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EXAMPLES OF POINCARÉ DUALITY GROUPS

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ABSTRACT. A closed aspherical manifold may have a fundamental group which is not residually finite and contains infinitely divisible elements.

The purpose of this note is to exhibit examples of Poincaré duality groups which are not residually finite and examples which contain infinitely divisible rank 1 abelian subgroups. In answer to a question, Gromov pointed out to me that examples with infinitely divisible rank 1 abelian subgroups were implicit in [D]. Here, I have enlarged slightly on this observation.

Let K be any finite aspherical polyhedron. Embed K in R^n, and let $N(K)$ be a regular neighbourhood of K. Triangulate the boundary of $N(K)$ and take the barycentric subdivision. Then there is a dual cell subdivision, and by introducing (as in [D]) an orbifold structure on the boundary such that adjacent top dimensional cells meet at right angles, we obtain an orbifold X whose fundamental group maps onto a Coxeter group $G(X)$ with kernel a free product of infinitely many copies of the fundamental group of K. $G(X)$ has a finite index torsion-free subgroup by Selberg's lemma, and the corresponding finite cover of the orbifold X is a closed n-manifold.

In particular we may take K to be the presentation complex of the Baumslag–Solitar presentation $\{a, b: abb = bbba\}$ or of the presentation $\{a, b: ab = bba\}$. The first presents a nonresidually finite group [BS] and the second contains the dyadic rationals as the centralizer of b. Both complexes are aspherical. Subgroups of residually finite groups are residually finite, completing the proof.

If we desire the universal cover of our aspherical examples to be R^n, it suffices to take the direct product of the given manifold with a circle.

By Lyndon's theorem [LS], the presentation complex of a torsion-free 1-relator group is aspherical. Gersten has shown in [G] that a (torsion-free) 1-relator group may have a Dehn function which grows faster than exponentially. The Dehn function $d(n)$ equals the supremum, over words of length n in the free group which are consequences of the relations of a given presentation, of the minimum number of 2-cells in a diagram representing a null-homotopy.

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It is easy to see that the Dehn function of the fundamental group of X grows at least as rapidly as that of the fundamental group of K, because there is a distance decreasing retraction of the universal cover of X onto the universal cover of K. This retraction is given by consecutive folds along the hypersurfaces which cover faces of X, as in [B]. So given a finite aspherical complex, there is an aspherical closed manifold with Dehn function growing at least as fast as that of the complex.

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