomial curve in P_k^n is a binomial set theoretic complete intersection in positive characteristic. And even more that all hypersurfaces involved are given by binomials. Finally in [R-V] L. Robbiano and G. Valla prove that all monomial

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space curves in P_k^3 which are arithmetically Cohen-Macaulay are binomial set theoretic complete intersections.

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Let k be an algebraically closed field and let C be a monomial curve. The associated ideal $I(C)$ of C is the kernel of $\varphi: k[X_0, X_1, X_2, X_3] \to k[t, u]$ given by $\varphi(X_0) = t^d$, $\varphi(X_1) = t^{a_1}u^{b_1}$, $\varphi(X_2) = t^{a_2}u^{b_2}$, $\varphi(X_3) = u^d$.

We endow $k[X_0, X_1, X_2, X_3]$ with two gradings, the usual grading ($\deg(X_i) = 1, i = 0, 1, 2, 3$) and the σ-grading in which $\deg(X_0) = d$, $\deg(X_1) = a_1$, $\deg(X_2) = a_2$ and $\deg(X_3) = 0$. Then the ideal $I(C)$ is homogeneous and σ-homogeneous.

Lemma 2.1. *If* $I(C) = \sqrt{(F, G)}$ *then we can choose both* F *and* G *to be irreducible.*

Proof. Let $F = F_1 \cdot F_2$, $F \in I(C)$ prime, so without loss of generality $F_1 \in I(C)$. $I(C) = \sqrt{(F, G)} = \sqrt{(F_1 \cdot F_2, G)} = \sqrt{(F_1, G)} \cap \sqrt{(F_2, G)} \subseteq \sqrt{(F_1, G)} \subseteq I(C)$ so $I(C) = \sqrt{(F_1, G)}$. □

Lemma 2.2. *If* $I(C) = \sqrt{(F, G)}$ *and* F *is both homogeneous and* σ*-homogeneous then we can choose* G *to be also homogeneous and* σ*-homogeneous.*

Proof. By Lemma 2.1 we can consider F to be irreducible.

Let's consider the canonical homomorphism of $k[X_0, X_1, X_2, X_3]$ onto $k[X_0, X_1, X_2, X_3]/(F)$. Since $I = \sqrt{(F, G)}$, we have $\overline{I} = \sqrt{(\overline{G})}$. Let $\overline{G} = \overline{G}_s + \cdots + \overline{G}_{s+p}$ in one of the two gradings; then $\overline{G} \in \sqrt{(\overline{G})} = \overline{I}$, $\overline{I}$ is homogeneous and σ-homogeneous ([Z-S, pg. 150]) so $\overline{G}_s \in \overline{I}$. There exists n such that

$$\overline{G}_s^n = (\overline{A}_l + \cdots + \overline{A}_{l+k}) \cdot (\overline{G}_s + \cdots + \overline{G}_{s+p}).$$

Since F is irreducible, $k[X_0, X_1, X_2, X_3]/(F)$ is an integral domain, so $k = 0$, $p = 0$. This implies that $\overline{G}$ is homogeneous and σ-homogeneous. So we can choose G to be homogeneous and σ-homogeneous. □

The homogeneous binomials belonging to $I(C)$ are in the following forms:

$$X_1^{n_1} - X_0^{n_0} X_2^{n_2} X_3^{n_3}, \tag{1}$$

$$X_2^{n_2} - X_0^{n_0} X_1^{n_1} X_3^{n_3}, \tag{2}$$

$$X_0^{n_0} X_1^{n_1} - X_2^{n_2} X_3^{n_3} \quad \text{or} \quad X_0^{n_0} X_2^{n_2} - X_1^{n_1} X_3^{n_3}, \tag{3}$$

$$X_1^{n_1} X_2^{n_2} - X_0^{n_0} X_3^{n_3} \tag{4}$$

for appropriate nonnegative integers n_0, n_1, n_2, n_3 in (1) and (2) and positive integers n_0, n_1, n_2, n_3 in (3) and (4).

Theorem 2.3. *If* $I(C) = \sqrt{(F, G)}$ *with* F *binomial then* F *is not of type* (3) *or* (4).

Proof. Let $F = X_0^{n_0}X_1^{n_1} - X_2^{n_2}X_3^{n_3}$ be of type (3). For a binomial $X_1^{m_1} - X_0^{m_0}X_2^{m_2}X_3^{m_3}$ of type (1) of $I(C) = \sqrt{(F, G)}$ we have that there exist N integer such that

$$(X_1^{m_1} - X_0^{m_0}X_2^{m_2}X_3^{m_3})^N = A(X_0^{n_0}X_1^{n_1} - X_2^{n_2}X_3^{n_3}) + B \cdot G$$

for some A, B in $k[X_0, X_1, X_2, X_3]$. Then if $G_{X_1X_2} = G(0, X_1, X_2, 0)$, by setting $X_0 = 0$, $X_3 = 0$ in the above equation we have $X_1^{m_1 \cdot N} = B_{X_1X_2}G_{X_1X_2}$.

Since $k[X_0, X_1, X_2, X_3]$ is a U.F.D. we have $G_{X_1X_2} = X_1^{N_1}$. Similarly by choosing a binomial of type (2) of $I(C)$ we have $G_{X_1X_2} = X_2^{N_2}$. Contradiction.

Now let $F = X_1^{n_1}X_2^{n_2} - X_0^{n_0}X_3^{n_3}$ by of type (4). F is binomial in $I(C)$, so it is also σ-homogeneous, then according to Lemma 2.2 we can choose G to be homogeneous and σ-homogeneous.

Again let us take a binomial of type (1) of $I(C)$. We have

$$(X_1^{m_1} - X_0^{m_0}X_2^{m_2}X_3^{m_3})^N = A \cdot (X_1^{n_1}X_2^{n_2} - X_0^{n_0}X_3^{n_3}) + BG$$

For some A, B in $k[X_0, X_1, X_2, X_3]$. By setting $X_0 = X_2 = X_3 = 0$ in the above equation, we have $X_1^{m_1 \cdot N} = B_{X_1}G_{X_1}$ which implies $G_{X_1} = X_1^k$ (k is the degree of G) so there is a term X_1^k in G. Similarly starting with a binomial of type (2) we conclude that there is a term X_2^k in G. But, since $a_1 \neq a_2$ that is a contradiction with the fact that G is σ-homogeneous. □

So we conclude that if $I(C)$ is set theoretic complete intersection on a binomial, then this binomial has to be of type (1) or (2). Since types (1) and (2) are essentially the same type, up to the rearrangement of the indices, in the sequent we will study monomial curves as complete intersections on a binomial of one of those types.

Let $\varphi: k[X_0, X_1, X_2, X_3] \to k[X_0, X_1, X_3]$ given by $\varphi(X_0) = X_0^{n_2}$, $\varphi(X_1) = X_1^{n_2}$, $\varphi(X_2) = X_0^{n_0}X_1^{n_1}X_3^{n_3}$ and $\varphi(X_3) = X_3^{n_2}$ where $n_i \in \mathbf{N}^0$, $n_0 + n_1 + n_3 = n_2$, g.c.d.$(n_0, n_1, n_3) = 1$ and $dn_0 + a_1n_1 = a_2n_2$ (and so $dn_3 + b_1n_1 = b_2n_2$). Then $\ker\varphi = (F) = (X_2^{n_2} - X_0^{n_0}X_1^{n_1}X_3^{n_3})$ and $F \in I(C)$.

Theorem 2.4. *C is a set theoretic complete intersection on F iff there exists a homogeneous polynomial $G \in I(C)$ such that* $\sqrt{(\varphi(G))} = \sqrt{(X_1^d - X_0^{a_1}X_3^{b_1})}$.

Proof. ($\to$) Suppose $I(C) = \sqrt{(F, G)}$. Let $g = \text{g.c.d.}(d, a_1)$, $d = gd^*$, $a_1 = ga_1^*$, $b_1 = gb_1^*$. Then from $a_2n_2 = dn_0 + a_1n_1$, g/d and g/a_1 we have that g/a_2n_2.

But $\text{g.c.d.}(g, a_2) = \text{g.c.d.}(d, a_1, a_2) = 1$ implies g/n_2 so $n_2 = g \cdot n_2^*$.

We consider the homogeneous ideal $I^\theta \subseteq k[X_0, X_1, X_3]$ of the curve C^θ of P_k^2 with generic zero $(t^{d^*}, \theta^{1/d^*}t^{a_1^*}u^{b_1^*}, u^{d^*})$ where θ is a root of $x^g - 1 = 0$.

Then $I^\theta = (X_1^{d^*} - \theta X_0^{a_1^*} X_3^{b_1^*})$. Let $f \in I(C)$, $\varphi(f(X_0, X_1, X_2, X_3)) = f(X_0^{n_2}, X_1^{n_2}, X_0^{n_0} X_1^{n_1} X_3^{n_3}, X_3^{n_2}) = \tilde{f}(X_0, X_1, X_3)$. We claim that $\tilde{f} \in I^\theta$.

$$\begin{aligned}
&\tilde{f}(t^{d^*}, \theta^{1/d^*} t^{a_1^*} u^{b_1^*}, u^{d^*}) \\
&\quad = f(t^{n_2 d^*}, \theta^{n_2/d^*} t^{n_2 a_1^*} u^{n_2 b_1^*}, \theta^{n_1/d^*} t^{n_0 d^* + n_1 a_1^*} u^{n_1 b_1^* + n_3 d^*}, u^{n_2 d^*}) \\
&\quad = f(t^{n_2 d^*}, \theta^{n_2/d^*} t^{n_2 a_1^*}, u^{n_2 b_1^*}, \theta^{n_1/d^*} t^{n_2^* a_2} u^{n_2^* b_2}, u^{n_2 d^*}).
\end{aligned}$$

but g.c.d.$(d, a_1, a_2) = 1$ implies that $1 = ka_1 + \lambda a_2 + \mu d$ for some $k, \lambda, \mu \in \mathbf{Z}$. Then using the facts that $\theta^g = 1$, $g = d/d^*$ and $dn_0 + a_1 n_1 = a_2 n_2$ we have:

$$\theta^{n_2/d^*} = \theta^{a_1(n_2 k + n_1 \lambda)/d^*} \quad \text{and} \quad \theta^{n_1/d^*} = \theta^{a_2(n_2 k + n_1 \lambda)/d^*}.$$

Set $\tilde{t} = \theta^{(n_2 k + n_1 \lambda)/d^*} t^{n_2^*}$ and $\tilde{u} = u^{n_2^*}$. Then

$$\tilde{f}(t^{d^*}, \theta^{1/d^*} t^{a_1^*} u^{b_1^*}, u^{d^*}) = f(\tilde{t}^d, \tilde{t}^{a_1} \tilde{u}^{b_1}, \tilde{t}^{a_2} \tilde{u}^{b_2}, \tilde{u}^d) = 0$$

so $\tilde{f} \in I^\theta$. So we have $\varphi(I) \subseteq I^\theta = (X_1^{d^*} - \theta X_0^{a_1^*} X_3^{b_1^*})$ for all θ such that $\theta^g - 1 = 0$ and so $\varphi(I) \subseteq \sqrt{(X_1^d - X_0^{a_1} X_3^{b_1})}$.

Note that it is possible to have multiple factors in $X_1^d - X_0^{a_1} X_3^{b_1}$ if characteristic p of k is positive and p/g.

If we look at

$$\begin{aligned}
\varphi(X_1^d - X_0^{a_1} X_3^{b_1}) &= X_1^{n_2 d} - X_0^{n_2 a_1} X_3^{n_2 b_1}, \\
\varphi(X_2^d - X_0^{a_2} X_3^{b_2}) &= X_0^{n_0 d} X_1^{n_1 d} X_3^{n_3 d} - X_0^{a_2 n_2} X_3^{b_2 n_2} \\
&= X_0^{n_0 d} X_3^{n_3 d} (X_1^{n_1 d} - X_0^{a_2 n_2 - n_0 d} X_3^{b_2 n_2 - n_3 d}) \\
&= X_0^{n_0 d} X_3^{n_3 d} (X_1^{n_1 d} - X_0^{n_1 a_1} X_3^{n_1 b_1})
\end{aligned}$$

and for $a_2 > a_1$

$$\begin{aligned}
\varphi(X_1^{a_2} - X_2^{a_1} X_3^{a_2 - a_1}) &= X_1^{a_2 n_2} - X_0^{n_0 a_1} X_1^{n_1 a_1} X_3^{n_3 a_1 + n_2(a_2 - a_1)} \\
&= X_1^{n_1 a_1} (X_1^{a_2 n_2 - n_1 a_1} - X_0^{n_0 a_1} X_3^{n_3 a_1 + n_2 a_2 - n_2 a_1}) \\
&= X_1^{n_1 a_1} (X_1^{d n_0} - X_0^{a_1 n_0} X_3^{n_3 a_1 + d n_0 + a_1 n_1 - n_2 a_1}) \\
&= X_1^{n_1 a_1} (X_1^{d n_0} - X_0^{a_1 n_0} X_3^{a_1(n_3 + n_1 - n_2) + d n_0}) \\
&= X_1^{n_1 a_1} (X_1^{d n_0} - X_0^{a_1 n_0} X_3^{d n_0 - a_1 n_0}) \\
&= X_1^{n_1 a_1} (X_1^{d n_0} - X_0^{a_1 n_0} X_3^{b_1 n_0})
\end{aligned}$$

we see that the only common factor is $(X_1^d - X_0^{a_1} X_3^{b_1})$, since g.c.d.$(n_0, n_1, n_3) = 1$.

So if $(\varphi(I)) \subseteq (l)$ then $(X_1^d - X_0^{a_1} X_3^{b_1}) \subseteq (l)$. But $(\varphi(I)) \subseteq \sqrt{(\varphi(G))} \Rightarrow (X_1^d - X_0^{a_1} X_3^{b_1}) \subseteq \sqrt{(\varphi(G))} \Rightarrow \sqrt{(X_1^d - X_0^{a_1} X_3^{b_1})} \subseteq \sqrt{(\varphi(G))}(1)$ and also $\varphi(G) \in \varphi(I) \subseteq \sqrt{(X_1^d - X_0^{a_1} X_3^{b_1}} \Rightarrow \sqrt{(\varphi(G))} \subseteq \sqrt{(X_1^d - X_0^{a_1} X_3^{b_1})}(2)$. So from (1) and (2) we have $\sqrt{(\varphi(G))} = \sqrt{(X_1^d - X_0^{a_1} X_3^{b_1})}$. □

$$(\leftarrow) \qquad (F, G) \subset I \Rightarrow \sqrt{(F,G)} \subset \sqrt{I} = I, \qquad I \text{ prime}.$$

So we have to prove that $I(C) \subset \sqrt{(F,G)}$. For $f \in I(C)$ we know $f(t^d, t^{a_1}u^{b_1}, t^{a_2}u^{b_2}, u^d) = 0$. Now let (x_0, x_1, x_2, x_3) be a common zero of F and G. Then $F(x_0, x_1, x_2, x_3) = 0 \Rightarrow x_2^{n_2} - x_0^{n_0}x_1^{n_1}x_3^{n_3} = 0 \Rightarrow x_2 = \zeta x_0^{n_0/n_2} x_2^{n_1/n_2} x_3^{n_3/n_2}$ for some ζ such that $\zeta^{n_2} = 1$.

But g. c. d.$(n_0, n_1, n_3) = 1$, so $\zeta = \zeta^{\lambda_0 n_0 + \lambda_1 n_1 + \lambda_3 n_3}$ for some $\lambda_0, \lambda_1, \lambda_3 \in \mathbf{Z}$ so $x_2 = (\zeta^{\lambda_0} x_0^{1/n_2})^{n_0} (\zeta^{\lambda_1} x_1^{1/n_2})^{n_1} (\zeta^{\lambda_3} x_3^{1/n_2})^{n_3}$. If we let $\xi_0 = \zeta^{\lambda_0} x_0^{1/n_2}$, $\xi_1 = \zeta^{\lambda_1} x_1^{1/n_2}$, $\xi_2 = x_2^{1/n_2}$, $\xi_3 = \zeta^{\lambda_3} x_3^{1/n_2}$ we have $x_0 = \xi_0^{n_2}$, $x_1 = \xi_1^{n_2}$, $x_2 = \xi_0^{n_0}\xi_1^{n_1}\xi_3^{n_3}$ and $x_3 = \xi_3^{n_2}$.

(x_0, x_1, x_2, x_3) is a zero also of G so

$$\begin{aligned} 0 = G(x_0, x_1, x_2, x_3) &= G(\xi_0^{n_2}, \xi_1^{n_2}, \xi_0^{n_0}\xi_1^{n_1}\xi_3^{n_3}, \xi_3^{n_2}) \\ &= G(\varphi(\xi_0), \varphi(\xi_1), \varphi(\xi_2), \varphi(\xi_3)) \\ &= \varphi(G(\xi_0, \xi_1, \xi_2, \xi_3)). \end{aligned}$$

But $\sqrt{(\varphi(G))} = \sqrt{(X_1^d - X_0^{a_1}X_3^{b_1})}$ from the hypothesis so $\varphi(G(\xi_0, \xi_1, \xi_2, \xi_3)) = 0 \Rightarrow \xi_1^d - \xi_0^{a_1}\xi_3^{b_1} = 0 \Rightarrow \xi_1 = \omega \xi_0^{a_1/d}\xi_3^{b_1/d}$ where $\omega^d = 1$.

So up to now we have

$$\begin{aligned} x_0 &= \xi_0^{n_2}, \qquad x_1 = \omega^{n_2}\xi_0^{a_1 n_2/d}\xi_3^{b_1 n_2/d}, \\ x_2 &= \xi_0^{n_0}(\omega\xi_0^{a_1/d}\xi_3^{b_1/d})^{n_1} \cdot \xi_3^{n_3} = \omega^{n_1}\xi_0^{\frac{dn_0 + a_1 n_1}{d}} \xi^{\frac{n_1 b_1 + n_3 d}{d}} \\ &= \omega^{n_1}\xi_0^{a_2 n_2/d}\xi_3^{b_2 n_2/d} \qquad \text{and} \quad x_3 = \xi_3^{n_2}. \end{aligned}$$

Since g. c. d.$(d, a_1, a_2) = 1 \Rightarrow (a_1, a_2) \equiv 1 \bmod d \Rightarrow \exists k,\ \lambda \in \mathbf{Z}$ such that $ka_1 + \lambda a_2 \equiv 1 \bmod d$. Then using the fact that $dn_0 + a_1 n_1 = a_2 n_2$ we have:

$$\begin{aligned} a_1(n_2 k + n_1\lambda) &\equiv a_1 n_2 k + a_1 n_1 \lambda \mod d \\ &\equiv a_1 n_2 k + a_2 n_2 \lambda \mod d \\ &\equiv n_2(a_1 k + a_2 \lambda) \mod d \\ &\equiv n_2 \mod d \end{aligned}$$

and

$$\begin{aligned} a_2(n_2 k + n_1\lambda) &\equiv a_2 n_2 k + a_2 n_1 \lambda \mod d \\ &\equiv a_1 n_1 k + a_2 n_1 \lambda \mod d \\ &\equiv n_1(a_1 k + a_2 \lambda) \mod d \\ &\equiv n_1 \mod d. \end{aligned}$$

So let $t = \omega^{n_2 k + n_1 \lambda}\xi_0^{n_2/d}$ and $u = \xi_3^{n_2/d}$; we have $x_0 = t^d$, $x_1 = t^{a_1}u^{b_1}$, $x_2 = t^{a_2}u^{b_2}$, $x_3 = u^d$ and so $f(x_0, x_1, x_2, x_3) = f(t^d, t^{a_1}u^{b_1}, t^{a_2}u^{b_2}, u^d) = 0$. From Hilbert's Nullstellensatz follows that $f \in \sqrt{(F,G)}$ and so $I(C) = \sqrt{(F,G)}$. □

3. Application

In [R-V] L. Robbiano and G. Valla prove that if $\text{char}(k) = 0$ then C_4 is not a set theoretic complete intersection on anyone of the three surfaces $X_0^2X_2 - X_1^3$, $X_0X_3 - X_1X_2$ and $X_1X_3^2 - X_2^3$, where C_4 is the smooth monomial curve of degree four. Here we extend this result to almost all smooth monomial curves and for all binomial surfaces.

Theorem 3.1. *In characteristic zero, smooth monomial curves are not binomial set theoretic complete intersections, except for the twisted cubic.*

Proof. A smooth monomial curve C is of the form $(t^d, t^{d_1}u, tu^{d-1}, u^d)$. Since C is symmetric with respect to interchanges $X_1 \leftrightarrow X_2$ and $X_0 \leftrightarrow X_3$, without loss of generality we can consider binomials of the form $F = X_2^{n_2} - X_0^{n_0}X_1^{n_1}X_3^{n_3}$. For the n_0, n_1, n_2, n_3 we have $n_2 = n_0 + n_1 + n_3$, $n_2(d-1) = n_1 + dn_3$ and $n_2 = dn_0 + (d-1)n_1$.

Then if C is binomial set theoretic complete intersection on F, from Theorem 2.4 we have $\sqrt{(\varphi(G))} = (X_1^d - X_0^{d-1}X_3)$; $(X_1^d - X_0^{d-1}X_3)$ is irreducible so

$$\begin{aligned}\varphi(G) &= (X_1^d - X_0^{d-1}X_3)^N \\ &= X_1^{Nd} - \binom{N}{1} X_1^{(N-1)d}X_0^{d-1}X_3 + \cdots + (-1)^N X_0^{N(d-1)}X_3^N.\end{aligned}$$

Let us denote by $\langle a, b\rangle$ the semigroup generated by a, b. Then since $\varphi(X_0) = X_0^{n_2}$, $\varphi(X_1) = X_1^{n_2}$, $\varphi(X_2) = X_0^{n_0}X_1^{n_1}X_3^{n_3}$ and $\varphi(X_3) = X_3^{n_2}$ we must have

$$d \in \langle n_2, n_1\rangle, \qquad d-1 \in \langle n_2, n_0\rangle, \qquad 1 \in \langle n_2, n_3\rangle.$$

From $1 \in \langle n_2, n_3\rangle$ and $n_2 > n_3$ we conclude $n_3 = 1$. But then $n_2 = n_0 + n_1 + 1$, $n_2(d-1) = n_1 + d$ so $(n_0 + n_1 + 1)(d-1) = n_1 + d$ which gives

$$n_0(d-1) + n_1(d-2) = 1;$$

since $d > 2$ and n_0, n_1, d are integers we have $n_0(d-1) = 0$ and $n_1(d-2) = 1$ or $n_0(d-1) = 1$ and $n_1(d-2) = 0$ the latter case is impossible since $d > 2$. From the first case we have $n_0 = 0$, $n_1 = 1$, $d = 3$. We conclude that the only smooth monomial curve which is a binomial set theoretic complete intersection in characteristic zero is the twisted cubic curve (t^3, t^2u, tu^2, u^3) and

$$I(C) = \sqrt{(X_2^2 - X_1X_3, X_1^3 - 2X_0X_1X_2 + X_3X_0^2)}. \quad \square$$

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