PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 112, Number 4, August 1991

A SECOND CATEGORY SET WITH ONLY FIRST CATEGORY FUNCTIONS

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(Communicated by Andreas R. Blass)

ABSTRACT. If the existence of a measurable cardinal is consistent then it is consistent in that there is a second category set $A \subseteq R$ such that every $A \to A$ function, as a subset of R^2, is of first category. Some other connected results are also proved.

0. INTRODUCTION

P. Erdös and the author proved in [4] that the plane admits a decomposition into countably many pieces none containing the three nodes of a right-angled triangle if and only if the continuum hypothesis holds. This implies that if the CH holds then there exists a second category set omitting right-angled triangles. The latter statement can actually be proved without any additional hypothesis by a transfinite recursion of length 2^ω. One only has to observe that $< 2^\omega$ lines and circles cannot cover a residual set, that is, the complement of a first category set. In fact this idea can be used to show that even in R^n, there exist second category/positive outer measure sets omitting rational distances, triangles of given shape/angle, or isosceles triangles, or any predetermined geometrical configurations. Returning to the result in [4], it implies the stronger statement, that (if CH holds) *every* second category set in R^2 contains a second category subset, omitting right-angled triangles. We show that this last statement is independent. For this, we prove that, assuming the consistency of a measurable cardinal, it is consistent that there exists a second category set $A \subseteq R$, such that the graph of every $f\colon A \to A$ function is of first category, as a subset of R^2 (it is convenient to identify functions with their graphs, as is done in axiomatic set theory). The measurable cardinal is not needed for the case of one-to-one functions. We show that for the general case at least an inaccessible cardinal is needed.

Using similar techniques, we prove a result complementary to an old theorem of Ulam; if the existence of a huge cardinal is consistent, then so is the existence of a second category set $A \subseteq R$ of size ω_3 such that every smaller subset is

Received by the editors January 9, 1990 and, in revised form, August 8, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 03E15; Secondary 03E35, 04A15, 54H05.

of first category and A cannot be decomposed into ω_3 many disjoint second category sets.

We use the standard set theory notions (see e.g. [5, 8]). Cardinals are identified with initial ordinals. For a set X, κ a cardinal,

$$[X]^{\kappa} = \{Y \subseteq X : |Y| = \kappa\},$$
$$[X]^{<\kappa} = \{Y \subseteq X : |Y| < \kappa\}.$$

1. One-to-one functions

Theorem 1. *It is consistent that $2^{\omega} = \omega_2$ and there exists a second category set $A \subseteq R$ such that every one-to-one function $f: A \to A$ is of first category.*

Proof. Let V be a model of ZFC + GCH. Let P add ω_1 Cohen reals. Let $A = \{r_\alpha : \alpha < \omega_1\}$ be the set of those Cohen reals. If $N \subseteq \omega_1$ is a nonstationary subset, let $Q(N)$ be the canonical partial ordering (see [5]) making $\{r_\alpha : \alpha \in N\}$ of first category. Let Q be the finite support product of these $Q(N)$'s. Our model will be $V^{P \star Q}$. It is well known that P, $P \star Q$ are ccc.

We first show that every one-to-one $f: A \to A$ is of first category. Let $g: \omega_1 \to \omega_1$ be the function such that $f(r_\alpha) = r_{g(\alpha)}$. By ccc, there is a closed unbounded set $C \subseteq \omega_1$ such that if $\alpha < \gamma \in C$, then $g(\alpha)$, $g^{-1}(\alpha) < \gamma$ is forced by 1. If $N = \omega_1 - C$, then the graph of f, $\{\{r_\alpha, f(r_\alpha)\} : \alpha < \omega_1\}$ is covered by

$$(\{r_\alpha : \alpha \in N\} \times R) \cup (R \times \{r_\alpha : \alpha \in N\}) \cup \{(x, x) : x \in R\},$$

and these sets are of first category.

We finally show that A is of second category in $V^{P\star Q}$. Assume that 1 forces A to be covered by $F_0, F_1, \ldots$, some nowhere dense, closed sets. Every nowhere dense, closed set can be described as the complement of some union of rational intervals. There are, by ccc, in V^P, countably many conditions $q_i \in Q$ $(i = 0, 1, \ldots)$ completely determining the F_i's. There are in V nonstationary sets $N_0, N_1, \ldots$ such that the supports of the q_i's are covered by $\{N_0, N_1, \ldots\}$. Select a $\gamma \in \omega_1 - (N_0 \cup N_1 \cup \cdots)$. We show that $1 \Vdash r_\gamma \notin \bigcup F_i$, and this gives the desired contradiction. It suffices to show that if $(p, q) \in P\star Q$, $i < \omega$, then there is an extension of (p, q) forcing that $r_\gamma \notin F_i$. Assume that the information p gives on r_γ is that $r_\gamma \in E$, where E is a diadic interval. In V^P it is true that there is an extension of q in Q forcing that $F_i \cap E' = \emptyset$ for some diadic subinterval E' of E. By the above remarks, we can find such a condition among those with supports from $\{N_0, N_1, \ldots\}$. Back in V we can find (p', q') forcing $F_i \cap E' = \emptyset$ where the γth coordinate of p' is still E, otherwise p' extends p. If we extend (p', q') to a condition (p'', q') where the only change is that the γth coordinate of p'' is E', then (p'', q') forces that r_γ is in the empty interval $E' \cap F_i$ which is impossible.

An easy argument gives

Theorem 2. *If CH holds, then every second category set has a second category one-to-one function onto itself.*

Proof. Assume that $A \subseteq R$ is a second category set.

Claim. $A \times A$ is of second category.

Proof of Claim. Assume that $A \times A = \bigcup\{T_n : n < \omega\}$, with T_n nowhere dense. Put $S(x, n) = \{y \in A : (x, y) \in T_n\}$ for $x \in A$. As $\bigcup\{S(x, n) : n < \omega\} = A$, for every $x \in A$, some $S(x, n)$ is dense in some rational interval I. If $n < \omega$, I is a rational interval, put $U(n, I) = \{x \in A : S(x, n)$ is dense in $I\}$. As $\cup\{U(n, I) : n, I\} = A$, for some n, I, $U(n, I)$ is dense in an interval J, so T_n is dense in $I \times J$, a contradiction.

Enumerate the first category F_σ subsets of R^2 as $\{B_\alpha : \alpha < \omega_1\}$. Select by transfinite recursion a_α, $b_\alpha \in A$ such that $a_\alpha \neq a_\beta$, $b_\alpha \neq b_\beta$ $(\beta < \alpha)$ and $(a_\alpha, b_\alpha) \notin B_\alpha$. As $A \times A$ is of second category, countably many lines plus B_α cannot cover it, so a_α, b_α may be selected. If we set $f(a_\alpha) = b_\alpha$, then f is not covered by any B_α, so it is of second category.

2. General functions

Theorem 3. *If the existence of a measurable cardinal is consistent then so is the existence of a second category set $A \subseteq R$ such that every $A \to A$ function is of first category. The same holds for every $A^n \to A$ function $(n = 1, 2, \ldots)$.*

Proof. Let V be a model of ZFC such that κ is measurable, and I is a κ-complete, normal, prime ideal on κ. As in the proof of Theorem 1, let P add κ Cohen reals, let their set be $A = \{r_\alpha : \alpha < \kappa\}$, and let Q make every $\{r_\alpha : \alpha \in B\}$ $(B \in I)$ of first category. As is the proof of Theorem 1, $P * Q$ is ccc and A is of second category, even in $V^{P,Q}$.

Assume first that $f : A \to A$ is a function in $V^{P,Q}$. Let $\underline{f}$ be a name for f such that $1 \Vdash \underline{f} : A \to A$. By ccc for every $\alpha < \kappa$ the set of potential values

$$T_\alpha = \{\xi < \kappa : \exists p \Vdash f(r_\alpha) = r_\xi\}$$

is countable. Say $T_\alpha = \{g_n(\alpha) : n < \omega\}$. It suffices to show that $S(g_n)$ is of first category, where $S(g) = \{(r_\alpha, r_{g(\alpha)}) : \alpha < \kappa\}$. If $g : \kappa \to \kappa$, $g \in V$ is given, by the properties of I, either $g(\alpha) = c$ holds for some $c < \kappa$, outside a set in I, which will be abbreviated by $g(\alpha) = c$ almost everywhere, or $g(\alpha) = \alpha$ almost everywhere, or else $g(\alpha) > \alpha$ almost everywhere. Let B be the set of those α values for which the appropriate (in) equality does not hold. As $B \in I$, $C = \{r_\alpha : \alpha \in B\}$ is of first category, in the final model.

The range of g is in I, in the first and the third cases. This is obvious for the first case, but in the third case there is a regressive function, namely g^{-1} on it, which does not have an unbounded set where it is constant. The normality of I gives the statement.

This implies that $D = \{r_{g(\alpha)} : \alpha < \kappa\}$ is of first category in $V^{P,Q}$. The graph in question is covered by

$$(C \times R) \cup (R \times \{c\}), \qquad (C \times R) \cup \{(x, x) : x \in R\}, \quad \text{and} \quad (C \times R) \cup (R \times D),$$

in the respective cases, so it is of first category.

To prove the general result about $f: A^n \to A$, it again suffices to show that if $g: \kappa^n \to \kappa$, then $T(g) = \{(r_{x_1}, \dots, r_{x_n}, r_{g(x_1, \dots, x_n)}) : x_i < \kappa\}$ is of first category in R^{n+1}. It is well known that there exists a set $B \in I$ such that $\kappa - B$ is a set of indiscernibles (see [5]). If $\pi: \{1, 2, \dots, n\} \to \{1, 2, \dots, n\}$ is a permutation, then $y = g(x_{\pi(1)}, \dots, x_{\pi(n)})$ lies in the same interval (depending only on π) with respect to $x_1 < \cdots < x_n$ for $x_1, \dots, x_n \in \kappa - B$. This implies that $y \in B$. Therefore, every vector in $T(g)$ has a coordinate in $\{r_\alpha : \alpha \in B\}$, which is of first cateogry, and so is $T(g)$.

Corollary 4. *If the existence of a measurable cardinal is consistent then so is the existence of a second category planar set that does not contain a second category subset omitting right angled triangles.*

Proof. $A \times A$ from Theorem 3 will work. Assume that $B \subseteq A \times A$ spans no right-angled triangles. Then, for $(x, y) \in B$ there is either no $z \in A$ such that $z \neq y$ and $(x, z) \in B$, or else there is no $z \in A$, $z \neq x$ such that $(z, y) \in B$. In other words, B is covered by a 'horizontal' and by a 'vertical' function.

Statement 5. *Corollary* 4 *is true with 'rectangles' in place of 'right-angled triangles.'*

Proof. We use the model of Theorem 3. Assume that $1 \Vdash B \subseteq A \times A$. For $\alpha < \beta < \kappa$, either $1 \Vdash (r_\alpha, r_\beta), (r_\beta, r_\alpha) \notin B$ or there is a $p(\alpha, \beta)$ forcing $(r_\alpha, r_\beta) \in B$ or forcing $(r_\beta, r_\alpha) \in B$. Fix such a $p(\alpha, \beta)$ (if it exists). If $\alpha < \beta_1 < \beta_2$ and $p(\alpha, \beta_1)$, $p(\alpha, \beta_2)$ are compatible, fix a common lower bound $p(\alpha, \beta_1, \beta_2)$. If, for $\alpha_1 < \alpha_2 < \beta_1 < \beta_2$, $p(\alpha_1, \beta_1, \beta_2)$ and $p(\alpha_2, \beta_1, \beta_2)$ are compatible, fix $q(\alpha_1, \alpha_2, \beta_1, \beta_2)$, a common lower bound.

Color the pairs, triples, and quadruples of κ according to which of the above-mentioned cases occur. By measurability, there is a measure one set $Z \subseteq \kappa$ on which the same cases hold. If, for $\alpha < \beta$ in Z, 1 forces $(r_\alpha, r_\beta), (r_\beta, r_\alpha) \notin B$, then B is covered by two first category sets as in the previous proofs, and we are done. We can therefore assume that, say, $p(\alpha, \beta)$ forces $(r_\alpha, r_\beta) \in B$ for $\alpha < \beta$ in Z. By the homogeneity of Z, either $p(\alpha, \beta_1)$, $p(\alpha, \beta_2)$ are incompatible for $\alpha < \beta_1 < \beta_2$ in Z, or they are always compatible. The former case is impossible, as the applied notion of forcing is ccc, and there are ω_1 different elements above an α in Z. Again, by homogeneity, either $p(\alpha_1, \beta_1, \beta_2)$, $p(\alpha_2, \beta_1, \beta_2)$ are incompatible whenever $\alpha_1 < \alpha_2 < \beta_1 < \beta_2$ in Z, or else they are always compatible. The former case is impossible, as there are $\beta_1 < \beta_2$ in Z, preceded by ω_1 elements of Z, and the notion of forcing is ccc. There are, therefore, $\alpha_1 < \alpha_2 < \beta_1 < \beta_2$ such that $q(\alpha_1, \alpha_2, \beta_1, \beta_2)$ exists, and forces the rectangle $\{r_{\alpha_1}, r_{\alpha_2}\} \times \{r_{\beta_1}, r_{\beta_2}\}$ to be in B.

We notice that if a set A satisfies Theorem 3, then $|A| > \omega_1$. For this, one takes the Sierpiński decomposition of $A \times A$ into countably many 'horizontal' and 'vertical' functions. One of them is of second category. (See e.g. [10, 11].)

It is possible that the conclusion of Theorem 3 implies the consistency of a measurable cardinal. Although we have not been able to prove this we have some evidence in its favor.

If κ is a cardinal, an ultrafilter U over κ is *uniform*, if $|A| = \kappa$ holds for every $A \in U$. An ultrafilter U is (ω, κ)-*regular*, if there exist $A_\alpha \in U$ $(\alpha < \kappa)$, such that the intersection of any infinitely many A_α's is empty.

Statement 6. *If a set as in Theorem* 3 *exists, then there is for some* κ *a uniform, non-*(ω, κ)*-regular ultrafilter over* κ.

Proof. Assuming that $A \subseteq R$ satisfies Theorem 3, we can assume that $\kappa = |A|$ is of minimal cardinality. Then every subset of A of size $< \kappa$ is of first category. Put $I = \{B \subseteq A\colon B \text{ is of first category}\}$. I is clearly a nontrivial, ω_1-complete ideal.

Claim 1. There are no mutually disjoint sets $B_\alpha \notin I$, $B_\alpha \subseteq A$, $\alpha < \kappa$.

Proof. Assume that $\{B_\alpha\colon \alpha < \kappa\}$ are disjoint, $A = \{a_\alpha\colon \alpha < \kappa\}$. Define $f\colon A \to A$ as follows: $f(x) = a_\alpha$ if $x \in B_\alpha$. If the graph of f is of first category, then by the Fubini theorem, for all but first category many $y \in R$, the set $\{x \in R\colon f(x) = y\}$ is of first category. There is, therefore, a $a_\alpha \in A$ such that $B_\alpha = f^{-1}(a_\alpha)$ is of first category; a contradiction.

To conclude, we need an argument of Silver (see [1]). Let I be an ω_1-complete ideal over some κ, $I \supseteq [\kappa]^{<\kappa}$. $I^+ = \{X \subseteq \kappa\colon X \notin I\}$, $I^\star = \{X \subseteq \kappa\colon \kappa - X \in I\}$.

Claim 2 (Silver). If there are no κ disjoint sets in I^+, then every ultrafilter $U \supseteq I^\star$ is (ω, κ)-irregular.

Proof. Assume that $X_\alpha \in U$ $(\alpha < \kappa)$. Put $Y \in G$ if there exists some $s \in [\kappa]^{<\kappa}$ such that $X_\alpha - Y \in I$ for all $\alpha \notin s$. Obviously, $\varnothing \notin G$.

Subclaim. G is ω_1-complete.

Proof. Assume that $Y_n \in G$, s_n, $n = 0, 1, \ldots$, witness this. Then $s = \bigcup\{s_n\colon n < \omega\}$ witnesses that $Y = \bigcap\{Y_n\colon n < \omega\} \in G$. If $\alpha \notin s$, then $\alpha \notin s_n$ $(n < \omega)$, so $X_\alpha - Y \subseteq \bigcup\{(X_\alpha - Y_n)\colon n < \omega\} \in I$, as I is ω_1-complete. As $[\kappa]^{<\kappa} \subseteq I$, and I is ω_1-complete, cf $(\kappa) > \omega$, so $|s| < \kappa$.

Select $\alpha(\xi) < \kappa$ as long as possible (say for $\xi < \theta$), such that $X_{\alpha(\xi)} - \bigcup\{X_{\alpha(\tau)}\colon \tau < \xi\} \in I^+$. By our hypotheses, $\theta < \kappa$. Let $S_0 = \{\alpha(\xi)\colon \xi < \theta\}$, then from $\kappa - S_0$ select similarly S_1, then S_2, etc. By the definition of G, $Y_n = \bigcup\{X_\alpha\colon \alpha \in S_n\} \in G$. Select $\gamma \in \bigcap\{Y_n\colon n < \omega\}$, then $\alpha_n \in S_n$ such that $\gamma \in X_{\alpha_n}$. We are done, as $\bigcap\{X_{\alpha_n}\colon n < \omega\} \neq \varnothing$.

H.-D. Donder obtained several results on nonregular ultrafilters [2]. Though he only proved that the existence of an ultrafilter as in Claim 2 implies the consistency of an inaccessible cardinal, it is very likely that those methods will lead to the proof of the consistency of a measurable cardinal, and giving the necessary counterpart to Theorem 3.

3. Decompositions of second category sets

An old result of Ulam [12] says that if

$$(*) \qquad \text{no cardinal } \leq 2^{\omega} \text{ is weakly inaccessible}$$

then every second category set is the disjoint union of ω_1 second category sets. We show in [6] that this statement is not true in general. Ulam's argument actually shows that under $(*)$ every second category $A \subseteq R$ is the disjoint union of τ second category sets, where τ is the least cardinal such that the union of τ many first category subsets of A may be of second category. Obviously, τ is uncountable and regular. One can ask if under $(*)$, τ can be replaced by $\theta = \min\{|B|: B \subseteq A \text{ 2nd category}\}$. We show that it is not the case.

Theorem 7. *If the existence of a huge cardinal is consistent then so is that* $2^{\omega} = \omega_4$ *and there exists a second category set* $A \subseteq R$ *with* $|A| = \omega_3$, *all* $B \subseteq A$ *with* $|B| \leq \omega_2$ *are of first category, and* A *is not the union of* ω_3 *disjoint second category sets.*

Proof. We are going to build a model similar to those in Theorems 1 and 3. Let I be a uniform ideal on ω_3 such that

$$(3.1) \qquad I \text{ is } \omega_1 - \text{complete};$$

$$(3.2) \qquad \text{if } A_{\xi} \in I^{+} (\xi < \omega_3), \text{ then there is a } Z \in [\omega_3]^{\omega_1} \text{ such that } \bigcap\{A_{\xi}: \xi \in Z\} \neq \emptyset.$$

Then let P be the notion of forcing which adds ω_3 Cohen reals, $\{r_{\alpha}: \alpha < \omega_3\}$. In V^{P}, for $B \in I$, let $Q(B)$ be the canonical notion of forcing that makes $\{r_{\alpha}: \alpha \in B\}$ of first category. Q is the finite support product of the $Q(B)$'s for all $B \in I$. We force by $P^{*}Q$. The proof that $A = \{r_{\alpha}: \alpha < \omega_3\}$ remains of second category is entirely the same as the corresponding proof in Theorem 1.

Assume that 1 forces $\{A_{\xi}: \xi < \omega_3\}$ to be a decomposition of A. Take $B_{\xi} = \{\alpha < \omega_3: \exists p \Vdash r_{\alpha} \in A_{\xi}\}$. If some B_{ξ} is in I, then A_{ξ} is made first category by $P \star Q$. If $B_{\xi} \notin I$ for all $\xi < \omega_3$, then by (3.2), there is a $Z \in [\omega_3]^{\omega_1}$ and a $\tau < \omega_3$ such that $\tau \in B_{\xi}$ for $\xi \in Z$. Choose $p(\xi) \in P \star Q$ forcing $r_{\tau} \in A_{\xi}$, then $\{p(\xi): \xi \in Z\}$ are ω_1 incompatible elements of $P \star Q$, a contradiction, since $P \star Q$ is ccc.

The remainder of the proof is dedicated to showing that a model of Magidor [9] contains an ideal I as described in (3.1, 3.2). Magidor's model can briefly be described as follows. Let κ be a huge cardinal, $j: V \to M$ an elementary embedding with κ as critical point, $j(\kappa) = \lambda$, $[M]^{\lambda} \subseteq M$. Let P be a notion of forcing such that

(3.3) $$|P| = \kappa\,,\ P \text{ is } \kappa\,\text{cc};$$

(3.4) $$V^P \Vdash \kappa = \omega_1\,;$$

(3.5) if $\alpha < \kappa$ is regular, $C \subseteq P$ is a regular subalgebra, $|C| < \kappa$, then $C * \mathrm{Col}(\alpha, \kappa) \subseteq P$.

A forcing notion with these properties can be built by diagonalization, see [8, 9]. In V^P, let Q be the canonical poset collapsing λ to $\kappa^{++} = \omega_3$ via conditions of size $\leq \kappa$. In $V^{P,Q}$ we are going to construct an ideal I over $[\lambda]^{<\kappa}$. As GCH holds, this suffices. In $M^{j(P)}$, $j(Q)$ is a partial ordering, which, over $M^{j(P)}$, collapses $j(\lambda)$ to λ^{++} with conditions of size $\leq \lambda$.

In $V^{j(P)}$ we are going to define $\{y_\alpha : \alpha < \lambda^+\}$, a decreasing sequence of elements of $j(Q)$. Let y_0 be a master condition for a generic $G \subseteq j(P)$. Then for every $A \subseteq [\lambda]^{<\kappa}$, $A \in V^{P,Q}$ let a y_α be selected that

(3.6) $$y_\alpha \Vdash j''\lambda \in j(A) \quad \text{or} \quad y_\alpha \Vdash j''\lambda \notin j(A).$$

In $V^{j(P)}$, let $A \in \mathscr{F}$ if for some α, $y_\alpha \Vdash j''\lambda \in j(A)$. This is an ultrafilter, in $V^{j(P)}$, over $P([\lambda]^{<\kappa}) \cap V^{P,Q}$. $\mathscr{F}$ is κ-complete, see [9].

In $V^{P,Q}$ we define I as follows. $A \subseteq [\lambda]^{<\kappa}$, $A \in V^{P,Q}$ is in I if $1 \Vdash A \in \mathscr{F}$. As $\mathscr{F}$ is κ-complete, I is κ-complete, so (3.1) is fulfilled.

For (3.2), let $A_\alpha \in I$, $(\alpha < \lambda)$ be given. Let $p(\alpha) \in j(P)/P * Q$ force $A \in \mathscr{F}$. $j(P)/P * Q$ is λ cc, and a well-known lemma with the regularity of λ ensures that some p forces that λ many $p(\alpha)$'s are in the generic set. Forcing by this element, we get that, in $V^{j(P)}$, λ many of the A_α's are in $\mathscr{F}$. For each of them, there is a y_ξ forcing $j''\lambda \in j(A_\alpha)$.

As M is closed under λ-sequences, in $M^{j(P),j(Q)}$ for $j(A_\alpha : \alpha < \lambda)$, it is true that for λ many elements of the sequence it is forced that $j''\lambda$ is contained in them, as $j(A)_{j(\alpha)} = j(A_\alpha)$ (for $\alpha < \lambda$). For λ many of them, the intersection is nonempty, therefore, by the elementarity of j, for κ many A_α, the intersection of them is nonempty in $V^{P,Q}$.

Acknowledgment

The author is grateful to the referee for several remarks and suggestions.

References

1. W. Boos, *Lectures on large cardinal axioms*, Lecture Notes in Math., vol. 669, Springer-Verlag, Berlin and New York, pp. 25–88.
2. H.-D. Donder, *Regularity of ultrafilters and the core model*, Israel J. Math. **63** (1988), 289–322.

3. P. Erdös, *Set theoretic, measure theoretic, combinatorial, and number theoretic problems concerning point sets in Euclidean space*, Real Analysis Exchange **4** (1978-79), 113–138.

4. P. Erdös and P. Komjáth, *Countable decompositions of R^2 and R^3*, Discrete Comput. Geom. **5** (1990), 325–331.

5. T. Jech, *Set theory*, Academic Press, New York, 1978.

6. P. Komjáth, *On second-category sets*, Proc. Amer. Math. Soc. **107** (1989), 653–654.

7. K. Kunen, *Set theory. An introduction to independence proofs*, North-Holland, Amsterdam, 1980.

8. ——, *Saturated ideals*, J. Symbolic Logic **43** (1978), 65–76.

9. M. Magidor, *On the existence of nonregular ultrafilters and the cardinality of ultrapowers*, Trans. Amer. Math. Soc. **249** (1979), 97–111.

10. W. Sierpiński, *Sur un théorème équivalent à l'hypothèse du continu*, Bull. Acad. Polon. Lett. Sér. A (1919), 1–3.

11. J. C. Simms, *Sierpiński's theorem* (to appear).

12. St. Ulam, *Über gewisse Zerlegungen von Mengen*, Fund. Math. **20** (1933), 221–223.

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