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PROPER KNOTS IN OPEN 3-MANIFOLDS HAVE LOCALLY UNKNOTTED REPRESENTATIVES

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ABSTRACT. Churchard and Spring [1] conjectured that all proper knots in open 3-manifolds are equivalent to (properly isotopic to) a locally unknotted proper knot. This paper proves the conjecture.

1. INTRODUCTION

The purpose of this paper is to prove the conjecture made in §3 of Churchard and Spring [1]; namely, that any proper knot in an arbitrary open 3-manifold is equivalent to (properly isotopic to) a locally unknotted proper knot. However, the suggested construction in [1] does not always work as the following example shows.

Consider the proper knot K defined as $^{*} \times (-1, 1)$ in $\mathbf{S}^2 \times (-1, 1)$. Note that this proper knot is not locally unknotted for the following reason: by the "lamp cord" trick, K is ambiently isotopic to a fiber with a trefoil knot tied in it, which is not locally unknotted.

In order to avoid such "global" knotting, we alter a proper knot K within a tubular neighborhood of K into the Fox proper knot [2] via a smooth proper isotopy (Figure 1, p. 564). We then show that this new proper knot, denoted by K_f, is locally unknotted.

2. DEFINITIONS AND MAIN THEOREM

Unless otherwise stated, all maps and manifolds are assumed to be piecewise linear (or smooth) and all intersections are assumed to be in general position. Let $\mathbf{M}^3$ denote an arbitrary open 3-manifold.

Definitions. A map $f: \mathbf{X} \to \mathbf{M}^3$ is said to be *proper* if for all compact subsets C, $\mathbf{M}^3 \supseteq C$, $f^{-1}(C)$ is compact in $\mathbf{X}$. A *proper knot* f is a proper embedding of $\mathbf{R}^1$ into $\mathbf{M}^3$. Two proper knots f_0 and f_1 are said to be *equivalent* if there is a proper isotopy $F: \mathbf{R}^1 \times [0, 1] \to \mathbf{M}^3$ such that $F_0 = f_0$ and $F_1 = f_1$. Note that this isotopy need not be an ambient isotopy, but it is either smooth or

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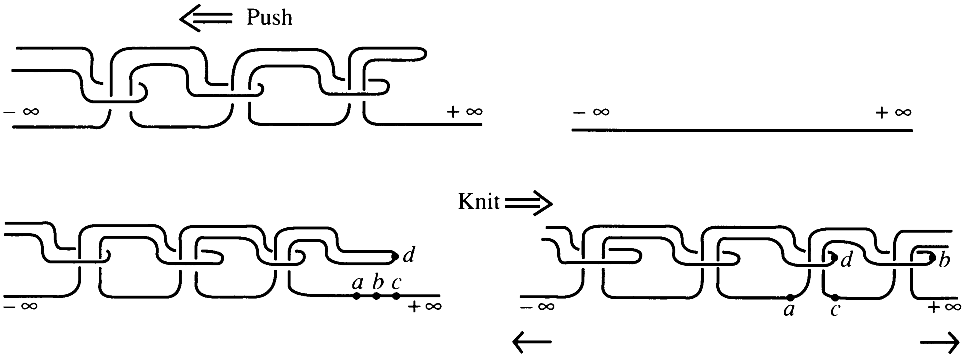

FIGURE 1

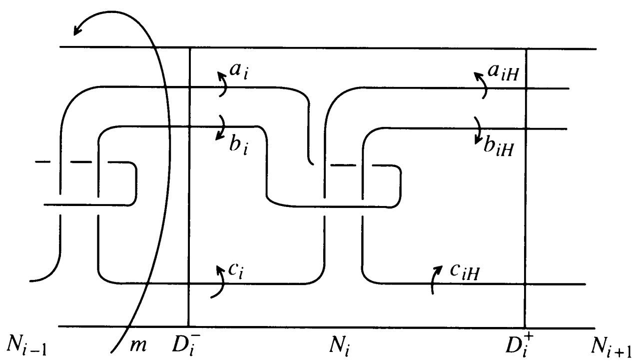

FIGURE 2

locally flat in the p.l. category (that is, the associated level preserving embedding $F \times \mathrm{Id}\colon \mathbf{R}^1 \times [0, 1] \to \mathbf{M}^3 \times [0, 1]$ is locally flat). By abuse of termiology, I will refer to the image $f(\mathbf{R}^1)$ of a proper knot f, denoted by K, as a proper knot. A proper knot K is *locally unknotted* if for every embedded 3-ball B^3 with ∂B^3 intersecting K in general position in exactly two points, $B^3 \cap K$ is unknotted in B^3. Note: this is equivalent to the existence of a disk D with $\partial D = L \cup P$ (where P is a simple arc on ∂B^3, and $L = B^3 \cap K$) and $\operatorname{int}(D) \cap B^3 = \varnothing$. If such a disk D exists for an arc L which intersects a 2-sphere S in exactly two points, we say that L *compresses* into S. Call a proper knot K *strongly locally unknotted* if such a D exists for all 2-spheres S (not just those 2-spheres that bound 3-balls) that intersect K in general position in exactly two points. Hence K is locally unknotted if it is strongly locally unknotted.

Let K be an arbitrary proper knot in $\mathbf{M}^3$. Let N be a tubular neighborhood of K in $\mathbf{M}^3$. In [1] it was shown that K is equivalent to the Fox proper knot K_f in N (Figure 2).

Theorem. *For any proper knot K in an open 3-manifold $\mathbf{M}^3$, K_f is strongly locally unknotted.*

One regards N as a union of $D^2 \times [0, 1]$ "chunks" N_i, $-\infty < i < +\infty$ (Figure 1). In ∂N_i let A_i denote the annulus $N_i \cap \partial N$ and let D_i^+, D_i^- denote the end disks. The intersections of K_f with the end disks D_i^+ are labeled a_{i+1}, b_{i+1}, and c_{i+1} respectively. Note that D_{i-1}^+ is identified with D_i^-. Let m denote the meridian of N, i.e., $[m]$ generates $\pi_1(\partial N)$. Let $i\colon \partial N \to (\mathbf{M}^3 - \mathrm{int}(N))$ be the standard inclusion and $i*\colon \pi_1(\partial N) \to \pi_1(\mathbf{M}^3 - \mathrm{int}(N))$ the induced map on fundamental groups. Note that by the Seifert–Van Kampen theorem:

$$\pi_1(\mathbf{M}^3 - K_f) \cong \pi_1(\mathbf{M}^2 - N) * \pi_1(N - K_f)/[m] = i * [m].$$

Let H the normal closure of the relation $[m] = 1$. Let G denote the group $\pi_1(N - K_f)/H$. Note that $\pi_1(N - K_f)/H$ injects into $\pi_1(\mathbf{M}^3 - K_f)$.

Proposition 1. *G is nontrivial.*

Proof. First, we calculate $\pi_1(N_i - K_f)$ by the Wirtinger method. We get the following relations:

$$a_{i+1}a_ia_{i+1}^{-1} = b_{i+1}b_ib_{i+1}^{-1}$$
$$b_i^{-1} = c_ib_i^{-1}a_{i+1}^{-1} = c_{i+1}b_i^{-1}b_{i+1}^{-1}.$$

Thus we eliminate c_i, c_{i+1}, and a_i. We get no other relations. Hence we have that $\pi_1(N_i - K_f)$ is isomorphic to a free group on three generators and is generated by $\langle a_{i+1}, b_i, b_{i+1}\rangle$. Now, by setting $[m] = 1$, we get

$$(1) \qquad [m] = 1 = b_i^{-1}b_{i+1}b_ib_{i+1}^{-1}a_{i+1}$$

which eliminates a_{i+1}. We now have that $\pi_1(N_i - K_f)/H$ is isomorphic to a free group on two generators; namely, $\langle b_{i+1}, b_i\rangle$. Note that this can be seen geometrically. Also note that $\pi_1(D_i^- - K_f)/H$ and $\pi_1(D_i^+ - K_f)/H$ are free groups on two generators generated by $\langle a_i, b_i\rangle$ and $\langle b_{i+1}, c_{i+1}\rangle$ respectively.

Lemma 2. *We have injections on fundamental groups*:

$$\pi_1(D_i^- - K_f)/H \to \pi_1(N_i - K_f)/H \quad \textit{and} \quad \pi_1(D_i^+ - K_f)/H \to \pi_1(N_1 - K_f)/H$$

induced by the natural inclusions.

Proof. Suppose these induced homomorphisms have a nontrivial kernel. Since the image (after replacing a_{i-1} by $b_{i+1}b_i^{-1}b_{i+1}^{-1}$ from (1)) of $\pi_1(D_i^- - K_f)/H$ is $\langle b_i, b_i^{-1}b_{i+1}b_1b_{i+1}b_i\rangle$ (respectively the image of $\pi_1(D_i^- - K_f)/H$ is $\langle b_i, b_i^{-1}b_{i+1}b_i\rangle$) is a subgroup of a need only check that it is not infinite cyclic.

Were it infinite cyclic, then the images of the generators of $\pi_1(D_i^- - K_f)/H$ would be equal in $\pi_1(N_i - K_f)/H$ (respectively the generators of $\pi_1(D_i^+ - K_f)/H$ would be equal in $\pi_1(N_i - K_f)/H$) since they would be conjugate elements in an abelian group. However this would induce relations in the free group $\pi_1(N_i - K_f)/H$, which is impossible.

Now, to prove the proposition, we put in the relations that join the N_i, we get that G is isomorphic to an infinite free product of free groups of rank two, amalgamated over a free group of rank two, i.e.

$$G \cong \cdots (Z * Z) *_{Z*Z} (Z * Z) *_{Z*Z} (Z * Z) \cdots .$$

Therefore G is nontrivial. (It can also be shown that there is a nontrivial representation of G onto a subgroup of A_5.)

Corollary 3. *There is no map $g: \mathbf{S}^2 \to \mathbf{M}^3$ such that $g(\mathbf{S}^2)$ meets K_f in general position in exactly one point (i.e., no singular 2-sphere hits K_f in exactly one point).*

Proof. First note that $\pi_1(\mathbf{M}^3 - K_f)/H$ has a nontrivial quotient; namely, $\pi_1(N - K_f)/H$. Were there such a sphere, then for any generator b_i, one could constuct a singular disk that is bounded by b_i by using a tubular neighborhood of K_f. This would imply that the above quotient is trivial, contradicting Proposition 1.

Corollary 4. *If a disk C is embedded in N in general position with respect to K_f and N such that ∂D is nontrivial in ∂N then C hit K_f in at least three points.*

Proof. Because $H_1(N - K_f) \cong Z$, C must meet K_f in an odd number of points. However, C cannot hit K_f only once, as ∂C corresponds to $[m]$, which is trivial in G whereas G is nontrivial.

We are now ready to start the proof of our theorem. Our strategy is as follows: we let S be any two sphere in our arbitrary open 3-manifold $\mathbf{M}^3$ that hits K_f in general position in exactly two points, say x_0 and x_1. We successively modify S in order to remove as many components of intersection as possible of S with ∂N into a new two sphere S' in such a way that if K_f had a subarc (with endpoints on S') which compressed into S', then the same subarc compressed into S. We then show that $S' \cup N$ with its regular neighborhood in $\mathbf{M}^3$, denoted by $R(S' \cup N)$, embeds into $\mathbf{S}^2 \times (-1, 1)$ via some p.l. homeomorphism Φ. Then we show that K_f is strongly locally unknotted in $\mathbf{S}^2 \times (-1, 1)$. Then, because $\mathbf{S}^2 \times (-1, 1)$ is actually (i.e., can be identified with) the union of $\Phi(R(S' \cup N))$ and a finite number of 3-balls (the end two being half open) that are attached to $\Phi(R(S' \cup N))$ via a neighborhood (on ∂N) of meridional "attaching 1-spheres" and boundary disk components of $\Phi(R(S' - N))$, the "compressing disk" can be ambiently isotoped into $\Phi(R(S' \cup N))$. It then can be mapped homeomorphically via Φ^{-1} into $\mathbf{M}^3$ where it still meets the necessary requirements (i.e. its boundary is still a union of subarcs on S' and K_f with endpoints x_0 and x_1). Note: henceforth, all maps and intersections are assumed to be in general position.

Lemma 5. (The disk swapping lemma). *Let K be an arc, S a two sphere, K and S p.l. (or smoothly) embedded in an open 3-manifold M, $K \cap S = \{x_0, x_1\}$ a two-point set. Let J be any disk in S that misses x_0 and x_1.*

Suppose J is replaced by another disk J', $M \supseteq J'$, $J' \cap K = \varnothing$, such that $\partial J'$, $(J' - \partial J') \cap (S - J) = \varnothing$ to yield a modified sphere S. If K compresses into the modified sphere S', then it compresses into S.

Proof. With no loss of generality we may assume that the compressing disk Δ is bounded by $K \cup \alpha$ where α is an arc that misses J' (we can isotope $\Delta \cap S' = \alpha$ to any other arc in S' with endpoints x_0 and x_1). Now assume that $\text{int}(J)$ intersects Δ. If it does not, we are done. The components of intersection are simple closed curves on Δ and J. Choose one that is innermost on J, say γ. γ is also a simple closed cure on Δ. We now modify Δ by slightly enlarging γ and then replacing the disk bounded by the enlarged γ by a disk missing J, S', and K. (This can be thought of as a "parallel" copy of the replaced disk.) This reduces the number of components of $J \cap \Delta$ by at least one, and by compactness there must have been only a finite number of components to begin with. Continue this process until all components of intersection are removed. The lemma is proved.

Let S be any two sphere that intersects K_f in exactly two points. Consider $S \cap \partial N$. This is a finite collection of simple closed curves on both S and ∂N. We call a simple closed curve γ *trivial* if it is homotopically trivial on ∂N. We call γ *separating* if it separates x_0 and x_1 on S.

Proposition 6. *S can be modified via disk swaps so that there are no separating simple closed curves in $S \cap \partial N$.*

Proof. We will prove this in four steps. Note: only Step 2 involves a modification of S; the other steps assert the nonexistence of various types of simple closed curves.

Step 1. There can be no separating, trivial simple closed curves. Suppose one existed, say α. α then bounds a disk on ∂N and a disk on S, which we may assume contains x_0 (since it is a separating curve). These two disks can be joined along α to form a sphere that hits K_f in exactly one point, x_0. This contradicts Corollary 3.

Step 2. Any trivial simple closed curve can be removed by a disk swap. Let α be a trivial simple closed curve that is innermost on ∂N. It bounds a disk F' on ∂N. By Step 1, α must bound a disk on S that does not contain either x_0 or x_1. Call this disk F. We can then swap an appropriately "pushed out" copy of F' for F, since neither disk meets K_f. Note that α need not have been innermost on S. Proceeding inductively, one removes the finite number of trivial simple closed curves on ∂N. Therefore, all simple closed curves of intersection are meridional on ∂N.

Step 3. There can be no innermost (on S), nontrivial sseparating simple closed curves of intersection.

Suppose α is such a curve. α then bounds a disk F on S that contains exactly one point of K_f, say x_0. Note that F must lie entirely in N as α is innermost. Then F is a disk such that ∂F is on $\partial N\,(\partial F = \alpha)$ that hits K_f

in only one point. This contradicts Corollary 4.

Step 4. There can be no nontrivial, separating simple closed curves of intersection.

Suppose α is such a curve. α then bounds a disk F_0 on S containing exactly one point of K_f, say x_0. α also bounds a disk F_1 on S containing x_1 and no other point of K_f. By Step 3, α is not innermost on S, therefore, there is an innermost simple closed curve of intersection, on say F_1, say β, which necessarily (by Step 3 it cannot be separating) bounds a disk F_1', on F_1 which contains no point of K_f. By Step 2, β must be meridional on ∂N. Hence, β and α bound an annulus on ∂N, called A'. Therefore, we may obtain a 2-sphere, possibly singular, by joining F_0 along α to the annulus (a suitable pushed copy) A' which is attached to the disk F_1', along β. The existence of this 2-sphere (possibly singular) contradicts Corollary 3 since it meets K_f in exactly one point, x_0. This completes Step 4 and the proof of the proposition.

Proposition 7. *S can be modified so that all simple closed curves of intersection are innermost on S.*

Proof. There are only a finite number of simple closed curves of intersection and all are meridional on ∂N. In addition, all must bound disks on S which do not meet K_f by the previous proposition. By finiteness, there must be on ∂N an adjacent pair of simple closed curves of intersection, one of which is innermost on S call it α, and one that is not innermost on S, say β (provided there are any noninnermost ones left). Call the annulus between α and β on $\partial N A'$. Now there are disks on S bounded by α (resp. β) F_α (resp. F_β) which are disjoint from K_f. Then we may perform a disk swap which replaces F_β by a slightly pushed out copy of A' which is joined to an appropriately modified (pushed out) copy of F_α. This ensures that β becomes an innermost curve of intersection. Note: all the modifications can take place in an appropriately small regular neighborhood of $S \cup N$. Also notice that we did not alter the intersection of S with K_f in any way. This proves the proposition.

Proposition 8. *S can be modified so that $S \cup N$ and its regular neighborhood in $\mathbf{M}^3$ can be embedded into $\mathbf{S}^2 \times (-1, 1)$.*

Proof. There are two cases. The first case has S not intersecting ∂N at all. In this case S is contained in N and N can be embedded into $\mathbf{S}^2 \times (-1, 1)$ as a regular neighborhood of a fiber $^* \times (-1, 1)$ where $^* \in \mathbf{S}^2$. Now suppose that the intersection of S and ∂N is not empty. From the previous propositions, the intersection consists of a finite collection of simple closed curves, say there are k of them, α_i, $1 \leq i \leq k$, each of which is meridional on ∂N. Furthrmore, each of the α_i are innermost on S and bound a disk F_i on S which does not intersect K_f. Hence, the interiors of each of the F_i lie outside of N. Order the α_i according to their least $\mathbf{R}^1$ coordinate with the first one labeled α_1. We denote the annulus on ∂N between α_i and α_{i+1} by A_i. Now go to α_2. If necessary, we can replace F_2 by a slightly modified A_1 joined to a slightly modified F_1. We can repeat this process for all i until all of the new disks bounded by the α_i are parallel. Now a regular neighborhood of this modified $S \cup N$, denoted by $R(S \cup N)$, is homeomorphic to a copy of $\mathbf{D}^2 \times \mathbf{R}^1$ with a finite number of 2-handles attached along meridional attaching 1-spheres, the α_i. (The cores of the handles are the F_i.) Then, letting the F_i correspond to the $\mathbf{S}^2$ factor, we can use any homeomorphism (p.l. or smooth) between $\mathbf{S}^2 \times \mathbf{R}^1$ embedding of $R(S \cup N)$ into $\mathbf{S}^2 \times (-1, 1)$ by Φ. Note that in $\mathbf{S}^2 \times (-1, 1)$, we can assume that

$$\Phi(R(S \cup N)) = (\mathbf{D} \times (-1, 1)) \bigcup_{i=1}^{k} (\mathbf{D_C} \times (t_i - \varepsilon, t_i + \varepsilon)),$$

(where $\mathbf{D}$ is a disk in the $\mathbf{S}^2$ factor which corresponds to $\Phi(D_i^+$, $\mathbf{D_C}$ is its complementary disk in the $\mathbf{S}^2$ factor and t_i corresponds to the $(-1, 1)$ factor associated with $\Phi(\alpha_i)$, and $\mathbf{D_C} \times t_i$ corresponds to the $\Phi(F_i)$.) In addition, $(\mathbf{S}^2 \times (-1, 1)) - \Phi(R(S \cup N)$ is now a union of $k + 1$ 3-balls, h_i, and h_1 and h_{k+1} being half open. The boundaries of the h_i are the following:

for $i = 1$, $\quad \partial h_i = (\partial \mathbf{D} \times [-1, (t_1 - \varepsilon)]) \cup (\mathbf{D_C} \times (t_1 - \varepsilon))$,

for $i = 2$ to k,

$$\partial h_i = (\partial \mathbf{D} \times [(t_{i-1} + \varepsilon), (t_i - \varepsilon)]) \cup \{(\mathbf{D_C} \times (t_{i-1} + \varepsilon)) \cup (\mathbf{D_C} \times (t_i - \varepsilon))\},$$

for $i = k + 1$, $\quad \partial h_i = (\partial \mathbf{D} \times [(t_k + \varepsilon), 1]) \cup (\mathbf{D_C} \times (t_k + \varepsilon))$.

Henceforth, when working in $\mathbf{S}^2 \times (-1, 1)$, I will suppress "$\Phi$" when referring to the image of S and N.

Proposition 9. *Let K be any proper knot running between the opposite ends of $\mathbf{S}^2 \times (-1, 1)$. Let S' be any 2-sphere that either misses K or intersects K in an even number of points. Then S' bounds a ball in $\mathbf{S}^2 \times (-1, 1)$.*

Proof. S' separates in $\mathbf{S}^2 \times (-1, 1)$. If S' separated the ends of $\mathbf{S}^2 \times (-1, 1)$ from one another, S' would hit K in an odd number of points. Hence S' bounds in $\mathbf{S}^2 \times (-1, 1)$ and therefore s' bounds a ball.

Proposition 10. *K_f is strongly locally unknotted in $\mathbf{S}^2 \times (-1, 1)$.*

Proof. First note that the boundaries of the end disks of the chunks N_i, ∂D_i (I will work with D_i^+ and suppress the "$+D$") bound disks P_i in the complement of N in $\mathbf{S}^2 \times (-1, 1)$. We can then form spheres $S_i = P_i \bigcup_{\partial D_i} D_i$. We can then form modified chunks N_i' whre N_i' is that part of $\mathbf{S}^2 \times (-1, 1)$ that is bounded by the "boundary spheres" S_{i-1} and S_i. (i.e. N_i' is obtained by gluing a "$P_i \times [0, 1]$" to N_i along ∂N_i in a standard way). It follows from Lemma 2 that $\pi_1(S_i - K_f)$ injects into both $\pi_1(N_i' - K_f)$ and $\pi_1(N_{i+1}' - K_f)$.

We start the proof of our proposition by showing that we can ambiently isotope S into a single N_i' in such a manner that K_f remains setwise fixed. Note that by compactness, $(\bigcup_i^\infty S_i) \cap S$, is a finite collection of simple closed curves on the S_i and on S. Choose one that is innermost on S, say β. β bounds an innermost disk F on S and two disks on a S_i. We show that all such curves can be removed. This process must terminate after a finite number of steps. We start off with those F that either contain either no or one point of K_f. Note that after these innermost simple closed curves of intersection are removed, there will be no F that contains two points of K_f.

Case 1. *F does not contain a point of K_F.* Now β bounds two disks on S_i. Neither disk can contain exactly one point of K_F because if one disk did, say C, then $F \bigcup_\beta C$ would form a 2-sphere that hits K_f exactly once; the existence of such a sphere contradicts Corollary 3. Thus β bounds a disk C on S_i which contains no points of K_f. By Proposition 9, the sphere formed by $F \bigcup_\beta C$ bounds a ball which can be used to guide an ambient isotopy which removes β and fixes K_f.

Case 2. *F contains one point of K_f* Consider β on S_i. β cannot bound a disk C on S_i which misses K_f else $F \bigcup_\beta C$ would form a 2-sphere hitting K_f in one point, contradicting Corollary 3. Hence β bounds a disk C on S_i that contains one point of K_f. Thus $F \bigcup_\beta C$ hits K_f twice and thus bounds a ball B in N_i' (Or N_{i+1}'). Employing the following lemma one easily shows that B can be used to guide an ambient isotopy which fixes K_F setwise and removes β.

Lemma 11. *Let B be any 3-ball contained in a single N_i' whose boundary intersect K_f twice. Then $B \cap K_f$ is unknotted.*

Proof. Because $[m] = 1$ in $\mathbf{S}^2 \times (-1, 1)$, as well as in N_i',

$$\pi_1(N_i' - K_f) \cong \pi_1(N_i - K_f)/H \cong Z^* Z.$$

I will denote this group by W. W contains no subgroup that is isomorphic to a nontrivial knot group; W is a free group and thus all of its subgroups are free. But the only free knot group is the trivial knot group. By the Seifert–Van Kampen theorem:

$$W \cong \pi_1(B - K_f)^* \pi_1((N_i' - B) - K_f)/i * [n] = [n],$$

where n is the meridian of $(B \cap K_f)$ and $i*$ is induced by the inclusion of $(\partial B - K_f)$ into N_i'. Because $i * [n]$ is either b_i or b_{i+1}, neither which is trivial in W, the knot group of $(B \cap K_f)$ is a nontrivial subgroup of W. Because it is both a knot group and a free group, it must be the trivial knot group. Lemma 11 is proved.

We have now shown that all innermost (on S) simple closed curves of intersection of S with the S_i can be removed. Hence, we may assume that S lies entirely in one modified chunk N_i'. Becuase S hits K_f twice, we can now apply Lemma 11 to show that, if B is the ball in $\mathbf{S}^2 \times (-1, 1)$ bounded by S (which evidently lies in N_i'), $B \cap K_f$ is unknotted. Since S was only changed by ambient isotopies, we can assume that K_f compresses into the original (unmodified) embedded S. Proposition 10 is proved.

We are ready to prove our main theorem. The following proposition will show that a "compressing" disk bounded by a subarc of K_f and a subarc of S can be ambiently isotoped into $\Phi(R(S \cup N))$. The disk can then be lifted back to $\mathbf{M}^3$ where it meets the necessary requirements. Let Δ be the disk that K_f uses to compress into S.

Proposition 12. Δ *can be isotoped into* $\Phi(R(S \cup N))$.

Proof. Recall the construction of $\mathbf{S}^2 \times (-1, 1)$ as given at the end of Proposition 8. Consider the intersection of the interior of Δ with the ∂h_i ($\partial\Delta$ is disjoint from the h_i). The components of intersection are simple closed curves on ∂h_i and on Δ. Choose one that is innermost on a ∂h_i, say γ. γ bounds a disk on ∂h_i and on Δ. One then can replace the disk on Δ by a suitably pushed out copy of the disk on ∂h_i. Δ is still a disk and its boundary has not been affected. This process terminates after a finite number of steps. Thus, Δ is modified to miss the h_i and therefore lies in $\Phi(R(S \cup N))$. The proposition is proved.

Acknowledgments

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References

1. P. Churchard and D. Spring, *Proper knot theory in open 3-manifolds*, Trans. Amer. Math. Soc. **308** (1988), 133–142.
2. R. H. Fox, *A remarkable simple closed curve*, Ann. of Math. **50** (1949), 264–265.
3. R. H. Fox and E. Artin, *Some wild cells and spheres in three dimensional space*, Ann. of Math. **49** (1948), 979–990.
4. D. Rolfsen, *Knots and links*, Publish or Perish, 1976.

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