ON BARELY $\alpha$-COMPACT SPACES AND REMOTE POINTS IN $\beta_{\alpha}X \setminus X$

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Abstract. Let $X$ be a Tychonoff space. It was proved by Terada that if the cellularity of $X$ is not Ulam-measurable, then no point of $\nu X \setminus X$ is a remote point of $X$. In this paper we generalize this result by proving that if $X$ is barely $\alpha$-compact, then no point in $\beta_{\alpha}X \setminus X$ is an $\alpha$-exotic point of $X$. This implies, in particular, that if no cellular family in $X$ has a $\alpha$-measurable cardinality, then no point of $\beta_{\alpha}X \setminus X$ is a remote point of $X$; the Terada theorem then follows as a corollary when $\alpha = \omega_1$.

1. Introduction and preliminaries

By a space we will always mean a Tychonoff space, and $\alpha$ and $\kappa$ will always denote infinite cardinals. By a mapping we will always mean a continuous mapping.

A subset $A$ of a space $X$ is regular closed if $A = \text{cl}_X \text{int}_X A$. The set $RC(X)$ of regular closed subsets of $X$ is a complete Boolean algebra when ordered by inclusion, and if $\mathcal{E} \subset RC(X)$, then $\bigwedge \mathcal{E} = \text{cl}_X \text{int}_X \bigcap \mathcal{E}$. A filter $\mathcal{F}$ of $RC(X)$ is a family $\mathcal{F} \subset RC(X)$ such that $\bigwedge \mathcal{E} \in \mathcal{F}$ for every finite $\mathcal{E} \subset \mathcal{F}$, $\emptyset \notin \mathcal{F}$ and $B \in \mathcal{F}$ whenever $A \in \mathcal{F}$ and $A \subset B \in RC(X)$. A filter $\mathcal{F}$ of $RC(X)$ is $\alpha$-complete if $\bigwedge \mathcal{E} \neq \emptyset$ for every $\mathcal{E} \subset \mathcal{F}$ such that $|\mathcal{E}| < \alpha$.

A family $\mathcal{A}$ of sets has the $\alpha$-intersection property if $\bigcap \mathcal{B} \neq \emptyset$ for every $\mathcal{B} \subset \mathcal{A}$ such that $|\mathcal{B}| < \alpha$. A family $\mathcal{A}$ with the $\omega$-intersection property is said to be finitely centered. The family $\mathcal{A}$ is free if $\bigcap \mathcal{A} = \emptyset$; otherwise $\mathcal{A}$ is fixed. A family $\mathcal{A}$ of subsets of a space $X$ converges to a point $p \in X$ if every neighborhood of $p$ contains a member of $\mathcal{A}$; a point $q \in X$ is a cluster point of $\mathcal{A}$ if $q \in cl_X A$ for every $A \in \mathcal{A}$.

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For a space $X$, we denote by $X^*$ the subspace $\beta X \setminus X$ of the Čech-Stone compactification $\beta X$ of $X$. A point $p \in X^*$ is a remote point of $X$ if $p \notin \text{cl}_{\beta X} D$ for every nowhere dense subset $D$ of $X$ (see van Douwen [6] for a detailed discussion of remote points).

We denote by $\mathcal{Z}(X)$ the set of zero-sets in $X$. The $\alpha$-compactification $\beta_\alpha X$ of $X$ is defined as $\beta_\alpha X = \{ p \in \beta X :$ the ultrafilter of $\mathcal{Z}(X)$ which converges to $p$ has the $\alpha$-intersection property}, and $X$ is $\alpha$-compact in the sense of Herrlich [12] if $X = \beta_\alpha X$ (see Comfort-Retta [5] for a detailed discussion of $\alpha$-compact spaces). In particular, the $\omega_1$-compact spaces are the realcompact spaces and the $\omega_1$-compactification of $X$ is the Hewitt realcompactification $\nu X$ of $X$ (see Gillman-Jerison [11] for the most important properties of realcompact spaces and realcompactifications).

A cellular family in $X$ is a family of pairwise disjoint nonempty open subsets of $X$, and the cellularity $c(X)$ of $X$ is defined as $c(X) = \sup\{|\mathcal{A}| : \mathcal{A} \text{ is a cellular family in } X\} + \omega$ (see, e.g., Engelking [8, 1.7.12]).

A cardinal $\kappa$ is measurable if there exists a free ultrafilter on the set $\kappa$ with the $\kappa$-intersection property (see, e.g., Comfort-Negrepontis [4, p. 186]). For a cardinal $\alpha$, we denote by $m(\alpha)$ the first measurable cardinal $\geq \alpha$ (if it exists); a cardinal $\kappa$ is $\alpha$-measurable if $\kappa \geq m(\alpha)$ (equivalently, there is a free ultrafilter on the set $\kappa$ with the $\alpha$-intersection property), and the $\omega_1$-measurable cardinals are the Ulam-measurable cardinals (Comfort-Negrepontis [4, p. 196]). It is consistent with the axioms of ZFC that there are no measurable cardinals, but it is not known whether the axioms of ZFC imply that there are not any.

A space $X$ is said to be barely $\alpha$-compact if every $\alpha$-complete ultrafilter of $RC(X)$ is fixed. General barely $\alpha$-compact spaces are studied in Blair-Polkowski-Swardson [3]; as noticed therein, they extend the class of almost $\alpha$-compact spaces introduced in Frolik ([9], [10]) in case $\alpha = \omega_1$ and in Bhaumik-Misra [1] in case $\alpha \neq \omega_1$. In particular, every $\alpha$-compact space is barely $\alpha$-compact.

We say that a point $p \in X^*$ is an $\alpha$-exotic point of $X$ if $\bigwedge \mathcal{E} \neq \emptyset$ whenever $\mathcal{E} \subset RC(X)$ is such that $|\mathcal{E}| < \alpha$, $\bigwedge \mathcal{H} \neq \emptyset$ for every finite $\mathcal{H} \subset \mathcal{E}$, and $p$ is a cluster point of $\mathcal{E}$ in $\beta X$.

Our main results are that if $X$ is barely $\alpha$-compact, then there is no $\alpha$-exotic point of $X$ in $X^*$ (2.7), that every $X$ with the property that no cellular family in $X$ has $\alpha$-measurable cardinality is barely $\alpha$-compact (2.11), and that every remote point of $X$ in $\beta_\alpha X \setminus X$ is $\alpha$-exotic (3.1). They imply, in particular, that if no cellular family in $X$ has $\alpha$-measurable cardinality, then no point of $\beta_\alpha X \setminus X$ is a remote point of $X$ (3.2). When $\alpha = \omega_1$, this last statement implies the following theorem of Terada [14, p. 264]: If $c(X)$ is not Ulam-measurable, then no point of $\nu X \setminus X$ is a remote point of $X$ (3.3).

We do not discuss in this paper some other interesting questions concerning $\alpha$-exotic points that are suggested by known properties of remote points; they will be discussed elsewhere.
We would also like to mention a recent paper by Dow [7], where an independent extension of the Terada theorem is given.

2. BARELY α-COMPACT SPACES AND α-EXOTIC POINTS

We begin with a well-known property of regular closed sets.

2.1. Lemma. If D ⊆ X is a dense subset of X, then the mapping A ↦ cl_\alpha A is a Boolean algebra isomorphism of RC(D) onto RC(X), with the inverse isomorphism given by A ↦ A ∩ D.

We rely on the following properties of filters of RC(X):

2.2. Lemma. If X and T are spaces, X ⊆ T and SF is a filter of RC(X), then (a) and (b), below, are equivalent for every p ∈ T:

(a) p is a cluster point of SF;
(b) there exists an ultrafilter \mathcal{U} of RC(X) such that SF ⊆ \mathcal{U} and \mathcal{U} converges to p.

Proof. Clearly, (b) implies (a). Assume (a) and let \mathcal{V} = \{V: V is a closed neighborhood of p in T\} and \mathcal{V}' = \{cl_\alpha (X ∩ \text{int}_T V): V ∈ \mathcal{V}\}. Then \mathcal{I} ∪ \mathcal{V}' is finitely centered in RC(X) and thus there exists an ultrafilter \mathcal{U} of RC(X) such that \mathcal{I} ∪ \mathcal{V}' ⊆ \mathcal{U}. Clearly, \mathcal{U} converges to p.

2.3. Corollary. If \mathcal{F} is an ultrafilter of RC(X), then \mathcal{F} converges to some p ∈ βX and, conversely, for each p ∈ βX there exists an ultrafilter \mathcal{F} of RC(X) which converges to p.

We let b_\alpha X = X ∪ \{p ∈ X^* : there exists an \alpha-complete ultrafilter of RC(X) which converges to p\}.

2.4. Proposition. (a) b_\alpha X is barely \alpha-compact;
(b) b_\alpha X is the smallest barely \alpha-compact subspace of βX which contains X;
(c) X is barely \alpha-compact if and only if X = b_\alpha X.

Proof. By 2.1 and 2.3, if \mathcal{F} is an \alpha-complete ultrafilter of RC(b_\alpha X), then \mathcal{F}|X = \{F ∩ X: F ∈ \mathcal{F}\} is an \alpha-complete ultrafilter of RC(X) and hence \mathcal{F}|X converges to some p ∈ βX. Thus p ∈ b_\alpha X and we have \cap \mathcal{F} = \{p\} ≠ ∅. It follows that b_\alpha X is barely \alpha-compact. Assume, now, that X ⊆ T ⊆ βX and that T is barely \alpha-compact. Let p ∈ b_\alpha X. There exists an \alpha-complete ultrafilter \mathcal{F} of RC(X) which converges to p. By 2.1, again, \mathcal{F} = \{cl_\alpha F: F ∈ \mathcal{F}\} is an \alpha-complete ultrafilter of RC(T) and, by 2.2, \mathcal{F} converges to some q ∈ T. Clearly, q = p and thus p ∈ T. It follows that b_\alpha X ⊆ T, which proves (b). Clearly, (b) implies (c).

A space X is extremally disconnected if, for every open set U ⊆ X, the closure cl_X U is open in X (see e.g., Engelking [8, p. 452]). A space X is extremally disconnected at a point p ∈ X if p ∈ cl_X U ∩ cl_X W implies
that \( U \cap W \neq \emptyset \) for each pair \( U, W \) of open subsets of \( X \) (van Douwen [6, 1.7]); clearly, \( X \) is extremally disconnected if and only if \( X \) is extremally disconnected at each of its points. We recall that \( \beta X \) is extremally disconnected whenever \( X \) is extremely disconnected (see, e.g., Engelking [8, 6.2.27]). For an open subset \( U \) of \( X \), we let \( \text{Ex}_X U = \beta X \setminus \text{cl}_{\beta X}(X \setminus U) \) (see van Douwen [6, 3.1, 3.2] for properties of the operator \( \text{Ex}_X \)).

2.5. **Lemma.** Suppose that \( \beta X \) is extremally disconnected at a point \( p \in \beta X \). If \( p \) is a cluster point of a family \( \mathcal{G} \subseteq RC(X) \), then \( p \in \text{cl}_{\beta X} \bigcap \mathcal{H} \) for every finite \( \mathcal{H} \subseteq \mathcal{G} \); in particular, there exists a unique ultrafilter of \( RC(X) \) which converges to \( p \).

**Proof.** Because of Lemma 2.2, it is sufficient to observe that as \( \beta X \) is extremally disconnected at \( p \), if

\[
p \in \text{cl}_{\beta X} U_1 \cap \text{cl}_{\beta X} U_2 \cap \cdots \cap \text{cl}_{\beta X} U_n,
\]

then

\[
p \in \text{cl}_{\beta X}(U_1 \cap U_2 \cap \cdots \cap U_n)
\]

for every family \( \{U_1, U_2, \ldots, U_n\} \) of open subsets of \( X \).

2.6. **Proposition.** For every \( p \in X^* \), (a), below, implies (b), and (a) and (b) are equivalent in case \( \beta X \) is extremally disconnected at \( p \):

(a) \( p \) is \( \alpha \)-exotic;

(b) if \( p \) is a cluster point of a filter \( \mathcal{F} \) of \( RC(X) \), then \( \mathcal{F} \) is \( \alpha \)-complete; in particular, if \( \mathcal{F} \) is an ultrafilter of \( RC(X) \) that converges to \( p \), then \( \mathcal{F} \) is \( \alpha \)-complete.

**Proof.** Clearly, (a) implies (b), and 2.2 and 2.5 imply that (a) follows from (b) when \( \beta X \) is extremally disconnected at \( p \).

Our main result can now be stated as follows:

2.7. **Theorem.** (a), below, implies (b), and (a) and (b) are equivalent in case \( X \) is extremally disconnected:

(a) \( X \) is barely \( \alpha \)-compact;

(b) no point of \( X^* \) is an \( \alpha \)-exotic point of \( X \).

**Proof.** To prove that (a) implies (b), assume that \( p \in X^* \) is an \( \alpha \)-exotic point of \( X \). By 2.3 and 2.6 there exists an \( \alpha \)-complete ultrafilter of \( RC(X) \) that converges to \( p \), and thus, by 2.4(c), \( X \) is not barely \( \alpha \)-compact. Assume, now, that \( X \) is extremally disconnected and \( X \) is not barely \( \alpha \)-compact. By 2.4(c), there exists \( p \in b_{\alpha} X \setminus X \) and thus there exists an \( \alpha \)-complete ultrafilter of \( RC(X) \) which converges to \( p \). By 2.5 and 2.6, \( p \) is \( \alpha \)-exotic.

We denote by \( E(X) \) the absolute of \( X \), and by \( k_X \) a perfect irreducible mapping of \( E(X) \) onto \( X \) (see Woods [15] for a survey of absolutes); we recall that \( E(X) \) is extremally disconnected (see, e.g., Woods [15, Theorem 2.1]). Clearly, \( c(X) = c(E(X)) \).
For a mapping \( f: X \to Y \), we denote by \( \beta f: \beta X \to \beta Y \) the Stone extension of \( f \).

2.8. Lemma (cf. Woods [15, 2.3]). If \( f: Z \to Y \) is a closed irreducible mapping of \( Z \) onto \( Y \), then \( A \mapsto f(A) \) is a Boolean algebra isomorphism of \( RC(Z) \) onto \( RC(Y) \).

2.9. Proposition. Let \( f: Z \to Y \) be a closed irreducible mapping of \( Z \) onto \( Y \). Then \( p \in b_a Y \) if and only if \( p = \beta f(q) \) for some \( q \in b_a Z \); that is, \( b_a Y = f(b_a Z) \).

Proof. By 2.8, \( \mathcal{F} \) is an \( \alpha \)-complete ultrafilter of \( RC(Y) \) if and only if \( \mathcal{F} = f(\mathcal{Z}) \) (\( = \{ f(A): A \in \mathcal{Z} \} \)) for some \( \alpha \)-complete ultrafilter \( \mathcal{Z} \) of \( RC(Z) \). Clearly, \( \mathcal{Z} \) converges to a point \( z \in \beta Z \) if and only if \( \mathcal{F} = f(\mathcal{Z}) \) converges to \( \beta f(z) \). It follows that \( p \in b_a Y \) if and only if \( p = \beta f(q) \) for some \( q \in b_a Z \).

Propositions 2.4(c) and 2.9, along with the fact that \( k_X: E(X) \to X \) is perfect irreducible, imply the following:

2.10. Corollary. \( X \) is barely \( \alpha \)-compact if and only if \( E(X) \) is barely \( \alpha \)-compact.

We can now prove the next result, which will provide (in 3.2 and 3.3) a link between the case \( \alpha = \omega_1 \) of 2.7 and the Terada theorem:

2.11. Theorem. If no cellular family in \( X \) has \( \alpha \)-measurable cardinality, then \( X \) is barely \( \alpha \)-compact.

Proof. By 2.10, it is sufficient to prove the theorem in case \( X \) is extremally disconnected, and in this case it is sufficient, by 2.7, to prove that no point of \( X^* \) is an \( \alpha \)-exotic point of \( X \). Assume, on the contrary, that \( p \in X^* \) is an \( \alpha \)-exotic point of \( X \). Consider a maximal cellular family \( \mathcal{U} \) in \( X \) with the property that \( p \notin \cl_{\beta X} U \) for every \( u \in \mathcal{U} \) (cf. the proof of the Theorem in Terada [14, p. 264]). Note that \( \bigcup \mathcal{U} \) is dense in \( X \) and let \( \mathcal{W} = \{ V \subseteq \mathcal{U}: p \in \cl_{\beta X} \bigcup V \} \). By the proof of 2.5, \( \mathcal{W} \) is a filter (in fact, an ultrafilter) on \( \mathcal{U} \), and clearly \( \mathcal{W} \) is free. Let \( \mathcal{A} \subseteq \mathcal{W} \) with \( |\mathcal{A}| < \alpha \) and let \( \mathcal{E} = \{ \cl_X \bigcup V : V \in \mathcal{A} \} \). Since \( p \) is \( \alpha \)-exotic, 2.6 yields \( \bigwedge \mathcal{E} \neq \emptyset \); and then, since \( \bigcup \mathcal{U} \) is dense in \( X \), it follows that \( \bigcap \mathcal{A} \neq \emptyset \). Thus \( \mathcal{W} \) has the \( \alpha \)-intersection property, so \( |\mathcal{W}| \) is \( \alpha \)-measurable, a contradiction.

3. Remote points in \( \beta_\alpha X \setminus X \)

Our main result in this section is the following:

3.1. Theorem. Every remote point of \( X \) in \( \beta_\alpha X \setminus X \) is an \( \alpha \)-exotic point of \( X \).
Proof. Let \( p \in \beta_nX \setminus X \) be a remote point of \( X \). As \( X \) is extremally disconnected at \( p \) (see van Douwen [6, 5.2]), it follows from 2.5 and 2.6 that it is sufficient to prove that the unique ultrafilter \( \mathcal{F} \) of \( RC(X) \) which converges to \( p \) is \( \alpha \)-complete. Now, let \( \mathcal{E} \subset \mathcal{F} \) with \( |\mathcal{E}| < \alpha \). For every \( A \in \mathcal{E} \), as \( p \in cl_{\beta_X} int_X A \) and \( p \) is a remote point of \( X \), we have \( p \in Ex_X int_X A \) (see van Douwen [6, 5.1(b)]). For each \( A \in \mathcal{E} \), select a zero-set neighborhood \( Z_A \) of \( p \) in \( \beta_X int_X A \) such that \( Z_A \subset Ex_X int_X A \). As \( \{|Z_A|: A \in \mathcal{E}\} < \alpha \), and \( p \in cl_{\beta_X} (Z_A \cap X) \), hence \( p \in \beta_nX \cap cl_{\beta_X} (Z_A \cap X) = cl_{\beta_nX} (Z_A \cap X) \) for every \( A \in \mathcal{E} \), we have \( p \in \bigcap_{A \in \mathcal{E}} cl_{\beta_nX} (Z_A \cap X) \). Thus
\[
p \in \bigcap_{A \in \mathcal{E}} (X \setminus Ex_X int_X A) = \bigcap_{A \in \mathcal{E}} (int_X A) \subset cl_{\beta_X} \bigcap_{A \in \mathcal{E}} \mathcal{E}.
\]
As \( p \) is a remote point of \( X \), \( int_X \mathcal{E} \neq \emptyset \) and thus \( \bigcap \mathcal{E} \neq \emptyset \). It follows that \( \mathcal{F} \) is \( \alpha \)-complete and thus \( p \) is an \( \alpha \)-exotic point of \( X \).

Theorems 2.7, 2.11 and 3.1 imply the following:

3.2. Corollary. If \( X \) is barely \( \alpha \)-compact (in particular, if no cellular family in \( X \) has \( \alpha \)-measurable cardinality), then no point in \( \beta_nX \setminus X \) is a remote point of \( X \).

3.3. Corollary (Terada [14]). If \( c(X) \) is not Ulam-measurable, then no point of \( \nu_X \setminus X \) is a remote point of \( X \).

3.4. Remark. Since every zero-set of \( X \) is a \( G_\delta \) in \( X \), one can use 2.6 to prove that if \( p \in X^* \) is \( \alpha \)-exotic and \( \mathcal{F} \) is the ultrafilter of \( \mathcal{E}(X) \) converging to \( p \), then \( int_X \mathcal{F} \neq \emptyset \) for every \( \mathcal{H} \subset \mathcal{F} \) such that \( |\mathcal{H}| < \alpha \); a fortiori, \( p \in \beta_nX \setminus X \).

3.5. Remarks. (1) The Alexandroff compactification of the discrete space \( D(\kappa) \) of cardinality \( \kappa \), where \( \kappa \geq m(\alpha) \), is barely \( \alpha \)-compact and has cellularity \( \geq m(\alpha) \). Thus, the converse of 2.11 is not true.

(2) Assume that \( \kappa \) has its natural order topology and that \( cf(\kappa) > \omega \) (so that \( \kappa^* = \{\kappa\} \)). Express the set \( D \) of successor ordinals of \( \kappa \) as \( \bigcup_{n \in \omega} D_n \), where the \( D_n \)'s are pairwise disjoint and cofinal in \( \kappa \), and let \( \mathcal{E} = \{cl_{\kappa} \bigcup_{m \geq n} D_n: n \in \omega\} \). Then \( \mathcal{E} \) is a subset of \( RC(\kappa) \) that witnesses that \( \kappa \) is not an \( \omega_1 \)-exotic (and hence not an \( \alpha \)-exotic) point of \( \kappa \). Assume, further, that \( \kappa = m(\alpha) \). Then \( D \) is not \( \alpha \)-compact (since \( |D| = m(\alpha) \) [5, 5.3]), and hence there is a free \( \alpha \)-complete ultrafilter \( \mathcal{F} \) on \( D \). By 2.1, \( \mathcal{F}' = \{cl_{\kappa} A: A \in \mathcal{F}\} \) is an \( \alpha \)-complete ultrafilter of \( RC(\kappa) \). Since \( \mathcal{F}' \) is easily seen to be free, \( \kappa \) is not barely \( \alpha \)-compact, and thus (a) and (b) of 2.7 are, in general, not equivalent.

(3) The space \( X = \beta_n D(\kappa) \setminus \{p\} \), where \( \kappa \geq m(\alpha) \) and \( p \) is the limit point in \( \beta D(\kappa) \) of a free \( \alpha \)-complete ultrafilter on \( D(\kappa) \), has no remote point in \( \beta_nX \setminus X = \{p\} \), while \( p \) is an \( \alpha \)-exotic point of \( X \). Thus, the converse of 3.1 does not hold.
(4) One can show that if \( p \in X^* \) is an \( \alpha \)-exotic point of \( X \) and \( k_X(q) = p \)
for some \( q \in E(X)^* \), then \( q \) is an \( \alpha \)-exotic point of \( E(X)^* \). The mapping
\( k_\kappa : E(\kappa) \to \kappa \), where \( \kappa \) is as in (2), above, does not preserve \( \alpha \)-exotic points.
(By 2.10 and (2), \( E(\kappa) \) is not barely \( \alpha \)-compact, and thus by 2.7 there are
\( \alpha \)-exotic points of \( E(\kappa) \) in \( E(\kappa)^* \), while there are no \( \alpha \)-exotic points of \( \kappa \) in
\( \kappa^* \).

(5) A subset \( S \subset X \) is cellularly embedded in \( X \) (Swardson [13, p. 664]) if every cellular
family in \( S \) can be extended to a cellular family in \( X \). In Blair [2, 2.3], the Terada theorem was
strengthened in yet another direction by showing that if no closed discrete cellularly embedded
subset of \( X \) has Ulam-measurable cardinality, then no point of \( vX \setminus X \) is a remote point of \( X \). Let
us observe that this last statement has no counterpart for \( \alpha \)-exotic points: every
closed discrete subset of the space \( X \) in (3), is finite, and yet there is an \( \alpha \)-exotic
point of \( X \) in \( X^* \).

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