A CORRECTION TO
"ON EXTENSIONS OF MODELS
OF STRONG FRAGMENTS OF ARITHMETIC"

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Theorem 3.1 in [Kos] says that if $M$ is a model of $I\Delta_0 + \text{exp}$ and $N$ is an end extension of $M$ such that $N \equiv I\Delta_0$ and for some $a \in M$ there is a function $f$ which is coded in $N$ whose domain is included in $< a$ and whose range is unbounded in $M$, then $M$ has continuum many automorphisms.

Richard Kaye has pointed out to me that, while the result is correct, the proof given in [Kos] is insufficient. The problem is discussed in [Kaye] where a full proof of a stronger theorem is also given. Here our purpose is to explain briefly where the gap is and how to fill it.

In the proof of [Kos, Theorem 3.1], I claimed that if $b \in M$ is such that $2^n < b$, for all standard $n$, then there is an automorphism of $M$ which is the identity function on $< a$ and which moves elements below $b$. But it is only proved that, under the above assumptions, there are $e_1, e_2 < b, e_1 \neq e_2$, which satisfy the same $\Delta_0$ formulas with parameters less than $a$. This is not enough, however, to prove the existence of the automorphism. (On the contrary, as shown in [Kaye], if $b$ is not large enough such automorphisms do not exist.)

Let me outline what has to be done.

Let $2_1(x) = 2^x$, and let $2_{n+1}(x) = 2_1(2_n(x))$. The correct version of the claim is: if $M, N$, and $a$ are as in Theorem 3.1 and for every standard $n$ we have $2_n(a) < b$, then there is an automorphism of $M$ which is the identity function on $< a$ and moves elements below $b$.

In the proof, Kaye uses a counting argument to show that there are $e_1, e_2 < b, e_1 \neq e_2$, which satisfy the same $\Delta_0$ formulas with parameters less than $2_n(a)$, for all standard $n$. But now this is enough to construct the automorphism (using an argument from [Kot, Lemma 4.4]).

To finish the proof of the theorem we need to consider two cases. Case 1: there is a $b \in M$ such that $2_n(a) < b$ for all standard $n$. Case 2: $M = \sup_{n \in \omega} 2_n(a)$.
In the first case, we apply the claim. In the second, there is a \( c \in N \) which codes the sequence \( 2_n(a), \ n \in \omega \). Then, instead of constructing an automorphism fixing \( < a \) pointwise, it is enough to find an automorphism fixing \( c \) and moving elements inside \( M \), and this is a much simpler task.

**Bibliography**


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