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THE HYPERSPACES OF SUBCONTINUA OF THE PSEUDO-ARC AND OF SOLENOIDS OF PSEUDO-ARCS ARE CANTOR MANIFOLDS

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ABSTRACT. New proofs of the above facts are based on specific homogeneity properties of the pseudo-arc and of solenoids of pseudo-arcs.

The reader is referred to [5] for hyperspace theory. It is known that if X is the pseudo-arc or a solenoid of pseudo-arcs (see [7] for the definition), then the hyperspace $C(X)$ of all nonvoid subcontinua of X is 2-dimensional. It is proved in [6] that if X is the pseudo-arc, then $C(X)$ is also a Cantor manifold, i.e., no 0-dimensional subset separates $C(X)$. In [2] a general theorem is presented that $C(X)$ has this property for an arbitrary metric, nondegenerate continuum X. Our proof of the theorem in the title is an application of the following result [3].

Lemma 1. *If X is an n-dimensional, locally compact, connected, homogeneous, metric space, then no $(n-2)$-dimensional subset separates $X (n \geq 1)$.*

Lemma 2. *If a dense, connected subset of a metric separable space X is separated by no n-dimensional subset, then the space X has the same property.* □

(1) Let X be the pseudo-arc. To show that $C(X)$ is a Cantor manifold it suffices to observe, by Lemmas 1 and 2, that the subspace $Y \subset C(X)$ of all nondegenerate, proper subcontinua of X is connected, locally compact, homogeneous (see [1]), as well as 2-dimensional and dense.

(2) Let X be a solenoid of pseudo-arcs with the continuous decomposition D into pseudo-arcs such that X/D is a solenoid S. The set D as a subspace of $C(X)$ is homeomorphic to S. As in (1) the open subspace Y of $C(X)$ is connected and dense. The set $Y \backslash D$ is dense in Y and is the union of two disjoint, open, connected, 2-dimensional subsets $M = \{y \in Y : d \neq y \subset d \in D\}$

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and $N = \{y \in Y: d \neq y \supset d \in D\}$. It follows from [4] and from properties of solenoids of pseudo-arcs [7] that for every pair $y_1, y_2 \in M (y_1, y_2 \in N)$ there exists a homeomorphism $h: X \to X$ such that $h(y_1) = h(y_2)$. The induced homeomorphism $\hat{h}: C(X) \to C(X)$ satisfies $\hat{h}(M) = M$, $\hat{h}(N) = N$ and $\hat{h}(y_1) = y_2$, so both M and N are homogeneous and, by Lemma 1, no 0-dimensional subset separates neither M nor N. Suppose a 0-dimensional subset C separates Y. Without loss of generality we may assume that C is a closed subset of Y. It means $Y \setminus C = A \cup B$, where A, B are nonvoid, disjoint and open subsets of $C(X)$. In view of the above properties of M and N we may assume $M \subset A$ and $N \subset B$. Thus $C \subset D$. If there is $d \in D \setminus C$, then some order arc $\alpha \subset C(X)$ passing through d joins M and N, which is impossible, since $\alpha \cap D = \{d\}$ and C separates Y between M and N. Therefore $C = D$, hence C is not 0-dimensional, a contradiction.

Remark. A similar proof works for X being a solenoid. However in this case $C(X)$ is the cone over X, which is evidently a Cantor manifold.

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