UNKNOTTING NUMBER ONE KNOTS ARE PRIME: A NEW PROOF

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Abstract. An alternative proof for unknotting number one knots being prime is given.

The unknotting number of a knot \( K \subset S^3 \), denoted by \( u(K) \), is the minimum number of crossing changes required to unknot \( K \). Obviously \( u(K) \) is a knot invariant, but surprisingly little is known about it. The following theorem of M. Scharlemann proves a long standing conjecture.

Theorem [S, Theorem]. A knot \( K \subset S^3 \) with \( u(K) = 1 \) is prime.

To prove the above theorem, Scharlemann developed certain combinatorics dealing with planar graphs coming from an intersection of two special planar surfaces. Later in [ST], Scharlemann and Thompson gave another proof of the theorem [ST, Corollary 3.4], that is based on a delicate application of the sutured manifold structure theory. In this note we point out a new proof, applying only some existing results. In fact the proof follows immediately from the following three known lemmas.

Lemma 1 [L, Lemma 1]. Let \( K \) be a knot in \( S^3 \) with \( u(K) = 1 \), and let \( M_K \) be the double cover of \( S^3 \) branched over \( K \). Then \( M_K \) can be obtained by \( n/2 \)-surgery on some knot in \( S^3 \), \( n \) being an odd integer.

Lemma 2 [GL, Theorem 1]. Let \( K \) be a knot in \( S^3 \), and let \( K(m/l) \) denote the manifold obtained by \( m/l \)-surgery on \( K \). Then \( K(m/l) \) is a prime manifold if \( |l| \neq 1 \).

Lemma 3 [KT, Corollary 4]. Let \( K \) be a knot in \( S^3 \). Then the double cover \( M_K \) of \( S^3 \) branched over \( K \) is a prime manifold iff \( K \) is a prime knot.

Proof of Theorem. Since \( u(K) = 1 \), \( M_K \) is a prime manifold by Lemmas 1 and 2. Hence \( K \) is a prime knot by Lemma 3.

The author has learned that a similar approach was pointed out by C. Gordon in a lecture given at Santa Barbara.

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