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WHAT MAKES $\operatorname{Tor}_1^R(R/I, I)$ FREE?

SHIRO GOTO AND NAOYOSHI SUZUKI

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ABSTRACT. Let I be a nonprincipal ideal in a Noetherian local ring R and let $H_1(I)$ be the first homology module of the Koszul complex $K.(I)$ associated with a minimal basis of I. Then $T := \operatorname{Tor}_1^R(R/I, I)$ is a free R/I-module if and only if both the R/I-modules I/I^2 and $H_1(I)$ are free. When this is the case, we have a canonical decomposition $T \cong \Lambda^2(I/I^2) \oplus H_1(I)$ as well as the equality $\operatorname{rank}_{R/I} T = \beta_2(R/I)$. (Here $\beta_2(R/I)$ denotes the second Betti number of the R-module R/I.) Some consequences are discussed too.

1. INTRODUCTION

Let I be an ideal of a Noetherian local ring R, and assume that I is minimally generated by n (≥ 1) elements $a_1, \ldots, a_n$. Let $H_1(I)$ denote the first homology module of the Koszul complex $K = K(a_1, \ldots, a_n; R)$ associated with the sequence $a_1, \ldots, a_n$. We set $T = \operatorname{Tor}_1^R(R/I, I)$. With this notation the purpose of this research is to investigate the conditions under which the R/I-module T is free, and our conclusion is summarized in the following.

Theorem (1.1). *Suppose that $n \geq 2$. Then the following two conditions are equivalent:*

(1) *T is a free R/I-module.*

(2) *Both the R/I-modules I/I^2 and $H_1(I)$ are free.*

When this is the case, one has the canonical decomposition

$$T = \bigwedge^2(I/I^2) \oplus H_1(I)$$

as well as the equality

$$\operatorname{rank}_{R/I} T = \beta_2(R/I),$$

where $\beta_2(R/I)$ denotes the second Betti number of the R-module R/I.

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In [5], A. Simis asked what happens on an ideal J of a Noetherian ring S if $\mathrm{Tor}_1^S(S/J, J)$ is S/J-free. His question originally suggested global Noetherian rings, and one must distinguish the former from the local case. J. Barja and A. G. Rodicio [2] linked his question with the generation problem relative to S-regular sequences and tried to solve it. They actually proved in the local case that with the above notation, provided T is R/I-free, I is generated by an R-regular sequence if I has finite projective dimension and 2 is invertible in R, or if I is a syzygetic ideal of $\mathrm{ht}_R I > 0$ whose conormal module I/I^2 has finite projective dimension over R/I. Our research began with an analysis of their proof and by trying to clarify what makes the R/I-module T free. Our Theorem (1.1) helps us somewhat simplify their results [2, Theorems 1 and 2], and as a result we get the following:

Corollary (1.2). *Suppose that T is R/I-free. Then I is generated by an R-regular sequence if I has finite projective dimension or if I is a syzygetic ideal with $n \geq 2$.*

Corollary (1.3). *Suppose that $n \geq 2$ and that T is a free R/I-module of* $\mathrm{rank}_{R/I} T = \binom{n}{2}$. *Then I is generated by an R-regular sequence.*

It should be noted that W. V. Vasconcelos [10] also dealt with the same problem and proved (1.2), under the hypothesis that T is free, I has finite projective dimension and I/I^2 has finite projective dimension over R/I.

The proof of Theorem (1.1) will be given in the next section. We must mention here the preceding works related to it. As is well known and is shown in [6, Proposition 1.1 and the remark there], there is a natural map

$$j\colon \bigwedge^2(I/I^2) \to T = \mathrm{Tor}_1^R(R/I, I)$$

with $T/\mathrm{Im}(j) \cong \delta(I)$, where $\delta(I) := (Z_1 \cap IK_1)/B_1$ is an invariant of I and, *if* 2 *is invertible in* R, j provides a splitting of T. It will be shown later that $\delta(I) \cong H_1(I)$ if I/I^2 is a free R/I-module of rank equal to the number of generators of I, so under the characteristic assumption, our (2.1) follows from it. But our discussion is characteristic-free throughout even if several ideas may stem from those in [6].

In §3 we will give a few consequences of (1.1) with proofs based essentially only on the freeness of $H_1(I)$ over R/I. If one could prove that *a Noetherian local ring R is Cohen-Macaulay whenever it has a parameter ideal I for which $H_1(I)$ is R/I-free*, we could greatly improve our results. Though for certain special local rings this statement is actually true (cf. (3.2)), the general answer is still open.

2. Proof of Theorem (1.1)

Let $I = (a_1, \ldots, a_n)\,(n \geq 1)$ be an ideal in a commutative ring R, and let $H_i(I)$, $Z_i = Z_i(I)$, and $B_i = B_i(I)$ denote respectively the ith homology module, cycle, and boundary of the Koszul complex $K. = K.(a_1, \ldots, a_n; R)$

associated with the sequence $a_1, \ldots, a_n$. Let $\{T_i\}_{i=1,\ldots,n}$ be a free basis of K_1 over R such that $\partial_1(T_i) = a_i$ for $i = 1, \ldots, n$ with the differential map ∂.

Let us give here a summary of [6, Proposition 1.1], where there was defined a linear map

$$\eta: \bigwedge^2 I \to (Z_1 \cap IK_1)/IZ_1 \cong \mathrm{Tor}_1^R(R/I, I),$$

with $\eta(a_i \wedge a_j) = a_iT_j - a_jT_i \pmod{IZ_1}$. Since $\mathrm{Im}(\eta) = B_1/IZ_1$,

$$\mathrm{Tor}_1^R(R/I, I)/\mathrm{Im}(\eta) \cong (Z_1 \cap IK_1)/B_1.$$

Furthermore, η induces j, as is referred to in §1, for I kills the image of η.

Then we have the following key lemma for our proof of Theorem (1.1):

Lemma (2.1). *Suppose that I/I^2 is a free R/I-module of rank n. Then there exists an exact sequence*

$$0 \to \bigwedge^2(I/I^2) \xrightarrow{j} \mathrm{Tor}_1^R(R/I, I) \to H_1(I) \to 0$$

of R/I-modules.

Proof. We already have the exact sequence

$$\bigwedge^2(I/I^2) \xrightarrow{j} \mathrm{Tor}_1^R(R/I, I) \to \delta(I) \to 0.$$

On the other hand, we have another exact sequence

$$0 \to \delta(I) \to H_1(I) \to Z_1/(Z_1 \cap IK_1) \to 0.$$

As I/I^2 is a free R/I-module of rank n, we have $IK_1 \subset Z_1$ and the isomorphism $\delta(I) \cong H_1(I)$. Also the injectivity of the map j follows from the assumption. In fact, let $z = \sum_{1\le i<j\le n} b_{ij}(a_i \wedge a_j) \in \ker \eta\,(b_{ij} \in R)$. Write $\sum_{1\le i<j\le n} b_{ij}(a_iT_j - a_jT_i) = \sum_{i=1}^n a_i(\sum_{j=1}^n c_{ij}T_j)$, with $\sum_{j=1}^n c_{ij}T_j \in Z_1$ for each $1 \le i \le n$. Then for $1 \le k \le n$, we have the equality

$$(*) \qquad \sum_{i<k}(b_{ik} - c_{ik})a_i + (-c_{kk})a_k + \sum_{k<i}(-b_{ki} - c_{ik})a_i = 0.$$

Since $\{a_i \bmod I^2\}_{1\le i\le n}$ form an R/I-free basis of I/I^2, and since $c_{ij} \in I$ for any $1 \le i, j \le n$, we see from the equality $(*)$ that $b_{ik} \in I\ (i < k)$ and $b_{ki} \in I\ (k < i)$. Thus $z \in I(\bigwedge^2 I)$, which completes the proof of (2.1).

We are now ready to prove Theorem (1.1).

Proof of Theorem (1.1). The implication (2) $\Rightarrow$ (1) follows from (2.1), even in the case $n = 1$.

As to the implication (1) $\Rightarrow$ (2), let us maintain the above notation. Then we have the commutative diagram

$$\begin{array}{ccc} 0 \to T = \mathrm{Tor}_1^R(R/I, I) & \longrightarrow & I \otimes_R I \xrightarrow{\mu} I \\ \eta \uparrow & \nearrow \xi & \\ \bigwedge^2 I & & \end{array}$$

with exact row, where $\mu(a \otimes b) = ab$ for $a, b \in I$ and $\xi: \bigwedge^2 I \to I \otimes_R I$ is the homomorphism defined by $\xi(a \wedge b) = a \otimes b - b \otimes a$ for $a, b \in I$. Let $\mathbf{k}$ denote the residue class field of R and put $\mathbf{V} = \mathbf{k} \otimes_R I$. Then, after identifying $\mathbf{k} \otimes_R \bigwedge^2 I = \bigwedge^2 \mathbf{V}$ and $\mathbf{k} \otimes_R (I \otimes_R I) = \mathbf{V} \otimes_{\mathbf{k}} \mathbf{V}$ the map $\mathbf{k} \otimes_R \xi$ coincides with the homomorphism $\xi_{\mathbf{v}}: \bigwedge^2 \mathbf{V} \to \mathbf{V} \otimes_{\mathbf{k}} \mathbf{V}$ defined as $\xi_{\mathbf{v}}(\mathbf{v}_1 \wedge \mathbf{v}_2) = \mathbf{v}_1 \otimes_{\mathbf{k}} \mathbf{v}_2 - \mathbf{v}_2 \otimes_{\mathbf{k}} \mathbf{v}_1$ for $\mathbf{v}_1, \mathbf{v}_2 \in V$. Therefore, as $\xi_{\mathbf{v}}$ is injective, so is the mapping $\mathbf{k} \otimes_R \xi$, whence $\{\eta(a_i \wedge a_j)\}_{1 \le i < j \le n}$ forms part of a minimal basis of the finitely generated R-module $\bar{T}$. Because T is free as an R/I-module, this implies that $\operatorname{Im}(\eta)$ is a direct summand of T with rank $\binom{n}{2}$ over R/I. Consequently, we get that $\bigwedge^2(I/I^2) = (R/I) \otimes_R (\bigwedge^2 I)$ is also, via j, a free direct summand of T. Thus I/I^2 is a free R/I-module (cf., e.g., [2, Lemma 2]) and we can now invoke (2.1). The fact that $H_1(I)$ is free and $T = \bigwedge^2(I/I^2) \oplus H_1(I)$ now readily follows from (2.1).

Let us check that $\operatorname{rank}_{R/I} T = \beta_2(R/I)$. To do this, we choose a minimal free resolution for I

$$F: \cdots \to F_3 \xrightarrow{\partial_3} F_2 \xrightarrow{\partial_2} F_1 \xrightarrow{\partial_1'} I \to 0.$$

Then tensor F by R/I to get a complex

$$\cdots \to R/I \otimes_R F_3 \xrightarrow{1 \otimes_R \partial_3} R/I \otimes_R F_2 \xrightarrow{1 \otimes_R \partial_2} R/I \otimes_R F_1 \xrightarrow{1 \otimes_R \partial_1'} \cdots$$

of R/I-modules. Notice that $1 \otimes_R \partial_2 = 0$, because $IF_1 \supset Z_1(F)$. Therefore with $B = \operatorname{Im}(1 \otimes_R \partial_3)$, we get a short exact sequence

$$0 \to B \to R/I \otimes_R F_2 \to T \to 0$$

of R/I-modules, which has to be split as T is free. However this forces B to be (0), because $\mathfrak{m}(R/I \otimes_R F_2) \supset B$ (here $\mathfrak{m}$ is the maximal ideal of R). Hence $IF_2 \supset Z_2(F)$, and $T = R/I \otimes_R F_2$. Thus $\operatorname{rank}_{R/I} T = \beta_2(R/I)$.

Corollary (2.2) (to the proof). *Let I be a nonprincipal ideal in a Noetherian local ring R and let*

$$\cdots \to F_3 \xrightarrow{\partial_3} F_2 \xrightarrow{\partial_2} F_1 \xrightarrow{\partial_1} F_0 \to R/I \to 0$$

denote a minimal free resolution of R/I. Then $\operatorname{Tor}_1^R(R/I, I)$ is free as an R/I-module if and only if $IF_{i-1} \supset \partial_i(F_i)$ for $i = 2, 3$.

Remark (2.3). The "only if " part of (2.2) is not true for general principal ideals; cf. (3.6).

We close this section with an example of ideals I for which $\operatorname{Tor}_1^R(R/I, I)$ is free.

Example (2.4). Let $X_1, X_2, \ldots, X_n, Y$ $(n \geq 1)$ be a regular sequence in a commutative ring S. We put $\mathfrak{a} = (Y) \cap (X_1^2, X_2^2, \ldots, X_n^2)$ and $R = S/\mathfrak{a}$. Let $a_i = X_i \bmod \mathfrak{a}$ and $I = (a_1, \ldots, a_n)$. Then both I/I^2 and $H_1(I)$ are free R/I-modules of rank n. Hence $\mathrm{Tor}_1^R(R/I, I)$ is free and of rank $\binom{n}{2} + n$. But any element of I is a zero-divisor in R.

Proof. Let $B_i \in S$ $(1 \leq i \leq n)$ be such that $\sum_{i=1}^n B_i X_i \in \mathfrak{a}$. Then as $\mathfrak{a} = (X_1^2 Y, X_2^2 Y, \ldots, X_n^2 Y)$, we have

$$(**) \qquad \sum_{i-1}^{n} (B_i - C_i X_i Y) X_i = 0$$

for some $C_i \in S$. Because $X_1, X_2, \ldots, X_n$ is an S-regular sequence, the equality $(**)$ implies that $B_i \in J := (X_1, X_2, \ldots, X_n)$ for all i. Hence I/I^2 is a free R/I-module of rank n. Let $K. = K.(a_1, \ldots, a_n; R)$. Then the above equality $(**)$ asserts that $Z_1(K) = B_1(K) + \sum_{i=1}^n R((a_i b) T_i)$, where $b = Y \bmod \mathfrak{a}$ and $\{T_i\}_{1 \leq i \leq n}$ is the free basis of K_1. Now consider $L = K.(X_1, \ldots, X_n; S)$ and choose the standard basis $\{V_i\}_{1 \leq i \leq n}$ of L_1. Let $C_{ij} \in S (1 \leq i < j \leq n)$ and $B_i \in S$ $(1 \leq i \leq n)$ and assume that

$$\sum_{1 \leq i < j \leq n} C_{ij}(X_i V_j - X_j V_i) + \sum_{i=1}^{n} B_i((X_i Y) V_i) \in \mathfrak{a} L_1.$$

Then for each $1 \leq k \leq n$ we get

$$\sum_{i<k} C_{ik} X_i + (B_k Y) X_k + \sum_{k<i} (-C_{ki}) X \equiv 0 \mod \mathfrak{a},$$

whence $B_k Y \in J$, as we have shown above. Thus $B_k \in J$ for all k, which guarantees that $H_1(I)$ is a free R/I-module with a free basis $\{(a_i b T_i \bmod B_1(K)\}_{1 \leq i \leq n}$. Hence, by (2.1), $\mathrm{Tor}_1^R(R/I, I)$ is a free R/I-module of rank $\binom{n}{2} + n$. Because $(\prod_{i=1}^n X_i) Y \notin \mathfrak{a}$, we see that the ideal I consists entirely of zero-divisors in R.

3. Some consequences

The purpose of this section is to give some consequences of Theorem (1.1).

Let R be a Noetherian local ring with maximal ideal $\mathfrak{m}$ and $d = \dim R$. We begin with the following:

Proof of Corollaries (1.2) *and* (1.3). We may assume that $n \geq 2$. Hence by (1.1), both of the R/I-modules I/I^2 and $H_1(I)$ are free. In the case where I has finite projective dimension, that assertion directly follows from [3, Proposition 1.4.9] or [9, Corollary 1]. Suppose that I is syzygetic, which means by its definition that $Z_1(K) \cap IK_1 = B_1(K)$ (cf. [6, p. 207]). Since $IK_1 \supset Z_1(K)$, we readily find $H_1(I) = (0)$; i.e., $a_1, \ldots, a_n$ is an R-regular sequence. If $\mathrm{rank}_{R/I} T = \binom{n}{2}$, we get by (1.1) that $H_1(I) = (0)$, and so the conclusion.

The next fact is well known and is a special case of (1.3).

Corollary (3.1). *Let* $n = \dim_{R/\mathfrak{m}} \mathfrak{m}/\mathfrak{m}^2$ *and assume that* $\beta_2(R/\mathfrak{m}) = \binom{n}{2}$. *Then* R *is a regular local ring.*

Theorem (3.2). *Suppose that* $d = \dim R \geq 2$, *and assume that* R *contains a parameter ideal* I *for which* $\operatorname{Tor}_1^R(R/I, I)$ *is* R/I*-free. Then* R *is Cohen-Macaulay if* R *satisfies one of the following conditions*:

(1) $\operatorname{depth} R \geq d - 1$;
(2) $\mathfrak{m} \cdot H^0_{\mathfrak{m}}(R) = (0)$ *and* $R/H^0_{\mathfrak{m}}(R)$ *is a Buchsbaum ring.*

Proof. First, suppose that $\operatorname{depth} R \geq d - 1$ and let $\mathbf{e}_I(R)$ (resp. $\lambda_R(H_i(I))$) denote the multiplicity of R relative to I (resp. the length of $H_i(I)$). Then $\mathbf{e}_I(R) = \sum_{i=0}^d (-1)^i \lambda_R(H_i(I))$ (cf. [1]), so we have

$$\mathbf{e}_I(R) = \lambda_R(R/I) - \lambda_R(H_1(I)),$$

because $H_i(R) = (0)$ for $i > 1$. Hence let $r = \operatorname{rank}_{R/I}(H_1(I))$, and we get $\mathbf{e}_I(R) = (1 - r) \cdot \lambda_R(R/I) > 0$, which implies $r = 0$; i.e., $H_1(I) = (0)$.

Let us consider case (2). Let $W := H^0_{\mathfrak{m}}(R)$ and $\overline{R} := R/W$. Assume that R is not Cohen-Macaulay, and let $I = (a_1, \ldots, a_d)$ be the parameter ideal of R. Let $x \in Z_1(I)$. Then, since I/I^2 is R/I-free, $x \in IK_1(I)$. Let $\overline{x}$ denote the image of x in $K_1(I; \overline{R})$. Since $\overline{R}$ is Buchsbaum, every s.o.p. forms a **d**-sequence, and hence I is syzygetic on $\overline{R}$, so we have $\overline{x} \in B_1(I; \overline{R})$. Thus $Z_1(I) \subset B_1(I; R) + WK_1(I; R)$ and we have $\mathfrak{m} \cdot H_1(I) = (0)$ (cf. [8]). By our assumption, $H_1(I)$ is R/I-free, which implies that $\mathfrak{m} \subset I$ and R must be regular. This is absurd.

Corollary (3.3). *Let* R *be a normal ring of* $\dim R = 3$. *Then* R *is Cohen-Macaulay if* R *contains a parameter ideal* I *for which* $\operatorname{Tor}_1^R(R/I, I)$ *is* R/I*-free.*

Proposition (3.4). *Suppose that* R *is a generalized Cohen-Macaulay ring and let* $a_1, \ldots, a_n$ $(2 \leq n < d = \dim R)$ *be a subsystem of parameters for* R. *Let* $I = (a_1, \ldots, a_n)$. *Then* $a_1, \ldots, a_n$ *form an* R*-regular sequence, if* $\operatorname{Tor}_1^R(R/I, I)$ *is* R/I*-free.*

Proof. Assume the contrary, and choose a minimal element $\mathfrak{p}$ of $V(I) = \operatorname{Supp}_R H_1(I)$. Then, as $n < d$, $R_{\mathfrak{p}}$ is a Cohen-Macaulay local ring of $\dim R_{\mathfrak{p}} = n$ (cf. [4]). Hence $a_1, \ldots, a_n$ is a regular sequence for $R_{\mathfrak{p}}$, and so we have $R_{\mathfrak{p}} \otimes_R H_1(I) = (0)$. This contradicts the choice of $\mathfrak{p}$.

Proposition (3.5). *Let* I *be a principal ideal of* $\operatorname{ht}_R I > 0$ *and suppose that* I/I^2 *is a free* R/I*-module. Then* I *is generated by a regular element, if* $\operatorname{Tor}_1^R(R/I, I)$ *is* R/I*-free.*

Proof. Assume the contrary, and choose R so that $\dim R$ is as small as possible among such counterexamples. Let $J = [(0) : I]$. Then J is a free R/I-module, because $I \supset J$ and $\operatorname{Tor}_1^R(R/I, I) \cong J \cap I$. Let $\mathfrak{p}$ be a minimal element of

$V(I) = \mathrm{Supp}_R J$. Then as $JR_\mathfrak{p} \neq (0)$, by the minimality of $\dim R$ we have $\mathfrak{p} = \mathfrak{m}$, whence $V(I) = \{\mathfrak{m}\}$. Thus $\dim R = 1$, and if we let $r = \mathrm{rank}_{R/I} J$, we get $\mathbf{e}_I(R) = (1 - r) \cdot \lambda_R(R/I) > 0$, which implies $J = (0)$. This is a contradiction.

The next example shows that in (3.5) the hypothesis that I/I^2 is free is essential.

Example (3.6). Let R be a Buchsbaum ring with $\mathrm{depth}\, R = 0 < \dim R = d$. Choose $a \in \mathfrak{m}$ so that $\dim R/(a) = d - 1$, and put $I = (a)$. Then I is a syzygetic ideal with $\mathrm{Tor}_1^R(R/I, I) = (0)$, but neither I/I^2 nor $H_1(I)$ is a free R/I-module.

Proof. Because R is Buchsbaum, we get $[(0): I] \cap I = (0)$ (cf. [7, Proposition 1.17]). Hence I is syzygetic. Furthermore, we have $\mathrm{Tor}_1^R(R/I, I) = (0)$, because $\mathrm{Tor}_1^R(R/I, I) \cong [(0): I] \cap I$. On the other hand, if I/I^2 were free, $[(0): I]$ would have to be contained in I, which would imply that $[(0): I] = (0)$. But as $\mathrm{depth}\, R = 0$, this is absurd. Recall that $H_1(I) = [(0): I]$ and that $\mathfrak{m} \cdot [(0): I] = (0)$. Then we readily find $H_1(I)$ is not R/I-free.

This example (3.6) shows that the assumption $n \geq 2$ in (1.1), (1.3), and (3.4), and the assumption $d \geq 2$ in (3.2), respectively, cannot be removed. Because I is a syzygetic ideal of $\mathrm{ht}_R I = 1$, the example also shows that the assumption $n \geq 2$ in the latter part of (1.2) is not superfluous.

References

1. M. Auslander and D. A. Buchsbaum, *Codimension and multiplicity*, Ann. of Math. (2) **68** (1958), 625–657.
2. J. Barja and A. G. Rodicio, *Syzygetic ideals, regular sequences, and a question of Simis*, J. Algebra **121** (1989), 310–314.
3. T. H. Gulliksen and G. Levin, *Homology of local rings*, Queen's Papers in Pure and Appl. Math. **20** (1969).
4. P. Schenzel, N. V. Trung, and N. T. Cuong, *Verallgemeinertte Cohen-Macaulay-Moduln*, Math. Nachr. **85** (1978), 57–73.
5. A. Simis, *Koszul homology and its syzygy-theoretic part*, J. Algebra **55** (1978), 28–42.
6. A. Simis and W. V. Vasconcelos, *The syzygies of the conormal module*, Amer. J. Math. **103** (1981), 203–224.
7. J. Stückrad and W. Vogel, *Buchsbaum rings and applications*, VEB Deutscher Verlag der Wissenschaften, 1987.
8. N. Suzuki, *On a basic theorem for quasi-Buchsbaum modules*, Bull. Dept. Gen. Ed. Shizuoka College of Pharmacy **11** (1982), 33–40.
9. W. V. Vasconcelos, *On the homology of I/I^2*, Comm. Algebra **6** (1978), 1801–1809.
10. ——, *Ideals generated by R-sequences*, J. Algebra **6** (1967), 309–316.

Department of Mathematics, Meiji University, Higashi-Mita 1-1-1, Tama-Ku, Kawasakishi, 214 Japan

Department of Mathematics, University of Shizuoka, Yata 395, Shizuoka-Shi, ShizuokaKen, 422 Japan