AN EASY EXAMPLE OF A 0-SPACE NOT ALMOST RIMCOMPACT

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Abstract. We construct an easy example of a space $X$ which is not almost rimcompact but for which $\beta X \setminus X$ is strongly 0-dimensional.

Recall that a space $X$ is rimcompact if $X$ possesses a base of open sets with compact boundaries, and almost rimcompact if $X$ has a compactification $KX$ in which points of $KX \setminus X$ have a base of open sets of $KX$ whose boundaries lie in $X$. Any rimcompact space is almost rimcompact (see, for example, VI, example 30, of [Is]); and any almost rimcompact space is clearly a 0-space, that is, has a compactification with 0-dimensional remainder. In VI, example 7, of [Is], Isbell indicates a construction involving the product of a 0-dimensional space and an ordinal space which yields an almost rimcompact space that is not rimcompact. Using different techniques, he goes on to produce a much more complicated example of a space $X$ which is not rimcompact (or even almost rimcompact), but has a compactification $KX$ with $KX \setminus X$ strongly 0-dimensional (that is, $\dim(KX \setminus X) = 0$). We show that a straightforward use of the easier construction can be used to produce a space $X$ having the latter properties and with $KX = \beta X$.

Let $\mathcal{R}$ denote a maximal almost disjoint collection of infinite subsets of the natural numbers $N$. The space $N \cup \mathcal{R}$ has the topology described in 51 of [GJ]; each point of $N$ is isolated, and $\lambda \in \mathcal{R}$ has an open base $\{\{\lambda\} \cup (\lambda \setminus F) : F$ is a finite subset of $N\}$. The space $N \cup \mathcal{R}$ is locally compact, pseudocompact, and 0-dimensional. According to 2.1 and the concluding remarks of [Te], given any first-countable separable compact Hausdorff space $T$, there is a family $\mathcal{R}$ so that $\beta(N \cup \mathcal{R}) \setminus (N \cup \mathcal{R})$ is homeomorphic to $T$. If we choose $T$ to be the unit interval $I$, then $N \cup \mathcal{R}$ is not strongly 0-dimensional, and $N \cup \mathcal{R} \cup \{0\}$ is totally disconnected but not 0-dimensional. The subspace $\mathcal{R}$ is discrete, so that the space $\mathcal{R} \cup \{0\}$ has only one nonisolated point; hence $\dim(\mathcal{R} \cup \{0\}) = 0$. The point 0 does not have an open neighborhood $U$ in $\beta(N \cup \mathcal{R})$ with $\frac{1}{2} \notin U$ and $\bd_{\beta(N \cup \mathcal{R})} U \cap \mathcal{R} = \emptyset$, for if such a $U$ exists, then $[\bd_{\beta(N \cup \mathcal{R})} U] \cap (N \cup \mathcal{R}) = \emptyset$, since points of $N$ are isolated in $\beta(N \cup \mathcal{R})$. 

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That is, \( U \cap (N \cup R) \) is open and closed in \( N \cup R \), and \( \text{cl}_{\beta(N \cup R)}[U \cap (N \cup R)] \) disconnects \( \beta(N \cup R) \setminus (N \cup R) \), a contradiction.

Let \( Y = \beta(N \cup R) \times (\omega_1 + 1) \), and \( X = Y \setminus [(R \cup \{0\}) \times \{\omega_1\}] \). It follows from Theorems 1 and 4 of [Gl] and 6.7 of [GJ] that \( \beta X = Y \). According to 2.8 of [Di], if \( X \) is almost rimcompact and \( \beta X \setminus X \) is 0-dimensional, points of \( \beta X \setminus X \) have bases in \( \beta X \) of open sets with boundaries in \( X \). The point \((0, \omega_1)\) does not have such a base, since the intersections of such sets with \( \beta(N \cup R) \times \{\omega_1\} \) would constitute a base for \((0, \omega_1)\) in \( \beta(N \cup R) \times \{\omega_1\} \) which cannot exist. Then the space \( X \) has the desired properties.

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**References**


