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ON BARELY α-COMPACT SPACES AND REMOTE POINTS IN $\beta_\alpha X \setminus X$

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ABSTRACT. Let X be a Tychonoff space. It was proved by Terada that if the cellularity of X is not Ulam-measurable, then no point of $\upsilon X \setminus X$ is a remote point of X. In this paper we generalize this result by proving that if X is barely α-compact, then no point in $\beta_\alpha X \setminus X$ is an α-exotic point of X. This implies, in particular, that if no cellular family in X has a α-measurable cardinality, then no point of $\beta_\alpha X \setminus X$ is a remote point of X; the Terada theorem then follows as a corollary when $\alpha = \omega_1$.

1. INTRODUCTION AND PRELIMINARIES

By a space we will always mean a Tychonoff space, and α and κ will always denote infinite cardinals. By a mapping we will always mean a continuous mapping.

A subset A of a space X is *regular closed* if $A = \mathrm{cl}_X \mathrm{int}_X A$. The set $RC(X)$ of regular closed subsets of X is a complete Boolean algebra when ordered by inclusion, and if $\mathscr{E} \subset RC(X)$, then $\bigwedge \mathscr{E} = \mathrm{cl}_X \mathrm{int}_X \bigcap \mathscr{E}$. A *filter* $\mathscr{F}$ of $RC(X)$ is a family $\mathscr{F} \subset RC(X)$ such that $\bigwedge \mathscr{E} \in \mathscr{F}$ for every finite $\mathscr{E} \subset \mathscr{F}$, $\varnothing \notin \mathscr{F}$ and $B \in \mathscr{F}$ whenever $A \in \mathscr{F}$ and $A \subset B \in RC(X)$. A filter $\mathscr{F}$ of $RC(X)$ is *α-complete* if $\bigwedge \mathscr{E} \neq \varnothing$ for every $\mathscr{E} \subset \mathscr{F}$ such that $|\mathscr{E}| < \alpha$.

A family $\mathscr{A}$ of sets has the *α-intersection property* if $\bigcap \mathscr{B} \neq \varnothing$ for every $\mathscr{B} \subset \mathscr{A}$ such that $|\mathscr{B}| < \alpha$. A family $\mathscr{A}$ with the ω-intersection property is said to be *finitely centered*. The family $\mathscr{A}$ is *free* if $\bigcap \mathscr{A} = \varnothing$; otherwise $\mathscr{A}$ is *fixed*. A family $\mathscr{A}$ of subsets of a space X *converges* to a point $p \in X$ if every neighborhood of p contains a member of $\mathscr{A}$; a point $q \in X$ is a *cluster point* of $\mathscr{A}$ if $q \in \mathrm{cl}_X A$ for every $A \in \mathscr{A}$.

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For a space X, we denote by X^* the subspace $\beta X \setminus X$ of the Čech-Stone compactification βX of X. A point $p \in X^*$ is a *remote point* of X if $p \notin \mathrm{cl}_{\beta X} D$ for every nowhere dense subset D of X (see van Douwen [6] for a detailed discussion of remote points).

We denote by $\mathscr{Z}(X)$ the set of zero-sets in X. The *α-compactification* $\beta_\alpha X$ of X is defined as $\beta_\alpha X = \{p \in \beta X :$ the ultrafilter of $\mathscr{Z}(X)$ which converges to p has the α-intersection property$\}$, and X is *α-compact* in the sense of Herrlich [12] if $X = \beta_\alpha X$ (see Comfort-Retta [5] for a detailed discussion of α-compact spaces). In particular, the ω_1-compact spaces are the realcompact spaces and the ω_1-compactification of X is the Hewitt realcompactification υX of X (see Gillman-Jerison [11] for the most important properties of realcompact spaces and realcompactifications).

A *cellular family* in X is a family of pairwise disjoint nonempty open subsets of X, and the *cellularity* $c(X)$ of X is defined as $c(X) = \sup\{|\mathscr{A}| : \mathscr{A}$ is a cellular family in $X\} + \omega$ (see, e.g., Engelking [8, 1.7.12]).

A cardinal κ is *measurable* if there exists a free ultrafilter on the set κ with the κ-intersection property (see, e.g., Comfort-Negrepontis [4, p. 186]). For a cardinal α, we denote by $m(\alpha)$ the first measurable cardinal $\geq \alpha$ (if it exists); a cardinal κ is *α-measurable* if $\kappa \geq m(\alpha)$ (equivalently, there is a free ultrafilter on the set κ with the α-intersection property), and the ω_1-measurable cardinals are the Ulam-measurable cardinals (Comfort-Negrepontis [4, p. 196]). It is consistent with the axioms of ZFC that there are no measurable cardinals, but it is not known whether the axioms of ZFC imply that there are not any.

A space X is said to be *barely α-compact* if every α-complete ultrafilter of $RC(X)$ is fixed. General barely α-compact spaces are studied in Blair-Polkowski-Swardson [3]; as noticed therein, they extend the class of almost α-compact spaces introduced in Frolík ([9], [10]) in case $\alpha = \omega_1$ and in Bhaumik-Misra [1] in case $\alpha \neq \omega_1$. In particular, every α-compact space is barely α-compact.

We say that a point $p \in X^*$ is an *α-exotic* point of X if $\bigwedge \mathscr{E} \neq \varnothing$ whenever $\mathscr{E} \subset RC(X)$ is such that $|\mathscr{E}| < \alpha$, $\bigwedge \mathscr{H} \neq \varnothing$ for every finite $\mathscr{H} \subset \mathscr{E}$, and p is a cluster point of $\mathscr{E}$ in βX.

Our main results are that if X is barely α-compact, then there is no α-exotic point of X in X^* (2.7), that every X with the property that no cellular family in X has α-measurable cardinality is barely α-compact (2.11), and that every remote point of X in $\beta_\alpha X \setminus X$ is α-exotic (3.1). They imply, in particular, that if no cellular family in X has α-measurable cardinality, then no point of $\beta_\alpha X \setminus X$ is a remote point of X (3.2). When $\alpha = \omega_1$, this last statement implies the following theorem of Terada [14, p. 264]: If $c(X)$ is not Ulam-measurable, then no point of $\upsilon X \setminus X$ is a remote point of X (3.3).

We do not discuss in this paper some other interesting questions concerning α-exotic points that are suggested by known properties of remote points; they will be discussed elsewhere.

We would also like to mention a recent paper by Dow [7], where an independent extension of the Terada theorem is given.

2. Barely α-compact spaces and α-exotic points

We begin with a well-known property of regular closed sets.

2.1. **Lemma.** *If $D \subset X$ is a dense subset of X, then the mapping $A \mapsto \mathrm{cl}_X A$ is a Boolean algebra isomorphism of $RC(D)$ onto $RC(X)$, with the inverse isomorphism given by $A \mapsto A \cap D$.*

We rely on the following properties of filters of $RC(X)$:

2.2. **Lemma.** *If X and T are spaces, $X \subset T$ and $\mathscr{F}$ is a filter of $RC(X)$, then* (a) *and* (b), *below, are equivalent for every $p \in T$:*

(a) *p is a cluster point of $\mathscr{F}$;*
(b) *there exists an ultrafilter $\mathscr{U}$ of $RC(X)$ such that $\mathscr{F} \subset \mathscr{U}$ and $\mathscr{U}$ converges to p.*

Proof. Clearly, (b) implies (a). Assume (a) and let $\mathscr{V} = \{V: V$ is a closed neighborhood of p in $T\}$ and $\mathscr{V}' = \{\mathrm{cl}_X(X \cap \mathrm{int}_T V): V \in \mathscr{V}\}$. Then $\mathscr{F} \cup \mathscr{V}'$ is finitely centered in $RC(X)$ and thus there exists an ultrafilter $\mathscr{U}$ of $RC(X)$ such that $\mathscr{F} \cup \mathscr{V}' \subset \mathscr{U}$. Clearly, $\mathscr{U}$ converges to p.

2.3. **Corollary.** *If $\mathscr{F}$ is an ultrafilter of $RC(X)$, then $\mathscr{F}$ converges to some $p \in \beta X$ and, conversely, for each $p \in \beta X$ there exists an ultrafilter $\mathscr{F}$ of $RC(X)$ which converges to p.*

We let $b_\alpha X = X \cup \{p \in X^*:$ there exists an α-complete ultrafilter of $RC(X)$ which converges to $p\}$.

2.4. **Proposition.** (a) *$b_\alpha X$ is barely α-compact;*

(b) *$b_\alpha X$ is the smallest barely α-compact subspace of βX which contains X;*
(c) *X is barely α-compact if and only if $X = b_\alpha X$.*

Proof. By 2.1 and 2.3, if $\mathscr{F}$ is an α-complete ultrafilter of $RC(b_\alpha X)$, then $\mathscr{F}|X = \{F \cap X: F \in \mathscr{F}\}$ is an α-complete ultrafilter of $RC(X)$ and hence $\mathscr{F}|X$ converges to some $p \in \beta X$. Thus $p \in b_\alpha X$ and we have $\cap\mathscr{F} = \{p\} \neq \varnothing$. It follows that $b_\alpha X$ is barely α-compact. Assume, now, that $X \subset T \subset \beta X$ and that T is barely α-compact. Let $p \in b_\alpha X$. There exists an α-complete ultrafilter $\mathscr{F}$ of $RC(X)$ which converges to p. By 2.1, again, $\mathscr{Z} = \{\mathrm{cl}_T F: F \in \mathscr{F}\}$ is an α-complete ultrafilter of $RC(T)$ and, by 2.2, $\mathscr{Z}$ converges to some $q \in T$. Clearly, $q = p$ and thus $p \in T$. It follows that $b_\alpha X \subset T$, which proves (b). Clearly, (b) implies (c).

A space X is *extremally disconnected* if, for every open set $U \subset X$, the closure $\mathrm{cl}_X U$ is open in X (see e.g., Engelking [8, p. 452]). A space X is *extremely disconnected at a point $p \in X$* if $p \in \mathrm{cl}_X U \cap \mathrm{cl}_X W$ implies that $U \cap W \neq \emptyset$ for each pair U, W of open subsets of X (van Douwen [6, 1.7]); clearly, X is extremally disconnected if and only if X is extremally disconnected at each of its points. We recall that βX is extremally disconnected whenever X is extremely disconnected (see, e.g., Engelking [8, 6.2.27]). For an open subset U of X, we let $\mathrm{Ex}_X U = \beta X \setminus \mathrm{cl}_{\beta X}(X \setminus U)$ (see van Douwen [6, 3.1, 3.2] for properties of the operator Ex_X).

2.5. **Lemma.** *Suppose that βX is extremally disconnected at a point $p \in \beta X$. If p is a cluster point of a family $\mathscr{E} \subset RC(X)$, then $p \in \mathrm{cl}_{\beta X} \bigwedge \mathscr{H}$ for every finite $\mathscr{H} \subset \mathscr{E}$; in particular, there exists a unique ultrafilter of $RC(X)$ which converges to p.*

Proof. Because of Lemma 2.2, it is sufficient to observe that as βX is extremally disconnected at p, if

$$p \in \mathrm{cl}_{\beta X} U_1 \cap \mathrm{cl}_{\beta X} U_2 \cap \cdots \cap \mathrm{cl}_{\beta X} U_n,$$

then

$$p \in \mathrm{cl}_{\beta X}(U_1 \cap U_2 \cap \cdots \cap U_n)$$

for every family $\{U_1, U_2, \ldots, U_n\}$ of open subsets of X.

2.6. **Proposition.** *For every $p \in X^*$,* (a), *below, implies* (b), *and* (a) *and* (b) *are equivalent in case βX is extremally disconnected at p:*

(a) *p is α-exotic;*
(b) *if p is a cluster point of a filter $\mathscr{F}$ of $RC(X)$, then $\mathscr{F}$ is α-complete; in particular, if $\mathscr{F}$ is an ultrafilter of $RC(X)$ that converges to p, then $\mathscr{F}$ is α-complete.*

Proof. Clearly, (a) implies (b), and 2.2 and 2.5 imply that (a) follows from (b) when βX is extremally disconnected at p.

Our main result can now be stated as follows:

2.7. **Theorem.** (a), *below, implies* (b), *and* (a) *and* (b) *are equivalent in case X is extremally disconnected:*

(a) *X is barely α-compact;*
(b) *no point of X^* is an α-exotic point of X.*

Proof. To prove that (a) implies (b), assume that $p \in X^*$ is an α-exotic point of X. By 2.3 and 2.6 there exists an α-complete ultrafilter of $RC(X)$ that converges to p, and thus, by 2.4(c), X is not barely α-compact. Assume, now, that X is extremally disconnected and X is not barely α-compact. By 2.4(c), there exists $p \in b_\alpha X \setminus X$ and thus there exists an α-complete ultrafilter of $RC(X)$ which converges to p. By 2.5 and 2.6, p is α-exotic.

We denote by $E(X)$ the absolute of X, and by k_X a perfect irreducible mapping of $E(X)$ onto X (see Woods [15] for a survey of absolutes); we recall that $E(X)$ is extremally disconnected (see, e.g., Woods [15, Theorem 2.1]). Clearly, $c(X) = c(E(X))$.

For a mapping $f\colon X \to Y$, we denote by $\beta f\colon \beta X \to \beta Y$ the Stone extension of f.

2.8. **Lemma** (cf. Woods [15, 2.3]). *If $f\colon Z \to Y$ is a closed irreducible mapping of Z onto Y, then $A \mapsto f(A)$ is a Boolean algebra isomorphism of $RC(Z)$ onto $RC(Y)$.*

2.9. **Proposition.** *Let $f\colon Z \to Y$ be a closed irreducible mapping of Z onto Y. Then $p \in b_\alpha Y$ if and only if $p = \beta f(q)$ for some $q \in b_\alpha Z$; that is, $b_\alpha Y = f(b_\alpha Z)$.*

Proof. By 2.8, $\mathscr{F}$ is an α-complete ultrafilter of $RC(Y)$ if and only if $\mathscr{F} = f(\mathscr{Z})$ $(= \{f(A)\colon A \in \mathscr{Z}\})$ for some α-complete ultrafilter $\mathscr{Z}$ of $RC(Z)$. Clearly, $\mathscr{Z}$ converges to a point $z \in \beta Z$ if and only if $\mathscr{F} = f(\mathscr{Z})$ converges to $\beta f(z)$. It follows that $p \in b_\alpha Y$ if and only if $p = \beta f(q)$ for some $q \in b_\alpha Z$.

Propositions 2.4(c) and 2.9, along with the fact that $k_X\colon E(X) \to X$ is perfect irreducible, imply the following:

2.10. **Corollary.** *X is barely α-compact if and only if $E(X)$ is barely α-compact.*

We can now prove the next result, which will provide (in 3.2 and 3.3) a link between the case $\alpha = \omega_1$ of 2.7 and the Terada theorem:

2.11. **Theorem.** *If no cellular family in X has α-measurable cardinality, then X is barely α-compact.*

Proof. By 2.10, it is sufficient to prove the theorem in case X is extremally disconnected, and in this case it is sufficient, by 2.7, to prove that no point of X^* is an α-exotic point of X. Assume, on the contrary, that $p \in X^*$ is an α-exotic point of X. Consider a maximal cellular family $\mathscr{U}$ in X with the property that $p \notin \mathrm{cl}_{\beta X} U$ for every $u \in \mathscr{U}$ (cf. the proof of the Theorem in Terada [14, p. 264]). Note that $\bigcup \mathscr{U}$ is dense in X and let $\mathscr{W} = \{\mathscr{V} \subset \mathscr{U}\colon p \in \mathrm{cl}_{\beta X} \bigcup \mathscr{V}\}$. By the proof of 2.5, $\mathscr{W}$ is a filter (in fact, an ultrafilter) on $\mathscr{U}$, and clearly $\mathscr{W}$ is free. Let $\mathscr{A} \subset \mathscr{W}$ with $|\mathscr{A}| < \alpha$ and let $\mathscr{E} = \{\mathrm{cl}_X \bigcup \mathscr{V}\colon \mathscr{V} \in \mathscr{A}\}$. Since p is α-exotic, 2.6 yields $\bigwedge \mathscr{E} \neq \varnothing$; and then, since $\bigcup \mathscr{U}$ is dense in X, it follows that $\bigcap \mathscr{A} \neq \varnothing$. Thus $\mathscr{W}$ has the α-intersection property, so $|\mathscr{U}|$ is α-measurable, a contradiction.

3. Remote points in $\beta_\alpha X \setminus X$

Our main result in this section is the following:

3.1. **Theorem.** *Every remote point of X in $\beta_\alpha X \setminus X$ is an α-exotic point of X.*

Proof. Let $p \in \beta_\alpha X \setminus X$ be a remote point of X. As X is extremally disconnected at p (see van Douwen [6, 5.2]), it follows from 2.5 and 2.6 that it is sufficient to prove that the unique ultrafilter $\mathscr{F}$ of $RC(X)$ which converges to p is α-complete. Now, let $\mathscr{E} \subset \mathscr{F}$ with $|\mathscr{E}| < \alpha$. For every $A \in \mathscr{E}$, as $p \in \mathrm{cl}_{\beta X} \mathrm{int}_X A$ and p is a remote point of X, we have $p \in \mathrm{Ex}_X \mathrm{int}_X A$ (see van Douwen [6, 5.1(b)]). For each $A \in \mathscr{E}$, select a zero-set neighborhood Z_A of p in βX such that $Z_A \subset \mathrm{Ex}_X \mathrm{int}_X A$. As $|\{Z_A \colon A \in \mathscr{E}\}| < \alpha$, and $p \in \mathrm{cl}_{\beta X}(Z_A \cap X)$, hence $p \in \beta_\alpha X \cap \mathrm{cl}_{\beta X}(Z_A \cap X) = \mathrm{cl}_{\beta_\alpha X}(Z_A \cap X)$ for every $A \in \mathscr{E}$, we have $p \in \bigcap_{A \in \mathscr{E}} \mathrm{cl}_{\beta_\alpha X}(Z_A \cap X) = \mathrm{cl}_{\beta_\alpha X} \bigcap_{A \in \mathscr{E}} (Z_A \cap X)$ (see Comfort-Retta [5, 2.3(h)]). Thus

$$p \in \mathrm{cl}_{\beta X} \bigcap_{A \in \mathscr{E}} (X \cap \mathrm{Ex}_X \mathrm{int}_X A) = \mathrm{cl}_{\beta X} \bigcap_{A \in \mathscr{E}} (\mathrm{int}_X A) \subset \mathrm{cl}_{\beta X} \bigcap \mathscr{E}.$$

As p is a remote point of X, $\mathrm{int}_X \cap \mathscr{E} \neq \varnothing$ and thus $\bigwedge \mathscr{E} \neq \varnothing$. It follows that $\mathscr{F}$ is α-complete and thus p is an α-exotic point of X.

Theorems 2.7, 2.11 and 3.1 imply the following:

3.2. **Corollary.** *If X is barely α-compact (in particular, if no cellular family in X has α-measurable cardinality), then no point in $\beta_\alpha X \setminus X$ is a remote point of X.*

3.3. **Corollary** (Terada [14]). *If $c(X)$ is not Ulam-measurable, then no point of $\upsilon X \setminus X$ is a remote point of X.*

3.4. *Remark.* Since every zero-set of X is a G_δ in X, one can use 2.6 to prove that if $p \in X^*$ is α-exotic and $\mathscr{F}$ is the ultrafilter of $\mathscr{Z}(X)$ converging to p, then $\mathrm{int}_X \bigcap \mathscr{H} \neq \varnothing$ for every $\mathscr{H} \subset \mathscr{F}$ such that $|\mathscr{H}| < \alpha$; a fortiori, $p \in \beta_\alpha X \setminus X$.

3.5. *Remarks.* (1) The Alexandroff compactification of the discrete space $D(\kappa)$ of cardinality κ, where $\kappa \geq m(\alpha)$, is barely α-compact and has cellularity $\geq m(\alpha)$. Thus, the converse of 2.11 is not true.

(2) Assume that κ has its natural order topology and that $cf(\kappa) > \omega$ (so that $\kappa^* = \{\kappa\}$). Express the set D of successor ordinals of κ as $\bigcup_{n \in \omega} D_n$, where the D_n's are pairwise disjoint and cofinal in κ, and let $\mathscr{E} = \{\mathrm{cl}_\kappa \bigcup_{m \geq n} D_m \colon n \in \omega\}$. Then $\mathscr{E}$ is a subset of $RC(\kappa)$ that witnesses that κ is not an ω_1-exotic (and hence not an α-exotic) point of κ. Assume, further, that $\kappa = m(\alpha)$. Then D is not α-compact (since $|D| = m(\alpha)$ [5, 5.3]), and hence there is a free α-complete ultrafilter $\mathscr{F}$ on D. By 2.1, $\mathscr{F}' = \{\mathrm{cl}_\kappa A \colon A \in \mathscr{F}\}$ is an α-complete ultrafilter of $RC(\kappa)$. Since $\mathscr{F}'$ is easily seen to be free, κ is not barely α-compact, and thus (a) and (b) of 2.7 are, in general, not equivalent.

(3) The space $X = \beta_\alpha D(\kappa) \setminus \{p\}$, where $\kappa \geq m(\alpha)$ and p is the limit point in $\beta D(\kappa)$ of a free α-complete ultrafilter on $D(\kappa)$, has no remote point in $\beta_\alpha X \setminus X = \{p\}$, while p is an α-exotic point of X. Thus, the converse of 3.1 does not hold.

(4) One can show that if $p \in X^*$ is an α-exotic point of X and $k_X(q) = p$ for some $q \in E(X)^*$, then q is an α-exotic point of $E(X)$. The mapping $k_\kappa: E(\kappa) \to \kappa$, where κ is as in (2), above, does not preserve α-exotic points. (By 2.10 and (2), $E(\kappa)$ is not barely α-compact, and thus by 2.7 there are α-exotic points of $E(\kappa)$ in $E(\kappa)^*$, while there are no α-exotic points of κ in κ^*.

(5) A subset $S \subset X$ is *cellularly embedded* in X (Swardson [13, p. 664]) if every cellular family in S can be extended to a cellular family in X. In Blair [2, 2.3], the Terada theorem was strengthened in yet another direction by showing that if no closed discrete cellularly embedded subset of X has Ulam-measurable cardinality, then no point of $\upsilon X \setminus X$ is a remote point of X. Let us observe that this last statement has no counterpart for α-exotic points: every closed discrete subset of the space X in (3), is finite, and yet there is an α-exotic point of X in X^*.

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