A FULL DESCRIPTION OF EXTREME POINTS IN $C(\Omega, L^p(\mu))$

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Abstract. Let $L^p(\mu)$ be an Orlicz space endowed with the Luxemburg norm. The main result of this paper reads as follows: $f$ is an extreme point of the unit ball of the space of continuous functions from a compact Hausdorff space $\Omega$ into $L^p(\mu)$ with the supremum norm if and only if the inverse image of the set of all extreme points of the unit ball of $L^p(\mu)$ under $f$ is dense in $\Omega$.

1. INTRODUCTION

Let $\Omega$ be a compact Hausdorff space, and let $X$ be a real Banach space. By $C(\Omega, X)$ we denote the Banach space of $X$-valued continuous functions on $\Omega$ equipped with the sup-norm $\|f\| = \sup_{\omega \in \Omega} \|f(\omega)\|$. By $B(X)$ we denote the set of all extremal points of the unit ball $B(X) = \{x : \|x\| \leq 1\}$ of $X$.

One of the most elegant characterizations of the set $\text{ext}B(C(\Omega, X))$ is the following

(1) $f \in \text{ext}B(C(\Omega, X)) \Leftrightarrow f(\omega) \in \text{ext}B(X)$ for every $\omega \in \Omega$

(the implication "$\Leftarrow"$ is trivial). It is evident that

$f \in \text{ext}B(C(\Omega, X)) \Rightarrow \|f(\omega)\| = 1$ for every $\omega \in \Omega$.

Therefore (1) is satisfied for every strictly convex Banach space $X$. Further, (1) remains reasonable only in case $\text{ext}B(X)$ is closed. An attempt in that direction has been done by Clausing and Papadopoulou [2, 11]. Although their papers have been devoted to the study of stability of convex sets, that property applied to $B(X)$ implies the closedness of $\text{ext}B(X)$ and, together with the Michael selection theorem [8], the equivalence (1). Since every Banach space $X$ that is strictly convex or satisfies 3.2 intersection property has stable unit ball, (1) appears as a trivial consequence of the above-mentioned selection theorem (original proofs are in [6, 8]).

D. Werner [14] proved that the equivalence (1) holds provided $X = L^1(\mu)$. Following that result, R. Grzaślewicz established (1) in the case $X = L^p(\mu)$.
Let \( L^\varphi(\mu) \) being an Orlicz space endowed with the Luxemburg norm, under assumptions that \( \varphi \) takes only finite values and satisfies the condition \( \Delta_2 \) depending on the measure \( \mu \) (in short: \( \varphi \in \Delta_2 \)). A. Suarez-Granero has observed that under the Grzaślewicz assumptions the unit ball of \( L^\varphi(\mu) \) is stable, so the before mentioned result can be deduced via the Michael selection theorem. Thus, the Grzaślewicz result has nothing to say about Orlicz spaces without stable unit ball.

However, “stability” is a wider notion than “Orlicz spaces with \( \varphi \in \Delta_2 \).” In fact, \( l^\infty \) can serve as an example of an Orlicz space with stable unit ball (it has 3.2 intersection property), which does not satisfy the Grzaślewicz assumptions \( (l^\infty \equiv l^\varphi, \text{ where } \varphi(u) = 0 \text{ for } |u| \leq 1 \text{ and } \varphi(u) = +\infty \text{ otherwise}) \). In [15, Theorem 4] there has been established that an Orlicz sequence space has stable unit ball if and only if either \( \varphi \in \Delta_2 \) or \( l^\varphi \) is isometric to \( l^\infty \).

The only way that allows us to drop the stability arguments is that of getting rid of the \( \Delta_2 \)-condition. Since the closedness of \( \text{ext } B(L^\varphi(\mu)) \) is no longer guaranteed, the more reasonable conjecture (and more general version) of (1) arises on replacing “for all \( \omega \in \Omega \)” by “on a dense subset of \( \Omega \).”

However, it should be pointed out that the equivalence (1) (even in the general version) does not hold in general. Extreme points of \( B(C(\Omega, X)) \) can have nothing to do with the set \( \text{ext } B(X) \). Blumenthal, Lindenstrauss, and Phelps [1] have presented an example of a four-dimensional space \( X \) and a function \( f \in \text{ext } C([0, 1], X) \) such that \( f(\omega) \notin \text{ext } B(X) \) for all \( \omega \in [0, 1] \). The aim of this paper is to give a full characterization of extreme points of \( B(C(\Omega, L^\varphi(\mu))) \) without \( \Delta_2 \)-arguments. The main result reads as follows.

1. **Theorem.** Let \( \Omega \) be a compact Hausdorff space. A function \( f \) is an extreme point of \( B(C(\Omega, L^\varphi(\mu))) \) if and only if \( f^{-1}(\text{ext } B(L^\varphi(\mu))) \) is a dense subset of \( \Omega \).

The sufficiency part of the proof is elementary. The necessity part requires some auxiliary results; it follows from Corollary 3(i) and Proposition 4 below. Besides, to avoid misunderstandings, we should also fix some notations, which is the aim of the next section.

### 2. Auxiliary definitions and results

Let \((T, \Sigma, \mu)\) be a \( \sigma \)-finite measure space and let \( \varphi : \mathbb{R} \to [0, \infty] \) be a convex, even function with \( \varphi(0) = 0 \). Although the condition \( \Delta_2 \) is not the key of importance in this paper, let us recall that \( \varphi \in \Delta_2 \) iff there is a constant \( M > 0 \) such that \( \varphi(2u) \leq M \varphi(u) \) for every \( u \) from some appropriate set \( D(\mu) \subseteq \mathbb{R} \) depending on the measure \( \mu \) (e.g. \( D(\mu) \) is a neighborhood of \( \infty \) if \( \mu(T) < \infty \), \( \mu \) atomless; \( D(\mu) = \mathbb{R} \) provided \( \mu(T) = \infty \), \( \mu \) atomless; \( D(\mu) \) is a neighborhood of 0 with \( \varphi(\sup D(\mu)) > 0 \) if \( \mu \) is the counting measure on \( T = \mathbb{N} \) etc.).

By the Orlicz space \( L^\varphi(\mu) \) we mean the set of all measurable functions \( x : T \to \mathbb{R} \) such that \( \int_T \varphi(\lambda x(t)) \, d\mu < \infty \) for some \( \lambda > 0 \) equipped with
the equality "almost everywhere" (a.e.) and the Luxemburg norm
\[ \|x\|_\varphi = \inf \left\{ \lambda > 0 : \int_T \varphi(\lambda^{-1} x(t)) \, d\mu \leq 1 \right\} \]
(cf. [7, 9, 10]). To simplify the notation we write \( I_\varphi(x) \) instead of \( \int_T \varphi(x(t)) \, d\mu \).
Those functions that satisfy \( I_\varphi(\lambda x) < \infty \) for every \( \lambda > 0 \) form a closed subspace of \( L^\varphi(\mu) \), which is denoted by \( E^\varphi(\mu) \). If \( \varphi \) takes only finite values and the measure \( \mu \) produces more than a finite number of atoms (i.e. the dimension of \( L^\varphi(\mu) \) is infinite), then we can characterize the condition \( \Delta_2 \) in the following way:
\( \varphi \in \Delta_2 \) iff \( L^\varphi(\mu) = E^\varphi(\mu) \) iff \( \|x\|_\varphi = 1 \Leftrightarrow I_\varphi(x) = 1 \) for every \( x \in L^\varphi(\mu) \). In the other case of measure, those equivalences hold true iff \( \varphi \) takes finite values.

Using standard methods and the Fatou lemma one can prove the lower semi-continuity of \( I_\varphi : L^\varphi(\mu) \to [0, \infty] \). Further, \( L^\varphi(\mu) \) is a Banach space with the following property:

(2) norm convergence implies convergence in measure \( \mu \) on every set of finite measure.

(If \( \mu \) is a purely atomic measure, then (2) simply says that the norm convergence implies the convergence with respect to each coordinate.)

Every extreme point \( x \) of \( B(L^\varphi(\mu)) \) is of norm one. Define \( c(\varphi) = \sup \{ u : \varphi(u) < \infty \} \). Let us recall that if \( \mu \) is an atomless (resp. purely atomic)
measure, then \( x \in \text{ext} B(L^\varphi(\mu)) \) if and only if

(i) \( I_\varphi(x) = 1 \) or \( |x(t)| = c(\varphi) < \infty \) for a.e. (resp. every) \( t \in T \);
(ii) \( (x(t), \varphi(x(t))) \) are points of strict convexity of \( \text{Graph} \varphi \) for a.e. (resp. every but one) \( t \in T \).

(c.f. [5, 16]). Thus the only extreme point of \( B(L^\varphi(\mu)) \) with \( x \geq 0 \), \( I_\varphi(x) < 1 \) can be the function \( e \) defined by \( e(t) = c(\varphi) \) for \( t \in T \).

2. Proposition. If \( \Omega \) is a compact Hausdorff space and \( f \) is an extreme point of \( B(C(\Omega, L^\varphi(\mu))) \), then for every \( \eta > 0 \), the set \( \{ \omega \in \Omega \setminus E_\eta : I_\varphi(f(\omega)) < 1 \} \)
is of the first Baire category in \( \Omega \setminus E_\eta \), where \( E_\eta = \{ \omega : \|f(\omega)| - e\|_\varphi < \eta \} \) if \( e \in \text{ext} B(L^\varphi(\mu)) \) and \( E_\eta = \emptyset \) otherwise.

Proof. Evidently
\[ \{ \omega \in \Omega \setminus E_\eta : I_\varphi(f(\omega)) < 1 \} = \bigcup_{m=1}^{\infty} A_m, \]
where \( A_m = \{ \omega \in \Omega \setminus E_\eta : I_\varphi(f(\omega)) \leq 1 - 1/m \} \). Since \( f \) is continuous and \( I_\varphi \) is lower semicontinuous, each \( A_m \) is closed.

Suppose that \( \bigcup_m A_m \) is not of the first Baire category in \( \Omega \setminus E_\eta \). Then \( \text{int} A_m \neq \emptyset \) for some \( m \in \mathbb{N} \), so we can find an open set \( W \) with
\[ \sup_{\omega \in W} I_\varphi(f(\omega)) \leq 1 - 1/m. \]
Let us fix an arbitrary element \( \omega_0 \) of the set \( W \). We have \( e \notin \text{ext} B(L^p(\mu)) \) if \( \|e\|_p > 1 \) (indeed, if \( \|e\|_p \leq 1 \) then \( c(\varphi) < \infty \) and \( \varphi(c(\varphi)) < \infty \); so \( (c(\varphi), \varphi(c(\varphi))) \) are points of strict convexity of Graph \( \varphi \), and by virtue of [5, 16], \( e \in \text{ext} B(L^p(\mu)) \)). Thus, regardless whether \( e \) is or is not an extreme point of \( B(L^p(\mu)) \), \( f(\omega_0) \neq e \). Hence, for some fixed \( n > 1 \), the set

\[
B = \{ t \in T : \|f(\omega_0)(t)\| < \min\{2n - 1, (1 - 1/n)c(\varphi)\}\}
\]
is of positive measure.

The rest of the proof is divided into two parts.

(1) \( B \) contains an atom \( \{a\} \). By (2), \( f(\cdot)(a) : \Omega \rightarrow \mathbb{R} \) is a continuous function. Further, \( \varphi(\cdot) \) is uniformly continuous on

\[
\{u : \|u\| < \min\{2n - 1, (1 - 1/n)c(\varphi)\}\},
\]
so there exists \( \delta > 0 \) such that \( \varphi(\|u\| + \delta) \leq \varphi(u) + 1/m\mu(\{a\}) \) for every \( u \) from that set. Therefore we can find a neighborhood \( U \subseteq W \) of \( \omega_0 \) such that

\[
\varphi(f(\omega)(a) + \delta) \leq \varphi(\|f(\omega)(a)\| + \delta) \leq \varphi(f(\omega)(a)) + 1/m\mu(\{a\})
\]
for every \( \omega \in U \).

\( \Omega \setminus U \) and \( \{\omega_0\} \) are compact disjoint subsets of \( \Omega \). Thus, the Urysohn lemma shows that there exists a continuous function \( s : \Omega \rightarrow [0, \delta] \) such that \( s(\omega_0) = \delta \) and \( s(\omega) = 0 \) for every \( \omega \notin U \). Now, let

\[
g(\omega) = f(\omega) + s(\omega)\chi_{\{a\}}, \quad h(\omega) = f(\omega) - s(\omega)\chi_{\{a\}}.
\]

Obviously, \( g, h \) are continuous, \( g \neq h \), and \( \frac{1}{2}(g + h) = f \). In order to prove that \( f \notin \text{ext} B(C(\Omega, L^p(\mu))) \) it suffices to show that \( g, h \in B(C(\Omega, L^p(\mu))) \) i.e., that \( I_\varphi(g(\omega)) \leq 1 \) and \( I_\varphi(h(\omega)) \leq 1 \) for every \( \omega \in \Omega \). These inequalities are evident if \( \omega \notin U \). Further, if \( \omega \in U \),

\[
I_\varphi(g(\omega)) = I_\varphi(f\chi_{\Omega \setminus \{a\}}(\omega)) + \varphi(f(\omega)(a))\mu(\{a\}) \leq I_\varphi(f\chi_{\Omega \setminus \{a\}}(\omega)) + \varphi(f(\omega)(a))\mu(\{a\}) + 1/m = I_\varphi(f(\omega)) + 1/m \leq 1
\]
and, analogously, \( I_\varphi(h(\omega)) \leq 1 \).

(2) No atom of the measure \( \mu \) is a subset of \( B \). Without loss of generality we can assume that \( 0 < \mu(B) < \infty \). Let \( \lambda = 2n/(2n - 1) \) (\( \lambda > 1 \)) and \( 0 < \delta \leq \min\{1, c(\varphi)/2n + 1\} \). Then for every \( t \in B \),

\[
\lambda(\|f(\omega_0)(t)\| + \delta) \leq \left\{ \begin{array}{ll}
2n + 1 & \text{if } c(\varphi) = \infty, \\
(1 - \frac{1}{4n - 1})c(\varphi) & \text{otherwise}.
\end{array} \right.
\]

Hence \( I_\varphi(\lambda(\|f(\omega_0)\| + \delta)\chi_B) < \infty \). Thus, we can find a subset \( C \subseteq B \) such that \( 0 < I_\varphi(\lambda(\|f(\omega_0)\| + \delta)\chi_C) \leq 1/m \). Moreover, by the continuity of \( f \), there is an open neighborhood \( U \subseteq W \) of \( \omega_0 \) such that \( \|f(\omega) - f(\omega_0)\|_p \leq \frac{1}{m} < 1 \). Then

\[
I_\varphi(\frac{1}{\lambda - 1}[f(\omega) - f(\omega_0)]) \leq \frac{1}{m} \quad \text{for } I_\varphi(x) \leq \|x\|_p \quad \text{provided } \|x\|_p \leq 1.
\]
By virtue of the Urysohn lemma there exists a continuous function \( s: \Omega \to [0, \delta] \) such that \( s(\omega_0) = \delta \) and \( s(\omega) = 0 \) for every \( \omega \notin U \). Let \( g(\omega) = f(\omega) + s(\omega) \chi_C \), \( h(\omega) = f(\omega) - s(\omega) \chi_C \). The functions \( g \) and \( h \) are continuous, \( g \neq h \), and \( \frac{1}{2}(g + h) = f \). Obviously \( I_\varphi(g(\omega)) \leq 1 \) and \( I_\varphi(h(\omega)) \leq 1 \) for every \( \omega \notin U \). Moreover, for every \( \omega \in U \),

\[
I_\varphi(g(\omega)) = I_\varphi(f(\omega) \chi_C) + I_\varphi(\frac{1}{2} \lambda |f(\omega) + s(\omega)| \chi_C) + (1 - \frac{1}{\lambda})(\frac{1}{\lambda - 1} |f(\omega) - f(\omega_0)| \chi_C)
\]

\[
\leq I_\varphi(f(\omega)) + \frac{1}{\lambda} \cdot I_\varphi(\lambda |f(\omega)| + \delta) \chi_C) + (1 - \frac{1}{\lambda}) \cdot I_\varphi(\frac{1}{\lambda - 1} |f(\omega) - f(\omega_0)|)
\]

\[
\leq 1 - \frac{1}{m} + \frac{1}{\lambda} \cdot \frac{1}{m} + (1 - \frac{1}{\lambda}) \frac{1}{m} = 1.
\]

Thus, \( f \) is not an extreme point of \( B(C(\Omega, L^\varphi(\mu))) \). The obtained contradiction completes the proof.

Applying the Baire category theorem [3, Theorem 3.9.3, p. 253], we obtain the following corollary.

3. Corollary. Let us assume that \( \Omega \) is a compact Hausdorff space and that \( f \in \text{ext} B(C(\Omega, L^\varphi(\mu))) \). Then the following assertions hold:

(i) \( \{ \omega : I_\varphi(f(\omega)) = 1 \text{ or } |f(\omega)| = e \} \) is dense in \( \Omega \);

(ii) for every \( \eta > 0 \) \( f(\Omega) \cap \{ x \in L^\varphi(\mu) : I_\varphi(x) < 1 \text{ and } ||| x || e || \varphi \geq \eta \} \) is of the first Baire category in \( f(\Omega) \) with the topology inherited from \( L^\varphi(\mu) \); and

(iii) \( f(\Omega) \cap \{ x : I_\varphi(x) = 1 \text{ or } |x| = e \} \) is dense in \( f(\Omega) \).

Let us turn back to the Grzaślewicz theorem [4]. The assumption \( \varphi \in \Delta_2 \) made there was necessary to conclude that \( I_\varphi(x) = 1 \) on every \( x \) with norm one. Further, the assumption that \( \varphi \) takes only finite values is superfluous there. Therefore, those assumptions can be omitted provided that the statement is reformulated as follows.

4. Proposition [4]. If \( \Omega \) is a compact Hausdorff space, \( f \) is an extreme point of \( B(C(\Omega, L^\varphi(\mu))) \), and \( I_\varphi(f(\omega)) = 1 \), then \( f(\omega) \in \text{ext} B(L^\varphi(\mu)) \).

Note that the equality \( |f(\omega)| = e \) for some \( \omega \in \Omega \) is possibly only if \( ||e|| = 1 \), i.e., \( e \in \text{ext} B(L^\varphi(\mu)) \). Thus the necessity part of the proof of Theorem 1 is an immediate consequence of Corollary 3(i) and Proposition 4. Applying Corollary 3(iii), Proposition 4 and Theorem 1, we get the following corollaries.

5. Corollary. Let \( \Omega \) be a compact Hausdorff space. Then \( f \) is an extreme point of \( B(C(\Omega, L^\varphi(\mu))) \) if and only if \( f(\Omega) \cap \text{ext} B(L^\varphi(\mu)) \) is dense in \( f(\Omega) \) with the topology inherited from \( L^\varphi(\mu) \).

6. Corollary. Let us assume that \( \Omega \) is a compact Hausdorff space and that the set \( \text{ext} B(L^\varphi(\mu)) \) is closed. Then

\( f \in \text{ext} B(C(\Omega, L^\varphi(\mu))) \iff f(\omega) \in \text{ext} B(L^\varphi(\mu)) \) for every \( \omega \in \Omega \).

Therefore, it is of interest to characterize those Orlicz spaces in which the set \( \text{ext} B(L^\varphi(\mu)) \) is closed. By the Suarez-Granero result [13], this is satisfied provided \( \varphi \in \Delta_2 \). However, the \( \Delta_2 \) condition is far from being necessary.
7. Example. Let \( \mu \) be a counting measure on \( \mathbb{N} \). Further, let \( a > 0 \) and assume that \( \varphi(u) = 0 \) for every \( |u| \leq a \). According to Theorem 2 in [15], the set \( \text{ext } B(l^\varphi) \) is closed if and only if \( \varphi \) is linear on some interval \( [a, a + \rho] \), \( \rho > 0 \). Put \( \varphi(u) = |u| - a \) for \( |u| > a \). Then the equivalence (1) with \( X = l^\varphi \) holds true although \( \varphi \notin \Delta_2 \) (and \( B(l^\varphi) \) is not stable).

Now let us assume that \( \varphi(u) = (|u| - a)^2 \) for \( |u| > a \) and that \( \Omega = [0, 1] \). Define
\[
\varphi_n(\omega) = \begin{cases} 
\frac{a}{(n-1)\omega^2} & \text{if } (n-1)\omega^2 \geq 1, \\
\frac{a + \min\{(1-(n-1)\omega^2)^{1/2}, \omega\}}{2} & \text{otherwise.}
\end{cases}
\]
Then \( \varphi_n \) are continuous functions and \( \varphi_n(0) = a \). Further, for every \( \omega \) we can find \( n_\omega \geq 1 \) such that \( \varphi_n(\omega) = a \) for \( n \geq n_\omega \), \( \varphi_n(\omega) = a + \omega \) for \( n \leq n_\omega - 2 \), and \( a < \varphi_{n_\omega - 1}(\omega) \leq a + \omega \).

Let \( f: [0, 1] \to l^\varphi \) be defined by \( f(\omega) = (\varphi_n(\omega))_{n=1}^\infty \). We claim that \( f \) is continuous. Fix \( \omega \in (0, 1] \). Then the set \( \{n_\omega : |\omega - \omega'| \leq \omega/2\} \) is bounded. Thus, for every \( \lambda > 0 \) and every sequence \( \omega_m \to \omega \), \( I_\varphi(\lambda[f(\omega_m) - f(\omega)]) \to 0 \) as \( m \to \infty \), i.e., \( \|f(\omega_m) - f(\omega)\|_\varphi \to 0 \). The continuity of \( f \) at 0 follows from the following inequalities:
\[
\|f(\omega) - (a)_{n=1}^\infty\|_\varphi \leq \|(a + \omega)_{n=1}^\infty - (a)_{n=1}^\infty\|_\varphi = \|(\omega)_{n=1}^\infty\|_\varphi = \inf\{\lambda > 0 : \sum_{n=1}^\infty \varphi(\omega/\lambda) \leq 1 \} = \inf\{\lambda > 0 : \varphi(\omega/\lambda) = 0 \} = \omega/a. \]

It is easy to check that \( I_\varphi(f(\omega)) = 1 = \|f(\omega)\|_\varphi \) for every \( \omega \in (0, 1] \). Further, for every \( \omega \in (0, 1] \) and \( n \in \mathbb{N} \), \( (\varphi_n(\omega), \varphi(\varphi_n(\omega))) \) are points of strict convexity of \( \text{Graph } \varphi \); so \( f(\omega) \) are extreme points of the ball \( B(l^\varphi) \) [16]. Thus \( f \in \text{ext } B(C([0, 1], l^\varphi)) \) although \( f(0) \) is not an extreme point of \( B(l^\varphi) \).

Let \( \alpha, \beta \in [0, 1] \), \( \alpha < \beta \), and define
\[
g_{\alpha, \beta}(\omega) = \begin{cases} 
f(\omega - \alpha) & \text{if } \alpha < \omega \leq (\alpha + \beta)/2, \\
f(\beta - \omega) & \text{if } (\alpha + \beta)/2 \leq \omega < \beta, \\
0 & \text{otherwise.}
\end{cases}
\]

Let us consider the Cantor ternary set \( C \), and let \( (\alpha_n, \beta_n) \) be the sequence of open intervals subtracted from \( [0, 1] \) during the process of constructing \( C \). Define
\[
h(\omega) = \sum_{n=1}^\infty g_{\alpha_n, \beta_n}(\omega) + (a)_{n=1}^\infty \cdot \chi_C(\omega). \]
One can easily prove the continuity of \( h \) at every point \( \omega \notin C \). Let \( \omega \in C \) and take a sequence \( (\omega_m) \) with \( \omega_m \to \omega \) as \( m \to \infty \). Without loss of generality we can assume that \( \omega_m \notin C \) for every \( m \), i.e., \( \omega_m \in (\alpha_n, \beta_n) \) for \( m = 1, 2, \ldots \). Since \( \omega \in C \),
\[
\lim_{m \to \infty} \min\{\omega_m - \alpha_n, \beta_n - \omega_m\} = 0.
\]
Thus
\[
\|h(\omega_m) - h(\omega)\|_\varphi = \|g_{\alpha_m, \beta_m}(\omega_m) - (a)_{m=1}^\infty\|_\varphi
\leq a^{-1} \cdot \min\{\omega_m - \alpha_m, \beta_m - \omega_m\} \rightarrow 0,
\]
i.e., \( h \) is a continuous function. Evidently, \( h(\omega) \in \text{ext} B(l^\varphi) \) for every \( \omega \in C \) and \( h(\omega) \notin \text{ext} B(l^\varphi) \) for \( \omega \in C \). Therefore \( h \) is an example of extreme point of \( B(C([0, 1], l^\varphi)) \) that takes uncountably many values outside the set \( \text{ext} B(l^\varphi) \).

D. Werner [14] has pointed out that the set of extreme points of \( B(C(\Omega, L^1(\mu))) \) is empty provided \( \mu \) is an atomless measure. Armed with Theorem 1 and results of [5, 16], we have:

8. **Corollary.** The following conditions are equivalent:
   (i) the set \( \text{ext} C(\Omega, L^\varphi(\mu)) \) is empty;
   (ii) the set \( \text{ext} B(l^\varphi(\mu)) \) is empty; and
   (iii) \( \mu \) is an atomless measure, \( c(\varphi) = \infty \) and \( \varphi \) is affine on \([a, \infty)\), where \( a = \sup\{u : \varphi(u) = 0\} \).

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**References**


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