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# AN EASY EXAMPLE OF A 0-SPACE NOT ALMOST RIMCOMPACT

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ABSTRACT. We construct an easy example of a space $X$ which is not almost rimcompact but for which $\beta X \setminus X$ is strongly 0-dimensional.

Recall that a space $X$ is *rimcompact* if $X$ possesses a base of open sets with compact boundaries, and *almost rimcompact* if $X$ has a conpactification $KX$ in which points of $KX \setminus X$ have a base of open sets of $KX$ whose boundaries lie in $X$. Any rimcompact space is almost rimcompact (see, for example, VI, example 30, of [Is]); and any almost rimcompact space is clearly a 0-space, that is, has a compactification with 0-dimensional remainder. In VI, example 7, of [Is], Isbell indicates a construction involving the product of a 0-dimensional space and an ordinal space which yields an almost rimcompact space that is not rimcompact. Using different techniques, he goes on to produce a much more complicated example of a space $X$ which is not rimcompact (or even almost rimcompact), but has a compactification $KX$ with $KX \setminus X$ strongly 0-dimensional (that is, $\dim(KX \setminus X) = 0$). We show that a straightforward use of the easier construction can be used to produce a space $X$ having the latter properties and with $KX = \beta X$.

Let $\mathscr{R}$ denote a maximal almost disjoint collection of infinite subsets of the natural numbers $N$. The space $N \cup \mathscr{R}$ has the topology described in 5I of [GJ]; each point of $N$ is isolated, and $\lambda \in \mathscr{R}$ has an open base $\{\{\lambda\} \cup (\lambda \setminus F) : F$ is a finite subset of $N\}$. The space $N \cup \mathscr{R}$ is locally compact, pseudocompact, and 0-dimensional. According to 2.1 and the concluding remarks of [Te], given any first-countable separable compact Hausdorff space $T$, there is a family $\mathscr{R}$ so that $\beta(N \cup \mathscr{R}) \setminus (N \cup \mathscr{R})$ is homeomorphic to $T$. If we choose $T$ to be the unit interval $I$, then $N \cup \mathscr{R}$ is not strongly 0-dimensional, and $N \cup \mathscr{R} \cup \{0\}$ is totally disconnected but not 0-dimensional. The subspace $\mathscr{R}$ is discrete, so that the space $\mathscr{R} \cup \{0\}$ has only one nonisolated point; hence $\dim(\mathscr{R} \cup \{0\}) = 0$. The point 0 does not have an open neighborhood $U$ in $\beta(N \cup \mathscr{R})$ with $\frac{1}{2} \notin U$ and $\mathrm{bd}_{\beta(N \cup \mathscr{R})} U \cap \mathscr{R} = \varnothing$, for if such a $U$ exists, then $[\mathrm{bd}_{\beta(N \cup \mathscr{R})} U] \cap (N \cup \mathscr{R}) = \varnothing$, since points of $N$ are isolated in $\beta(N \cup \mathscr{R})$.

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That is, $U \cap (N \cup \mathscr{R})$ is open and closed in $N \cup \mathscr{R}$, and $cl_{\beta(N \cup \mathscr{R})}[U \cap (N \cup \mathscr{R})]$ disconnects $\beta(N \cup \mathscr{R}) \setminus (N \cup \mathscr{R})$, a contradiction.

Let $Y = \beta(N \cup \mathscr{R}) \times (\omega_1 + 1)$, and $X = Y \setminus [(\mathscr{R} \cup \{0\}) \times \{\omega_1\}]$. It follows from Theorems 1 and 4 of [Gl] and 6.7 of [GJ] that $\beta X = Y$. According to 2.8 of [Di], if $X$ is almost rimcompact and $\beta X \setminus X$ is 0-dimensional, points of $\beta X \setminus X$ have bases in $\beta X$ of open sets with boundaries in $X$. The point $(0, \omega_1)$ does not have such a base, since the intersections of such sets with $\beta(N \cup \mathscr{R}) \times \{\omega_1\}$ would constitute a base for $(0, \omega_1)$ in $\beta(N \cup \mathscr{R}) \times \{\omega_1\}$ which cannot exist. Then the space $X$ has the desired properties.

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