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COSIMPLICIAL HOMOTOPIES

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ABSTRACT. A very general theorem is obtained which shows that, under certain conditions, 2 cosimplicial objects (constructed from triples) are homotopy-equivalent; cosimplicial homotopies are used. A number of applications are given.

1. INTRODUCTION

The use of cosimplicial spaces in algebraic topology is increasing rapidly; yet the most simple-minded technique for showing that two cosimplicial mappings, upon passage to total spaces, yield homotopic mappings, has been completely neglected. I am referring to the construction of a cosimplicial homotopy between them. This is probably due to the fact that cosimplicial homotopies (see Definition 2.1) have $n+1$ components in dimension n. Nevertheless, in many cases, cosimplicial spaces are defined inductively and it turns out that cosimplicial homotopies also can often be constructed inductively.

In Section 2, we give the definition of a cosimplicial homotopy and show that it induces an ordinary homotopy at the total space level. In Section 3, we present the main theorem: *if $\mathscr{C}$, $\mathscr{D}$ are categories, T a triple on $\mathscr{C}$, U a triple on $\mathscr{D}$, $F: \mathscr{C} \to \mathscr{D}$ and $\theta: FT \to UF$, $\psi: UF \to FT$ natural transformations satisfying simple conditions, then θ, ψ induce cosimplicial mappings $\theta^*: FT^* \to U^*F$, $\psi^*: U^*F \to FT^*$ and, furthermore, $\theta^*\psi^*$ and $\psi^*\theta^*$ are cosimplicially homotopic to* 1. The conditions, (3.1), (3.2), refer to the behavior of θ, ψ with respect to the structure maps η, μ of T and U. Section 4 gives a number of consequences of the main theorem. In §5, we sketch briefly the result dual to our main theorem and dealing with simplicial cotriple resolutions.

The formulas for cosimplicial homotopies used in the proof of the main theorem have their roots in earlier work of May, [3], and the author, [5]; the immediate impetus for the present work, however, stems from the study of [2] by Dwyer, Miller and Neisendorfer. I am grateful to Jean Lannes whose

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lectures reawakened my interest in cosimplicial resolutions and who showed me a manuscript copy of [2].

2. Cosimplicial homotopies

In this section we define cosimplicial homotopies and show that they induce homotopies of the total spaces.

2.1. **Definition.** Let f^*, $g^*: X^* \to Y^*$ be cosimplicial mappings. A *cosimplicial homotopy* from f^* to g^* is a collection of mappings $k_n^i: X^{n+1} \to Y^n$, $n \geq 0$, $0 \leq i \leq n$, satisfying the identities

$$k_n^0 d^0 = f^n, \qquad k_n^n d^{n+1} = g^n,$$

$$k_n^j d^i = \begin{cases} d^i k_{n-1}^{j-1}, & i < j, \\ k_n^{j-1} d^j, & i = j > 0, \\ d^{i-1} k_{n-1}^j, & i > j+1, \end{cases}$$

$$k_n^j s^i = \begin{cases} s^i k_{n+1}^{j+1}, & i \leq j, \\ s^{i-1} k_{n+1}^j, & i > j. \end{cases}$$

We then write $f^* \simeq g^*$. This is the obvious dual of the notion of simplicial homotopy, [3, 9.1]. In order to relate this notion of cosimplicial homotopy to that of homotopy, we require another definition.

2.2. **Definition.** Let K_* be a simplicial set and Y^* a cosimplicial $\mathscr{T}$-object. Let H: Sets$^* \times \mathscr{T} \to \mathscr{T}$ be the usual exponential functor sending (S, X) to X^S, where X^S is the product (which is assumed to exist in $\mathscr{T}$) of copies of X, one for each element of S. Then $H^*(K_*, Y^*)$ is the cosimplicial $\mathscr{T}$-object with $H^n(K_*, Y^*) = H(K_n, Y^n)$ and cosimplicial operators defined in the obvious fashion by $(\theta: m \to n$ in $\underline{\Delta})$

$$H(K_m, Y^m) \stackrel{H(\theta^*, \theta_*)}{\longrightarrow} H(K_n, Y^n).$$

2.3. **Proposition.** *If $\{k_n^i\}$ is a cosimplicial homotopy from f^* to g^*, then there exists a cosimplicial map*

$$K^*: X^* \to H^*(\Delta[1], Y^*)$$

such that $\pi_0 K^* = f^*$, $\pi_1 K^* = g^*$, where π_i are the mappings $H^*(\Delta[1], Y^*) \to H^*(\Delta[0], Y^*) \approx Y^*$ induced by the inclusions $j_i: \Delta[0] \to \Delta[1]$, $i = 0, 1$. Furthermore cosimplicial homotopies $\{k_n^i\}$ are in one-to-one correspondence with such simplicial maps K^*.

Proof. $\Delta[1]_n$ consists of the simplices $s_{n-1} \cdots s_1 s_0 0$, $s_{n-1} \cdots s_1 s_0 1$ and $s_{n-1} \cdots \hat{s}_i \cdots s_0 I$, $0 \leq i \leq n-1$. We must therefore define the coordinates of K^n which are indexed by these simplices. We define $K^n_{s_{n-1} \cdots s_1 s_0 0} = f^n$, $K^n_{s_{n-1} \cdots s_1 s_0 1} = g^n$

and $K^n_{s_{n-1}\cdots\hat{s}_i\cdots s_0 I} = k^i_n d^{i+1}$. Conversely, given K^* as above, define k^i_n, $0 \leq i \leq n$, by $k^i_n = s^i K^{n+1}_{s_n\cdots\hat{s}_i\cdots s_0 I}$.

It is a long but straightforward exercise, using (2.1) and the cosimplicial identities, to verify that K^*, and $\{k^i_n\}$, so defined, have the required properties and that the correspondence $\{k^i_n\} \to K^*$ is one-to-one.

Let now $\mathscr{T} = \mathscr{S}$ or Top, i.e., either the category of simplicial sets or a suitable version of topological spaces. We can now define the *geometrical realization* or *total space*, Tot Y^*, of a cosimplicial $\mathscr{T}$-object by

$$\operatorname{Tot} Y^* = \operatorname{end}_n h(\phi(n), Y^n),$$

where h is the internal hom-functor of $\mathscr{T}$ and $\phi\colon \underline{\Delta} \to \mathscr{T}$ are the models, [1], [5].

2.4. **Theorem.** $\operatorname{Tot} H^*(K_*, Y^*) \approx h(RK_*, \operatorname{Tot} Y^*)$, *where* R *is the usual geometric realization of simplicial sets.*

The proof will be omitted. It is very similar to that of [6, A.2.3]. It involves the manipulation of ends and coends, the fact that simplicial $\mathscr{T}$-objects can be represented as coends of generalized simplices $P(K_n, \Delta[n])$ and cosimplicial $\mathscr{T}$-objects as ends of generalized "cosimplices" $H(\Delta[n], Y^n)$. This theorem holds in a much more general setting than that stated here; see [6, Appendix], for the pure simplicial analogue.

Combining (2.3) and (2.4), we see that cosimplicial homotopies give rise to ordinary (simplicial or topological) homotopies of the associated total spaces.

2.5. **Definition.** X^*, Y^* are said to be of the *same cosimplicial homotopy type* if there exist cosimplicial maps $f^*\colon X^* \to Y^*$, $g^*\colon Y^* \to X^*$ such that $f^*g^* \simeq 1$, $1 \simeq g^*f^*$. We then write, $\operatorname{Tot} X^* \simeq \operatorname{Tot} Y^*$.

Note that this is very different from the notion of f^* being a homotopy-equivalence of cosimplicial objects, [1]. Nevertheless, (2.4) shows that, upon passage to total spaces, our notion yields (strong) homotopy-equivalences, while [1, p. 277], shows that their notion yields (weak) homotopy-equivalences.

3. The main theorem

One of the standard methods for constructing cosimplicial spaces is to use resolutions obtained from triples. In this section we will obtain a very general theorem which enables us to exhibit homotopy-equivalences between the total spaces of such cosimplicial spaces. This is done by the use of appropriate cosimplicial homotopies.

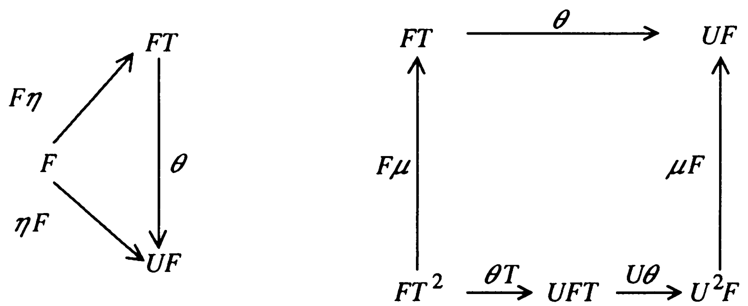
Let $\mathscr{C}$ be a category and (T, η, μ) a triple defined on $\mathscr{C}$. Recall that the *standard resolution* of an object X of $\mathscr{C}$ (with respect to T) is the cosimplicial $\mathscr{C}$-object T^*X with

$$(T^*X)^n = T^{n+1}X,$$

$$\begin{aligned} d_n^i &: (T^*X)^{n-1} \to (T^*X)^n \text{ is } T^i\eta T^{n-1}: T^nX \to T^{n+1}X, \\ s_n^i &: (T^*X)^{n+1} \to (T^*X)^n \text{ is } T^i\mu T^{n-i}: T^{n+2}X \to T^{n+1}X, \end{aligned} \qquad i = 0, 1, \ldots, n.$$

Note that $(T^*X)^{n+1} = T(T^*X)^n$, $d_n^i = Td_{n-1}^{i-1} = d_{n-1}^iT$ (clearly, the first identity holds only for $i > 0$ and the second for $n - i > 0$). Similarly, $s_n^i = Ts_{n-1}^{i-1} = s_{n-1}^iT$. The standard resolution T^*X is *co-augmented* over X, i.e., there is $d_0^0: X \to (T^*X)^0 = TX$ with $d_0^0 = \eta X$ and $d_1^0d_0^0 = d_1^1d_0^0$.

3.1. **Proposition.** *If T is a triple on $\mathscr{C}$, U a triple on $\mathscr{D}$, $F: \mathscr{C} \to \mathscr{D}$ a functor and $\theta: FT \to UF$ a natural transformation satisfying*



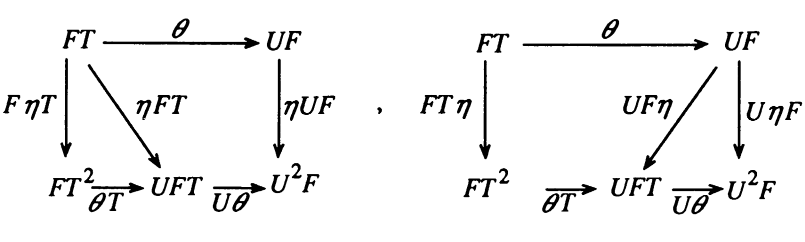
then θ induces a cosimplicial map $\theta^*: FT^* \to U^*F$.

Proof. We define $\theta^n: FT^{n+1} \to U^{n+1}F$ by

$$FT^{n+1} \overset{\theta T^n}{\to} UFT^n \overset{U\theta T^{n-1}}{\to} U^2FT^{n-1} \to \cdots U^nFT \overset{U^n\theta}{\to} U^{n+1}F$$

or, inductively, by $\theta^n = U\theta^{n-1} \cdot \theta T^n = U^n\theta \cdot \theta^{n-1}T$. It is understood that the cosimplicial operators of FT^* are those of T^* multiplied on the left by F, and similarly for U^*F.

The commutative diagrams



$$\begin{array}{ccc}
FT^{n-1} & \xrightarrow{\theta^{n-2}} & U^{n-1}F \\
{\scriptstyle FT^i\eta T^{n-i-1}}\downarrow & & \downarrow{\scriptstyle U^i\eta U^{n-i-1}F} \\
FT^{n} & \xrightarrow{\theta^{n-1}} & U^{n}F
\end{array}\quad ,$$

show that $d_1^0\theta^0 = \theta^1 d_1^0$, $d_1^1\theta^0 = \theta^1 d_1^1$. Even more easily, one sees that $\theta^0 s_0^0 = s_0^0\theta^1$.

Assume, inductively, that $\theta^{n-1}\cdot d_{n-1}^i = d_{n-1}^i\cdot\theta^{n-2}$, $i = 0, 1, \ldots n-1$, so that

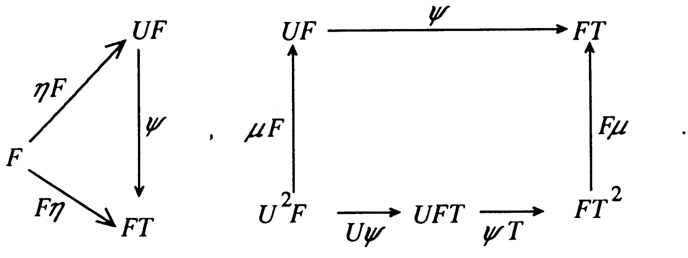
Apply, respectively, T on the right and U on the left, and complete the diagrams as shown:

$$\begin{array}{ccccc}
FT^{n} & \xrightarrow{\theta^{n-2}T} & U^{n-1}FT & \xrightarrow{U^{n-1}\theta} & U^{n}F \\
{\scriptstyle FT^i\eta T^{n-i}}\downarrow & & \downarrow{\scriptstyle U^i\eta U^{n-i-1}FT} & & \downarrow{\scriptstyle U^i\eta U^{n-i}F} \\
FT^{n+1} & \xrightarrow{\theta^{n-1}T} & U^{n}FT & \xrightarrow{U^{n}\theta} & U^{n+1}F
\end{array}\quad ,$$

$$\begin{array}{ccccc}
FT^{n} & \xrightarrow{\theta T^{n-1}} & UFT^{n-1} & \xrightarrow{U\theta^{n-2}} & U^{n}F \\
{\scriptstyle FT^{i+1}\eta T^{n-i-1}}\downarrow & & \downarrow{\scriptstyle UFT^i\eta T^{n-i-1}} & & \downarrow{\scriptstyle U^{i+1}\eta U^{n-i-1}F} \\
FT^{n+1} & \xrightarrow{\theta T^{n}} & UFT^{n} & \xrightarrow{U\theta^{n-1}} & U^{n+1}F
\end{array}\quad .$$

Together, these show that $\theta^n\cdot d_n^i = d_n^i\cdot\theta^{n-1}$, $i = 0, 1, \ldots, n$. The codegeneracies are handled similarly.

3.2. **Proposition.** *If* T *is a triple on* $\mathscr{C}$, U *a triple on* $\mathscr{D}$, $F: \mathscr{C} \to \mathscr{D}$ *a functor and* $\psi: UF \to FT$ *a natural transformation satisfying*



Then ψ *induces a cosimplicial map* $\psi^*: U^*F \to FT^*$.

Proof. We define $\psi^n: U^{n+1}F \to FT^{n+1}$ by

$$U^{n+1}F \overset{U^n\psi}{\to} U^nFT \overset{U^{n-1}\psi T}{\to} U^{n-1}FT^2 \to \cdots UFT^n \overset{\psi T^n}{\to} FT^{n+1}$$

or, inductively, by $\psi^n = \psi^{n-1}T \cdot U^n\psi = \psi T^n \cdot U\psi^{n-1}$. The proof that ψ^* is cosimplicial is similar to that for θ^*.

3.3. *Remark.* A natural transformation θ (resp., ψ), together with F, satisfying the conditions of (3.1) (resp. (3.2)) is what Street [7], or [4], calls a *monad opfunctor* (resp., *monad functor*).

Assume, now, that both θ and ψ exist and satisfy the conditions of (3.1) and (3.2), then we have

$$U^{n+2}F \overset{U(\theta^n\psi^n)}{\to} U^{n+2}F \overset{s_n^0}{\to} U^{n+1}F,$$

$$FT^{n+2} \overset{(\psi^n\theta^n)T}{\to} FT^{n+2} \overset{s_n^n}{\to} FT^{n+1}.$$

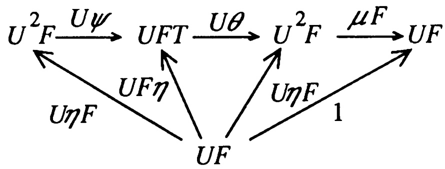
Here, of course, s_n^0 is the operator in U^*F, i.e., μU^nF, while s_n^n is the operator in FT^*, i.e., $FT^n\mu$.

3.4. **Theorem.** (i) *The formulas* $k_n^0 = s_n^0 \cdot U(\theta^n\psi^n)$ *and, for* $i > 0$, $k_n^i = Uk_{n-1}^{i-1}$ *define a cosimplicial homotopy from* $\theta^*\psi^*$ *to* 1.

(ii) *The formulas* $l_n^n = s_n^n \cdot (\psi^n\theta^n)T$ *and, for* $i < n$, $l_n^i = l_{n-1}^iT$ *define a cosimplicial homotopy from* 1 *to* $\psi^*\theta^*$.

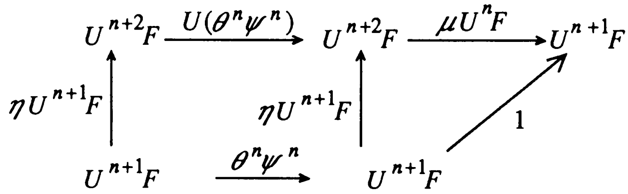
Proof. The proofs of (i) and (ii) are clearly very similar, so we shall treat (i) only.

The diagram



shows that $k_0^0 d_1^1 = 1$. Applying U^n on the left yields $k_n^n d_{n+1}^{n+1} = 1$.

The diagram



shows that $k_n^0 d_{n+1}^0 = \theta^n \psi^n$.

Inductively, if $j > 0$ and $i > 0$, the correct formulas for $k_n^j d_{n+1}^i$ follow from those for $k_{n-1}^{j-1} d_n^{i-1}$. Thus we need only treat the cases (i) $j > 0$, $i = 0$, (ii) $j = 0$, $i = 1$, (iii) $j = 0$, $i > 1$. Each of these is verified by a computation using the triple identities, the cosimplicial identities and the fact that θ^*, ψ^* are cosimplicial.

The formulas for $k_n^j s_{n+1}^i$ are handled similarly. Induction takes care of $j > 0$, $i > 0$ and we need only consider cases (i) $j > 0$, $i = 0$, (ii) $j = 0$, $i > 0$.

3.5. **Corollary.** *If $\mathcal{D}$ is $\mathcal{S}$ or* Top *(or more generally, whenever* Tot *can be defined for cosimplicial $\mathcal{D}$-objects), then* $\mathrm{Tot}(U^* F) \simeq \mathrm{Tot}(F T^*)$.

3.6. *Remark.* The formulas in (3.4) for the cosimplicial homotopies were inspired by the formulas for simplicial homotopies in [3, 9.8 and 9.9], and [5, 6.4 and 6.6], which deal with similar but simpler situations.

4. Applications

In this section we consider various consequences of the results of §3. We follow the notation of [1] and denote $\mathrm{Tot}\, T^*$ by T_∞, whenever it is defined.

4.1. **Proposition.** *If T, U are triples on the same category and maps of triples $\theta\colon T \to U$, $\psi\colon U \to T$ exist, then $T_\infty X \simeq U_\infty X$.*

This is simply the case $F = 1$ of (3.4).

4.2. **Proposition.** *If* $F: \mathscr{C} \to \mathscr{D}$, U *is a triple on* $\mathscr{D}$ *and* $\psi: UF \to F$ *is a natural transformation satisfying* $\psi \cdot \eta F = 1$, $\psi \cdot U\psi = \psi \cdot \mu F$, *then* $U_\infty FX \simeq FX$.

This is simply the case $T = 1$ of (3.4); θ can be taken here to be ηF.

4.3. **Proposition.** *If* $F: \mathscr{C} \to \mathscr{D}$, T *is a triple on* $\mathscr{C}$, $\theta: FT \to F$ *is a natural transformation satisfying* $\theta \cdot F\eta = 1$, $\theta \cdot \theta T = \theta \cdot F\mu$, *and* Tot *commutes with* F, *then* $FT_\infty X \simeq FX$.

This is simply the case $U = 1$ of (3.4); ψ can be taken here to be $F\eta$. These last two propositions are implicit in [2]. The still more special cases $F = U$ or T yield $U_\infty UX \simeq UX$ and $TT_\infty X \simeq TX$ which are the cosimplicial analogues of the well-known (simplicial) homotopy-equivalences $B(F, C, CY) \simeq FY$ and $B(C, C, X) \simeq X$, [3].

4.4. **Proposition.** *Let* U *be a triple on* $\mathscr{D}$, *then* $T = U \times U$ *is a triple on* $\mathscr{C} = \mathscr{D} \times \mathscr{D}$. *If* $F: \mathscr{D} \times \mathscr{D} \to \mathscr{D}$ *is a functor and* $\theta: F(U \times U) \to UF$, $\psi: UF \to F(U \times U)$ *satisfy the conditions of* (3.1), (3.2), *and* F *commutes with* Tot, *then* $F(U \times U)_\infty \simeq U_\infty F$.

4.5. **Corollary.** *Let* U *be a triple on* $\mathscr{D}$, $\theta: UX \times UY \to U(X \times Y)$ *a natural transformation satisfying the condition of* (3.1), *and* Tot *commute with products. Then* $U_\infty X \times U_\infty Y \simeq U_\infty(X \times Y)$.

Let, in (4.4), F be the product; then $\psi: U(X \times Y) \to UX \times UY$ always exists, induced by the projections $U(X \times Y) \to UX$, $U(X \times Y) \to UY$.

4.6. **Corollary.** *Let* R *be a commutative ring with unit, and* X, Y *be simplicial sets; then* $R_\infty X \times R_\infty Y \simeq R_\infty(X \times Y)$.

This is [1, I.7.2]; θ, already defined in [1], is given by $\theta(\sum_i r_i x_i, \sum_j s_j y_j) = \sum_{ij} r_i s_j (x_i, y_j)$. The conditions of (3.1) are easily verified.

4.7. **Corollary.** *Let* R *be a commutative ring with unit, and* X, Y *be simplicial sets over* B; *then* $\dot{R}_\infty X \times_B \dot{R}_\infty Y \simeq \dot{R}_\infty(X \times_B Y)$.

Here, $\dot{R}$ denotes the triple used by Bousfield-Kan to define fibre-wise completion, [1, p. 40]. It is only necessary to define θ, which is given by the same formula as above.

Next we consider the special case whose study led to finding (3.3); it is due to Dwyer, Miller and Neisendorfer, [2]. Let $\mathscr{C} = \mathscr{S}$, B a simplicial set, $\mathscr{D} = \mathscr{S}_B$, $T = R$, $F = B\times-$ and $U = BR$. The triple BR, [2], is defined as follows: if $p: X \to B$ is an object of $\mathscr{D}$, then $BR(p: X \to B)$ is the simplicial set (over B) consisting of all $(b, \sum_i r_i x_i)$, with $b \in B$, and $\sum_i r_i x_i \in RX$, i.e., $B \times RX$. The triple structure maps are given by $\eta(x) = (p(x), 1 \cdot x)$ and $\mu(b, \sum_i r_i(b_i, \sum_j r_{ij} x_{ij})) = (b, \sum_{ij} r_i r_{ij} x_{ij})$. The natural transformations θ, ψ, already defined in [2], are given by $\theta(b, \sum_i r_i x_i) = (b, \sum_i r_i(b, x_i))$,

$\psi(b, \sum_i r_i(b_i, x_i)) = (b, \sum_i r_i x_i)$ and are easily seen to satisfy the conditions of (3.1), (3.2). This yields the important result of [2]:

4.8. **Corollary.** $BR_\infty(B \times X) \simeq B \times R_\infty X$.

5. The dual situation

Given a *cotriple* G on a category $\mathscr{C}$, with $\varepsilon: G \to I$, $\delta: G \to G^2$, the *standard resolution* of an object X (with respect to G) is the *simplicial* $\mathscr{C}$-object G_*X with $(G_*X)_n = G^{n+1}X$, $d_i^n = G^i\varepsilon G^{n-i}: G^{n+1}X \to G^nX$, $s_i^n = G^i\delta G^{n-i}: G^{n+1}X \to G^{n+2}X$. If $F: \mathscr{C} \to \mathscr{D}$, H is a cotriple on $\mathscr{D}$ and we have natural transformations $\theta: FG \to HF$, $\psi: HF \to FG$ satisfying the obvious conditions dual to those of (3.1), (3.2), then θ, ψ induce simplicial maps $\theta_*: FG_* \to H_*F$, $\psi_*: H_*F \to FG_*$. Finally, the formulas $k_n^0 = H(\theta_n\psi_n) \cdot \delta H^n F$ and $k_n^i = Hk_{n-1}^{i-1}$ for $i > 0$, as well as $1_n^n = (\psi_n\theta_n)G \cdot FG^n\delta$ and $1_n^i = 1_{n-1}^i G$ for $i < n$, define *simplicial* homotopies from $\theta_*\psi_*$ to 1 and from 1 to $\psi_*\theta_*$.

We will leave to the reader the statement of special cases dual to those of §4. We conclude by noting that [3, 9.8] is a corollary of our theorem.

Suppose (T, η, μ) is a triple on $\mathscr{C}$. Then recall that a T-algebra (X, ν) consists of an object X of $\mathscr{C}$ and a morphism $\nu: TX \to X$ satisfying certain conditions. These T-algebras form a category $\mathscr{C}_T$ and we have the forgetful functor $U: \mathscr{C}_T \to \mathscr{C}$. We have the identity cotriple on $\mathscr{C}$ and a cotriple $\overline{T}$ on $\mathscr{C}_T$ defined as follows: $\overline{T}(X, \nu) = (TX, \mu)$, $\varepsilon: (TX, \mu) \to (X, \nu)$ is ν, and $\delta: (TX, \mu) \to (T^2X, \mu T)$ is $T\eta$. It is easily verified that $\overline{T}_*(X, \nu) = B_*(T, T, X)$ and that our main theorem is applicable, yielding $B_*(T, T, X) \simeq X_*$, which is the required result.

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