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A FULL DESCRIPTION OF EXTREME POINTS IN $C(\Omega, L^{\varphi}(\mu))$

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ABSTRACT. Let $L^{\varphi}(\mu)$ be an Orlicz space endowed with the Luxemburg norm. The main result of this paper reads as follows: f is an extreme point of the unit ball of the space of continuous functions from a compact Hausdorff space Ω into $L^{\varphi}(\mu)$ with the supremum norm if and only if the inverse image of the set of all extreme points of the unit ball of $L^{\varphi}(\mu)$ under f is dense in Ω.

1. INTRODUCTION

Let Ω be a compact Hausdorff space, and let X be a real Banach space. By $C(\Omega, X)$ we denote the Banach space of X-valued continuous functions on Ω equipped with the sup-norm $\|f\| = \sup_{\omega\in\Omega} \|f(\omega)\|$. By $\operatorname{ext} B(X)$ we denote the set of all extremal points of the unit ball $B(X) = \{x : \|x\| \leq 1\}$ of X.

One of the most elegant characterizations of the set $\operatorname{ext} B(C(\Omega, X))$ is the following

$$(1) \qquad f \in \operatorname{ext} B(C(\Omega, X)) \Leftrightarrow f(\omega) \in \operatorname{ext} B(X) \quad \text{for every } \omega \in \Omega$$

(the implication "$\Leftarrow$" is trivial). It is evident that

$$f \in \operatorname{ext} B(C(\Omega, X)) \Rightarrow \|f(\omega)\| = 1 \quad \text{for every } \omega \in \Omega.$$

Therefore (1) is satisfied for every strictly convex Banach space X. Further, (1) remains reasonable only in case $\operatorname{ext} B(X)$ is closed. An attempt in that direction has been done by Clausing and Papadopoulou [2, 11]. Although their papers have been devoted to the study of stability of convex sets, that property applied to $B(X)$ implies the closedness of $\operatorname{ext} B(X)$ and, together with the Michael selection theorem [8], the equivalence (1). Since every Banach space X that is strictly convex or satisfies 3.2 intersection property has stable unit ball, (1) appears as a trivial consequence of the above-mentioned selection theorem (original proofs are in [6, 8]).

D. Werner [14] proved that the equivalence (1) holds provided $X = L^1(\mu)$. Following that result, R. Grząślewicz established (1) in the case $X = L^{\varphi}(\mu)$,

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$L^{\varphi}(\mu)$ being an Orlicz space endowed with the Luxemburg norm, under assumptions that φ takes only finite values and satisfies the condition Δ_2 depending on the measure μ (in short: $\varphi \in \Delta_2$). A. Suarez-Granero has observed that under the Grząślewicz assumptions the unit ball of $L^{\varphi}(\mu)$ is stable, so the before mentioned result can be deduced via the Michael selection theorem. Thus, the Grząślewicz result has nothing to say about Orlicz spaces without stable unit ball.

However, "stability" is a wider notion than "Orlicz spaces with $\varphi \in \Delta_2$." In fact, l^{∞} can serve as an example of an Orlicz space with stable unit ball (it has 3.2 intersection property), which does not satisfy the Grząślewicz assumptions ($l^{\infty} \equiv l^{\varphi}$, where $\varphi(u) = 0$ for $|u| \leq 1$ and $\varphi(u) = +\infty$ otherwise). In [15, Theorem 4] there has been established that an Orlicz sequence space has stable unit ball if and only if either $\varphi \in \Delta_2$ or l^{φ} is isometric to l^{∞}.

The only way that allows us to drop the stability arguments is that of getting rid of the Δ_2-condition. Since the closedness of $\operatorname{ext} B(L^{\varphi}(\mu))$ is no loner guaranteed, the more reasonable conjecture (and more general version) of (1) arises on replacing "for all $\omega \in \Omega$" by "on a dense subset of Ω."

However, it should be pointed out that the equivalence (1) (even in the general version) does not hold in general. Extreme points of $B(C(\Omega, X))$ can have nothing to do with the set $\operatorname{ext} B(X)$. Blumenthal, Lindenstrauss, and Phelps [1] have presented an example of a four-dimensional space X and a function $f \in \operatorname{ext} C([0, 1], X)$ such that $f(\omega) \notin \operatorname{ext} B(X)$ for all $\omega \in [0, 1]$. The aim of this paper is to give a full characterization of extreme points of $B(C(\Omega, L^{\varphi}(\mu)))$ without Δ_2-arguments. The main result reads as follows.

1. **Theorem.** *Let Ω be a compact Hausdorff space. A function f is an extreme point of $B(C(\Omega, L^{\varphi}(\mu)))$ if and only if $f^{-1}(\operatorname{ext} B(L^{\varphi}(\mu)))$ is a dense subset of Ω.*

The sufficiency part of the proof is elementary. The necessity part requires some auxiliary results; it follows from Corollary 3(i) and Proposition 4 below. Besides, to avoid misunderstandings, we should also fix some notations, which is the aim of the next section.

2. Auxiliary definitions and results

Let (T, Σ, μ) be a σ-finite measure space and let $\varphi: \mathbb{R} \to [0, \infty]$ be a convex, even function with $\varphi(0) = 0$. Although the condition Δ_2 is not the key of importance in this paper, let us recall that $\varphi \in \Delta_2$ iff there is a constant $M > 0$ such that $\varphi(2u) \leq M\varphi(u)$ for every u from some appropriate set $D(\mu) \subseteq \mathbb{R}$ depending on the measure μ (e.g. $D(\mu)$ is a neighborhood of ∞ if $\mu(T) < \infty$, μ atomless; $D(\mu) = \mathbb{R}$ provided $\mu(T) = \infty$, μ atomless; $D(\mu)$ is a neighborhood of 0 with $\varphi(\sup D(\mu)) > 0$ if μ is the counting measure on $T = \mathbb{N}$ etc.).

By *the Orlicz space* $L^{\varphi}(\mu)$ we mean the set of all measurable functions $x: T \to \mathbb{R}$ such that $\int_T \varphi(\lambda x(t))\, d\mu < \infty$ for some $\lambda > 0$ equipped with the equality "almost everywhere" (a.e.) and the Luxemburg norm

$$\|x\|_{\varphi} = \inf\left\{\lambda > 0 : \int_T \varphi(\lambda^{-1}x(t))\,d\mu \le 1\right\}$$

(cf. [7, 9, 10]). To simplify the notation we write $I_{\varphi}(x)$ instead of $\int_T \varphi(x(t))d\mu$. Those functions that satisfy $I_{\varphi}(\lambda x) < \infty$ for every $\lambda > 0$ form a closed subspace of $L^{\varphi}(\mu)$, which is denoted by $E^{\varphi}(\mu)$. If φ takes only finite values and the measure μ produces more than a finite number of atoms (i.e. the dimension of $L^{\varphi}(\mu)$ is infinite), then we can characterize the condition Δ_2 in the following way: $\varphi \in \Delta_2$ iff $L^{\varphi}(\mu) = E^{\varphi}(\mu)$ iff $(\|x\|_{\varphi} = 1 \Leftrightarrow I_{\varphi}(x) = 1$ for every $x \in L^{\varphi}(\mu))$. In the other case of measure, those equivalences hold true iff φ takes finite values.

Using standard methods and the Fatou lemma one can prove the lower semicontinuity of $I_{\varphi} : L^{\varphi}(\mu) \to [0, \infty]$. Further, $L^{\varphi}(\mu)$ is a Banach space with the following property:

(2) norm convergence implies convergence in measure μ on every set of finite measure.

(If μ is a purely atomic measure, then (2) simply says that the norm convergence implies the convergence with respect to each coordinate.)

Every extreme point x of $B(L^{\varphi}(\mu))$ is of norm one. Define $c(\varphi) = \sup\{u : \varphi(u) < \infty\}$. Let us recall that if μ is an atomless (resp. purely atomic) measure, then $x \in \operatorname{ext} B(L^{\varphi}(\mu))$ if and only if

(i) $I_{\varphi}(x) = 1$ or $|x(t)| = c(\varphi) < \infty$ for a.e. (resp. every) $t \in T$;

(ii) $(x(t), \varphi(x(t)))$ are points of strict convexity of $\operatorname{Graph}\varphi$ for a.e. (resp. every but one) $t \in T$

(cf. [5, 16]). Thus the only extreme point of $B(L^{\varphi}(\mu))$ with $x \ge 0$, $I_{\varphi}(x) < 1$ can be the function e defined by $e(t) = c(\varphi)$ for $t \in T$.

2. **Proposition.** *If Ω is a compact Hausdorff space and f is an extreme point of $B(C(\Omega, L^{\varphi}(\mu)))$, then for every $\eta > 0$, the set $\{\omega \in \Omega \setminus E_{\eta} : I_{\varphi}(f(\omega)) < 1\}$ is of the first Baire category in $\Omega \setminus E_{\eta}$, where $E_{\eta} = \{\omega : \|\,|f(\omega)| - e\|_{\varphi} < \eta\}$ if $e \in \operatorname{ext} B(L^{\varphi}(\mu))$ and $E_{\eta} = \varnothing$ otherwise.*

Proof. Evidently

$$\{\omega \in \Omega \setminus E_{\eta} : I_{\varphi}(f(\omega)) < 1\} = \bigcup_{m=1}^{\infty} A_m,$$

where $A_m = \{\omega \in \Omega \setminus E_{\eta} : I_{\varphi}(f(\omega)) \le 1 - 1/m\}$. Since f is continuous and I_{φ} is lower semicontinuous, each A_m is closed.

Suppose that $\bigcup_m A_m$ is not of the first Baire category in $\Omega \setminus E_{\eta}$. Then $\operatorname{int} A_m \ne \varnothing$ for some $m \in \mathbb{N}$, so we can find an open set W with

$$\sup_{\omega \in W} I_{\varphi}(f(\omega)) \le 1 - 1/m.$$

Let us fix an arbitrary element ω_0 of the set W. We have $e \notin \operatorname{ext} B(L^{\varphi}(\mu))$ iff $\|e\|_{\varphi} > 1$ (indeed, if $\|e\|_{\varphi} \leq 1$ then $c(\varphi) < \infty$ and $\varphi(c(\varphi)) < \infty$; so $(c(\varphi), \varphi(c(\varphi)))$ are points of strict convexity of $\operatorname{Graph}\varphi$, and by virtue of [5, 16], $e \in \operatorname{ext} B(L^{\varphi}(\mu))$). Thus, regardless whether e is or is not an extreme point of $B(L^{\varphi}(\mu))$, $f(\omega_0) \neq e$. Hence, for some fixed $n > 1$, the set

$$B = \{t \in T : |f(\omega_0)(t)| < \min\{2n-1, (1-1/n)c(\varphi)\}\}$$

is of positive measure.

The rest of the proof is divided into two parts.

(1) B contains an atom $\{a\}$. By (2), $f(\cdot)(a) \colon \Omega \to \mathbb{R}$ is a continuous function. Further, $\varphi(|\cdot|)$ is uniformly continuous on

$$\{u : |u| \leq \min\{2n-1, (1-1/n)c(\varphi)\}\},$$

so there exists $\delta > 0$ such that $\varphi(|u| + \delta) \leq \varphi(u) + 1/m\mu(\{a\})$ for every u from that set. Therefore we can find a neighborhood $U \subseteq W$ of ω_0 such that

$$\varphi(f(\omega)(a) + \delta) \leq \varphi(|f(\omega)(a)| + \delta) \leq \varphi(f(\omega)(a)) + 1/m\mu(\{a\})$$

for every $\omega \in U$.

$\Omega \setminus U$ and $\{\omega_0\}$ are compact disjoint subsets of Ω. Thus, the Urysohn lemma shows that there exists a continuous function $s \colon \Omega \to [0, \delta]$ such that $s(\omega_0) = \delta$ and $s(\omega) = 0$ for every $\omega \notin U$. Now, let

$$g(\omega) = f(\omega) + s(\omega)\chi_{\{a\}}, \qquad h(\omega) = f(\omega) - s(\omega)\chi_{\{a\}}.$$

Obviously, g, h are continuous, $g \neq h$, and $\frac{1}{2}(g+h) = f$. In order to prove that $f \notin \operatorname{ext} B(C(\Omega, L^{\varphi}(\mu)))$ it suffices to show that $g, h \in B(C(\Omega, L^{\varphi}(\mu)))$ i.e., that $I_{\varphi}(g(\omega)) \leq 1$ and $I_{\varphi}(h(\omega)) \leq 1$ for every $\omega \in \Omega$. These inequalities are evident if $\omega \notin U$. Further, if $\omega \in U$,

$$\begin{aligned} I_{\varphi}(g(\omega)) &= I_{\varphi}(f\chi_{T\setminus\{a\}}(\omega)) + \varphi(f(\omega)(a) + s(\omega))\mu(\{a\}) \\ &\leq I_{\varphi}(f\chi_{T\setminus\{a\}}(\omega)) + \varphi(f(\omega)(a))\mu(\{a\}) + 1/m \\ &= I_{\varphi}(f(\omega)) + 1/m \leq 1 \end{aligned}$$

and, analogously, $I_{\varphi}(h(\omega)) \leq 1$.

(2) No atom of the measure μ is a subset of B. Without loss of generality we can assume that $0 < \mu(B) < \infty$. Let $\lambda = 2n/(2n-1)$ $(\lambda > 1)$ and $0 < \delta \leq \min\{1, c(\varphi)/2n+1\}$. Then for every $t \in B$,

$$\lambda(|f(\omega_0)(t)| + \delta) \leq \begin{cases} 2n+1 & \text{if } c(\varphi) = \infty, \\ \left(1 - \frac{1}{4n^2-1}\right) c(\varphi) & \text{otherwise}. \end{cases}$$

Hence $I_{\varphi}(\lambda(|f(\omega_0)| + \delta)\chi_B) < \infty$. Thus, we can find a subset $C \subseteq B$ such that $0 < I_{\varphi}(\lambda(|f(\omega_0)| + \delta)\chi_C) \leq 1/m$. Moreover, by the continuity of f, there is an open neighborhood $U \subseteq W$ of ω_0 such that $\|\frac{\lambda}{\lambda-1}[f(\omega) - f(\omega_0)]\|_{\varphi} \leq \frac{1}{m} < 1$. Then $I_{\varphi}(\frac{\lambda}{\lambda-1}[f(\omega) - f(\omega_0)]) \leq \frac{1}{m}$ for $I_{\varphi}(x) \leq \|x\|_{\varphi}$ provided $\|x\|_{\varphi} \leq 1$.

By virtue of the Urysohn lemma there exists a continuous function $s\colon \Omega \to [0, \delta]$ such that $s(\omega_0) = \delta$ and $s(\omega) = 0$ for every $\omega \notin U$. Let $g(\omega) = f(\omega)+s(\omega)\chi_C$, $h(\omega) = f(\omega)-s(\omega)\chi_C$. The functions g and h are continuous, $g \neq h$, and $\frac{1}{2}(g+h) = f$. Obviously $I_\varphi(g(\omega)) \leq 1$ and $I_\varphi(h(\omega)) \leq 1$ for every $\omega \notin U$. Moreover, for every $\omega \in U$,

$$\begin{aligned} I_\varphi(g(\omega)) &= I_\varphi(f(\omega)\chi_{T\setminus C}) + I_\varphi(\tfrac{1}{\lambda}\lambda[f(\omega_0) + s(\omega)]\chi_C \\ &\qquad + (1 - \tfrac{1}{\lambda})(\tfrac{\lambda}{\lambda-1})[f(\omega) - f(\omega_0]\chi_C) \\ &\leq I_\varphi(f(\omega)) + \tfrac{1}{\lambda} \cdot I_\varphi(\lambda[|f(\omega_0)| + \delta]\chi_C) + (1 - \tfrac{1}{\lambda}) \cdot I_\varphi(\tfrac{\lambda}{\lambda-1}[f(\omega) - f(\omega_0]) \\ &\leq 1 - \tfrac{1}{m} + \tfrac{1}{\lambda} \cdot \tfrac{1}{m} + (1 - \tfrac{1}{\lambda})\tfrac{1}{m} = 1. \end{aligned}$$

Thus, f is not an extreme point of $B(C(\Omega, L^{\varphi}(\mu)))$. The obtained contradiction completes the proof.

Applying the Baire category theorem [3, Theorem 3.9.3, p. 253], we obtain the following corollary.

3. **Corollary.** *Let us assume that Ω is a compact Hausdorff space and that $f \in \operatorname{ext} B(C(\Omega, L^{\varphi}(\mu)))$. Then the following assertions hold*:

(i) $\{\omega : I_\varphi(f(\omega)) = 1$ *or* $|f(\omega)| = e\}$ *is dense in* Ω;

(ii) *for every* $\eta > 0$ $f(\Omega) \cap \{x \in L^{\varphi}(\mu) : I_\varphi(x) < 1$ *and* $\| |x| - e\|_\varphi \geq \eta\}$ *is of the first Baire category in* $f(\Omega)$ *with the topology inherited from* $L^{\varphi}(\mu)$; *and*

(iii) $f(\Omega) \cap \{x : I_\varphi(x) = 1$ *or* $|x| = e\}$ *is dense in* $f(\Omega)$.

Let us turn back to the Grzaślewicz theorem [4]. The assumption $\varphi \in \Delta_2$ made there was necessary to conclude that $I_\varphi(x) = 1$ on every x with norm one. Further, the assumption that φ takes only finite values is superfluous there. Therefore, those assumptions can be omitted provided that the statement is reformulated as follows.

4. **Proposition** [4]. *If Ω is a compact Hausdorff space, f is an extreme point of $B(C(\Omega, L^{\varphi}(\mu)))$, and $I_\varphi(f(\omega)) = 1$, then $f(\omega) \in \operatorname{ext} B(L^{\varphi}(\mu))$.*

Note that the equality $|f(\omega)| = e$ for some $\omega \in \Omega$ is possibly only if $\|e\| = 1$, i.e., $e \in \operatorname{ext} B(L^{\varphi}(\mu))$. Thus the necessity part of the proof of Theorem 1 is an immediate consequence of Corollary 3(i) and Proposition 4. Applying Corollary 3(iii), Proposition 4 and Theorem 1, we get the following corollaries.

5. **Corollary.** *Let Ω be a compact Hausdorff space. Then f is an extreme point of $B(C(\Omega, L^{\varphi}(\mu)))$ if and only if $f(\Omega) \cap \operatorname{ext} B(L^{\varphi}(\mu))$ is dense in $f(\Omega)$ with the topology inherited from $L^{\varphi}(\mu)$.*

6. **Corollary.** *Let us assume that Ω is a compact Hausdorff space and that the set $\operatorname{ext} B(L^{\varphi}(\mu))$ is closed. Then*

$$f \in \operatorname{ext} B(C(\Omega, L^{\varphi}(\mu))) \quad \textit{iff} \quad f(\omega) \in \operatorname{ext} B(L^{\varphi}(\mu)) \quad \textit{for every } \omega \in \Omega.$$

Therefore, it is of interest to characterize those Orlicz spaces in which the set $\operatorname{ext} B(L^{\varphi}(\mu))$ is closed. By the Suarez-Granero result [13], this is satisfied provided $\varphi \in \Delta_2$. However, the Δ_2 condition is far from being necessary.

7. **Example.** Let μ be a counting measure on $\mathbb{N}$. Further, let $a > 0$ and assume that $\varphi(u) = 0$ for every $|u| \leq a$. According to Theorem 2 in [15], the set $\operatorname{ext} B(l^\varphi)$ is closed if and only if φ is linear on some interval $[a, a+\rho]$, $\rho > 0$. Put $\varphi(u) = |u| - a$ for $|u| > a$. Then the equivalence (1) with $X = l^\varphi$ holds true although $\varphi \notin \Delta_2$ (and $B(l^\varphi)$ is not stable).

Now let us assume that $\varphi(u) = (|u| - a)^2$ for $|u| > a$ and that $\Omega = [0, 1]$. Define

$$f_n(\omega) = \begin{cases} a & \text{if } (n-1)\omega^2 \geq 1, \\ a + \min\{(1-(n-1)\omega^2)^{1/2}, \omega\} & \text{otherwise}. \end{cases}$$

Then f_n are continuous functions and $f_n(0) = a$. Further, for every ω we can find $n_\omega \geq 1$ such that $f_n(\omega) = a$ for $n \geq n_\omega$, $f_n(\omega) = a + \omega$ for $n \leq n_\omega - 2$, and $a < f_{n_\omega - 1}(\omega) \leq a + \omega$.

Let $f\colon [0, 1] \to l^\varphi$ be defined by $f(\omega) = (f_n(\omega))_{n=1}^\infty$. We claim that f is continuous. Fix $\omega \in (0, 1]$. Then the set $\{n_{\omega'} : |\omega - \omega'| \leq \omega/2\}$ is bounded. Thus, for every $\lambda > 0$ and every sequence $\omega_m \to \omega$, $I_\varphi(\lambda[f(\omega_m) - f(\omega)]) \to 0$ as $m \to \infty$, i.e., $\|f(\omega_m) - f(\omega)\|_\varphi \to 0$. The continuity of f at 0 follows from the following inequalities:

$$\begin{aligned} \|f(\omega) - (a)_{n=1}^\infty\|_\varphi &\leq \|(a+\omega)_{n=1}^\infty - (a)_{n=1}^\infty\|_\varphi = \|(\omega)_{n=1}^\infty\|_\varphi \\ &= \inf\left\{\lambda > 0 : \sum_{n=1}^\infty \varphi(\omega/\lambda) \leq 1\right\} \\ &= \inf\{\lambda > 0 : \varphi(\omega/\lambda) = 0\} = \omega/a. \end{aligned}$$

It is easy to check that $I_\varphi(f(\omega)) = 1 = \|f(\omega)\|_\varphi$ for every $\omega \in (0, 1]$. Further, for every $\omega \in (0, 1]$ and $n \in \mathbb{N}$, $(f_n(\omega), \varphi(f_n(\omega)))$ are points of strict convexity of $\operatorname{Graph}\varphi$; so $f(\omega)$ are extreme points of the ball $B(l^\varphi)$ [16]. Thus $f \in \operatorname{ext} B(C([0, 1], l^\varphi))$ although $f(0)$ is not an extreme point of $B(l^\varphi)$.

Let $\alpha, \beta \in [0, 1]$, $\alpha < \beta$, and define

$$g_{\alpha,\beta}(\omega) = \begin{cases} f(\omega - \alpha) & \text{if } \alpha < \omega \leq (\alpha+\beta)/2, \\ f(\beta - \omega) & \text{if } (\alpha+\beta)/2 \leq \omega < \beta, \\ 0 & \text{otherwise}. \end{cases}$$

Let us consider the Cantor ternary set C, and let (α_n, β_n) be the sequence of open intervals subtracted from $[0, 1]$ during the process of constructing C. Define

$$h(\omega) = \sum_{n=1}^\infty g_{\alpha_n, \beta_n}(\omega) + (a)_{n=1}^\infty \cdot \chi_C(\omega).$$

One can easily prove the continuity of h at every point $\omega \notin C$. Let $\omega \in C$ and take a sequence (ω_m) with $\omega_m \to \omega$ as $m \to \infty$. Without loss of generality we can assume that $\omega_m \notin C$ for every m, i.e., $\omega_m \in (\alpha_{n_m}, \beta_{n_m})$ for $m = 1, 2, \ldots$. Since $\omega \in C$,

$$\lim_{m\to\infty} \min\{\omega_m - \alpha_{n_m}, \beta_{n_m} - \omega_m\} = 0.$$

Thus

$$\|h(\omega_m) - h(\omega)\|_\varphi = \|g_{\alpha_{n_m}, \beta_{n_m}}(\omega_m) - (a)_{n=1}^\infty\|_\varphi \\ \leq a^{-1} \cdot \min\{\omega_m - \alpha_{n_m}, \beta_{n_m} - \omega_m\} \xrightarrow[m\to\infty]{} 0,$$

i.e., h is a continuous function. Evidently, $h(\omega) \in \operatorname{ext} B(l^\varphi)$ for every $\omega \in C$ and $h(\omega) \notin \operatorname{ext} B(l^\varphi)$ for $\omega \in C$. Therefore h is an example of extreme point of $B(C([0, 1], l^\varphi))$ that takes uncountably many values outside the set $\operatorname{ext} B(l^\varphi)$.

D. Werner [14] has pointed out that the set of extreme points of $B(C(\Omega, L^1(\mu)))$ is empty provided μ is an atomless measure. Armed with Theorem 1 and results of [5, 16], we have:

8. **Corollary.** *The following conditions are equivalent*:

(i) *the set* $\operatorname{ext} B(C(\Omega, L^\varphi(\mu)))$ *is empty*;

(ii) *the set* $\operatorname{ext} B(L^\varphi(\mu))$ *is empty*; *and*

(iii) μ *is an atomless measure,* $c(\varphi) = \infty$ *and* φ *is affine on* $[a, \infty)$, *where* $a = \sup\{u : \varphi(u) = 0\}$.

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