PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 114, Number 2, February 1992

ON THE POROSITY OF THE SET OF ω-NONEXPANSIVE MAPPINGS WITHOUT FIXED POINTS

J. MYJAK AND R. SAMPALMIERI

(Communicated by Palle E. T. Jorgensen)

ABSTRACT. Let C be a nonempty closed convex bounded subset of a Banach space E. Let $\mathscr{M}$ denote the family of all multivalued mappings from C into E which are nonempty weakly compact convex valued, ω-nonexpansive and weakly-weakly u.s.c., endowed with the metric of uniform convergence. Let $\mathscr{M}_0$ be the set of all $F \in \mathscr{M}$ for which the fixed point problem is well posed. It is proved that the set $\mathscr{M}\backslash\mathscr{M}_0$ is σ-porous (in particular meager). A similar result is given for weak properness.

1. INTRODUCTION AND NOTIONS

Let E be a Banach space with norm $\| \ \|$. Let τ_ω denote the weak topology of E (i.e. $\tau_\omega = \sigma(E, E^*)$). For $A \subset E$, $\overline{\text{co}}(A)$, $\text{diam}(A)$, $\overline{A}$, and $\overline{A}^{\tau_\omega}$ stands for the closed convex hull of A, the diameter of A, the closure of A in the norm topology, and the closure of A in the weak topology, respectively. Let S denote the unit open ball in E. Moreover, for $x \in E$, set $d(x, A) = \inf\{\|a - x\| | a \in A\}$.

Define

$$\mathfrak{C}_\omega(E) = \{A \subset E | A \text{ is nonempty, convex, and weakly compact}\}.$$

For $A \subset E$ the measure $\omega(A)$ of the noncompactness of A in the weak topology is defined (see [2]) by

$$\omega(A) = \inf\{t > 0 | \text{ there exists a weakly compact subset } K \text{ of } E \text{ such that } A \subset K + tS\}.$$

Let C be a nonempty closed convex bounded subset of E (with $\text{diam}(C) > 0$). In this note we study the fixed point problem for multifunctions with weakly compact convex values.

Recall that a multifunction $F: C \to \mathfrak{C}_\omega(E)$ is said to be:

ω-strict contraction if there is $\gamma \in [0, 1)$ such that $\omega(F(A)) \le \gamma\omega(A)$ for every $A \subset C$:

ω-condensing if $\omega(F(A)) < \omega(A)$ for every $A \subset C$ with $\omega(A) \neq 0$;

Received by the editors January 21, 1990 and, in revised form, July 18, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 47H09; Secondary 47H10.
Key words and phrases. Multifunctions, fixed point, well-posedness, weak properness, porosity.

ω-nonexpansive if $\omega(F(A)) \leq \omega(A)$ for every $A \subset C$;

weakly-weakly u.s.c. (weakly-weakly upper semicontinuous) if the set $F^{-}(D) = \{x \in C | F(x) \cap D \neq \varnothing\}$ is closed in (C, τ_ω) for every D closed in (E, τ_ω):

weakly proper if the set $\overline{F^{-}(K)}^{\tau_\omega}$ is weakly compact for every weakly compact subset K of E.

Note that if $F = f$ is a single-valued function, the weak-weak u.s.c. of F is equivalent to the weak-weak continuity of f (i.e. f is continuous as a function from (C, τ_ω) into (E, τ_ω)).

Define

$$\mathfrak{M} = \{F: C \to \mathfrak{C}_\omega(C) | F \text{ is } \omega\text{-nonexpansive and weakly-weakly u.s.c.}\}.$$

Suppose that $\mathfrak{M}$ is endowed with the metric of uniform convergence ρ, given by $\rho(F, G) = \sup\{h(F(x), G(x)) | x \in C\}$, $F, G \in \mathfrak{M}$, h denotes the Hausdorff metric generated by the norm.

For $F \in \mathfrak{M}$ we consider the fixed point problem

$$x \in F(x). \tag{1.1}$$

Let $\mathscr{S}_F$ be the set of all solutions of (1.1), i.e. $\mathscr{S}_F = \{x \in C | x \in F(x)\}$.

Definition 1.1. The problem (1.1) is said to be *weakly well posed* if (i) the set $\mathscr{S}_F$ is nonempty and weakly compact and (ii) every sequence $\{x_n\} \subset C$ such that $\lim_{n \to +\infty} d(x_n, F(x_n)) = 0$ is weakly compact.

Let X be a complete metric space. For $x \in X$ and $r > 0$, $S_X(x, r)$ stands for the open ball in X with center in x and radius r.

A subset Y of X is said to be *porous* (in X) if there is $0 < \alpha \leq 1$ and $r_0 > 0$ such that for every $x \in X$ and every $0 < r \leq r_0$ there is $y \in X$ such that $S_X(y, \alpha r) \subset S_X(x, r) \cap (X \setminus Y)$. A set Y is said to be *σ-porous* (in X) if it is a countable union of porous (in X) sets.

Note that every σ-porous set is of Baire first category. The converse in general is not true. There are sets of Baire first category that are not σ-porous; on the other hand, it follows from the Lebesgue density theorem, that every σ-porous subset of $\mathbb{R}^n$ is of Lebesgue measure zero.

A property is said to be P-generic (resp. generic) in X if the set of all $x \in X$ for which this property fails is σ-porous (resp. of Baire first category). Clearly every P-generic property is also generic. The converse, as follows from above remarks, is in general not true.

This note consists of five sections (with the introduction). In §2 we recall the properties of ω-measure of noncompactness in the weak topology and some properties concerning ω-strict contractions. In §3 we prove that well posedness of the problem (1.1) is a P-generic property in the space $(\mathfrak{M}, \rho)$. It follows, in particular, that nonemptyness and weak compactness of the set $\mathscr{S}_F$ is a P-generic property in $(\mathfrak{M}, \rho)$. This generalizes the result of [8], where it is proved that such property is generic in $(\mathfrak{M}, \rho)$. In §4 we prove that the weak properness of the operator $I - F$, where I is the identity and $F \in \mathfrak{M}$, is a P-generic property in $(\mathfrak{M}, \rho)$. Analogous results hold for single-valued mappings. Our proofs are quite elementary, in particular, considerably more simple than those of [1, 8]. In the last section some P-generic results, corresponding to generic results from [1, 3, 6], are reviewed. For further related generic results see [1–6, 8, 9].

2. Preliminaries

The measure ω of the noncompactness in the weak topology has the following properties.

Proposition 2.1 [2]. *Let A and B be subsets of E. Then*

(i) $\omega(A) = 0$ *if and only if* $\overline{A}^{\tau_\omega}$ *is weakly compact*;

(ii) $\omega(\overline{A}^{\tau_\omega}) = \omega(A)$;

(iii) $A \subset B$ *implies* $\omega(A) \le \omega(B)$;

(iv) $\omega(A \cup B) = \max\{\omega(A), \omega(B)\}$;

(v) $\omega(A + B) \le \omega(A) + \omega(B)$;

(vi) $\omega(tA) = t\omega(A)$, $t \ge 0$;

(vii) $\omega(\overline{\mathrm{co}}(A)) = \omega(A)$;

(viii) $\omega(S) = 0$ *provided S reflexive*; $\omega(S) = 1$ *provided* $\dim E = +\infty$ *and E nonreflexive.*

(ix) *If $\{A_n\}_{n\ge1}$ is a decreasing sequence of nonempty weakly closed subsets of E such that* $\lim_{n\to+\infty} \omega(A_n) = 0$, *then* $\bigcap_{n=1}^\infty A_n$ *is nonempty and weakly compact.*

Define

$$\mathfrak{N} = \{F \in \mathfrak{M} | F \text{ is } \omega\text{-strict contraction}\}.$$

Lemma 2.1. *For every $F \in \mathfrak{N}$ the set $\mathscr{S}_F$ is nonempty and weakly compact.*

Proof. Put $D_1 = \overline{\mathrm{co}}\, F(C)$, $D_n = \overline{\mathrm{co}}\, F(D_{n-1})$, $n = 2, 3, \ldots$. Clearly $D_1 \supset D_2 \supset \cdots$ and $\omega(D_n) \to 0$ as $n \to +\infty$. By Proposition 2.1(ix) the set $D = \bigcap_{n=1}^\infty D_n$ is nonempty and weakly compact. Since D is convex and $F(D) \subset D$, by Schauder–Tychonov Theorem, there is $x \in D$ such that $x \in F(x)$. Thus $\mathscr{S}_F \neq \varnothing$. Since $\mathscr{S}_F$ is weakly closed and contained in D, it is weakly compact.

Lemma 2.2. $\mathfrak{N}$ *is dense in* $(\mathfrak{M}, \rho)$.

Proof. Let $F \in \mathfrak{M}$ and let $\varepsilon > 0$. Put $r = \max\{\varepsilon, \|C\|\}$, where $\|C\| = \sup\{\|x\| \,|x \in C\}$. Take $x_0 \in C$ and consider the map $\widetilde{F}: C \to \Sigma_\omega(C)$ given by

$$\widetilde{F}(x) = \frac{\varepsilon}{2r} x_0 + \left(1 - \frac{\varepsilon}{2r}\right) F(x).$$

For $x \in C$ we have

$$h(\widetilde{F}(x), F(x)) = h\left(\frac{\varepsilon}{2r}x_0 + \left(1 - \frac{\varepsilon}{2r}\right) F(x), F(x)\right) \le h\left(\frac{\varepsilon}{2r}x_0, 0\right) + h\left(\left(1 - \frac{\varepsilon}{2r}\right) F(x), F(x)\right) \le \frac{\varepsilon}{2r} \cdot r + \frac{\varepsilon}{2r} \cdot r = \varepsilon,$$

whence it follows that $\rho(\widetilde{F}, F) \le \varepsilon$. On the other hand, from Proposition 2.1(v), (vi), it follows that $\widetilde{F}$ is a ω-strict contraction (with $\gamma \le 1 - \varepsilon/2r$). This completes the proof.

3. Weakly well-posed fixed point problems

For $F \in \mathfrak{M}$ and $\sigma > 0$ define

$$D_F(\sigma) = \{x \in C | d(x, F(x)) \le \sigma\}.$$

Lemma 3.1. *The set* $D_F(\sigma)$ *is nonempty weakly closed and* $D_F(\sigma') \subset D_F(\sigma)$ *if* $0 < \sigma' < \sigma$.

Proof. Let $F \in \mathfrak{M}$ and $\sigma > 0$. From Lemmas 2.1 and 2.2 it follows immediately that $D_F(\sigma) \neq \varnothing$. Clearly $D_F(\sigma') \subset D_F(\sigma)$ if $0 < \sigma' < \sigma$. It remains to prove that $D_F(\sigma)$ is weakly closed.

Let a net $\{x_\lambda\} \subset D_F(\sigma)$ converge weakly to x_0. Let V be an open (in τ_ω) neighborhood of the origin, and let U be an open (in τ_ω) balanced neighborhood of the origin such that $2U \subset V$. Since F is weakly-weakly u.s.c. and $x_\lambda \to x_0$, there is λ_0 such that $F(x_\lambda) \subset F(x_0) + U$ and $x_0 \in x_\lambda + U$ for every $\lambda \geq \lambda_0$. Consequently $x_0 \in x_\lambda + U \subset F(x_\lambda) + \sigma S + U \subset F(x_0) + \sigma S + 2U \subset F(x_0) + \sigma S + V$. Since $F(x_0) + \sigma \overline{S}$ is weakly closed and V is an arbitrary neighborhood of the origin, it follows that $x_0 \in F(x_0) + \sigma\overline{S}$, which means that $x_0 \in D_F(\sigma)$. Thus $D_F(\sigma)$ is weakly closed.

Proposition 3.1. *Let* $F \in \mathfrak{M}$. *If* $\lim_{\sigma\to 0} \omega(D_F(\sigma)) = 0$, *then the problem* (1.1) *is weakly well posed.*

Proof. By Lemma 3.1 and Proposition 2.1(ix), the set $D = \bigcap_{\sigma>0} D_F(\sigma)$ is nonempty and weakly compact. Clearly $\mathscr{S}_F \subset D$. By virtue of the weak-weak u.s.c. of F, the set $\mathscr{S}_F$ is weakly closed. Consequently $\mathscr{S}_F$ is weakly compact.

Let $\{x_n\} \subset C$ be such that $d(x_n, F(x_n)) \to 0$. For given $\sigma > 0$, there is n_σ such that $x_n \in D_F(\sigma)$ for $n \geq n_\sigma$. From this, Proposition 2.1(v), (iii) and the hypotheses of Proposition 3.1, it follows that $\omega(\{x_n\}) = 0$, which means that $\{x_n\}$ is weakly compact. By Lemmas 2.1 and 2.2 there is a sequence $\{x_n\} \subset C$ such that $d(x_n, F(x_n)) \to 0$ as $n \to +\infty$. Since such sequence must be weakly compact, without loss of generality, we can assume that $\{x_n\}$ converges weakly to x_0. Let V be an open (in τ_ω) neighborhood of the origin. Let U be an open (in τ_ω) balanced neighborhood of the origin such that $3U \subset V$. Let n_0 be such that $x_0 \in x_n + U$, $F(x_n) \subset F(x_0) + U$, and $x_n \in F(x_n) + U$ for every $n \geq n_0$. Consequently $x_0 \in F(x_0) + 3U \subset F(x_0) + V$. It follows that $x_0 \in F(x_0)$. Thus $\mathscr{S}_F \neq \varnothing$ and the proof is complete.

Lemma 3.2. *Let* $G \in \mathfrak{N}$, *and let* $\varepsilon > 0$. *Let* $\delta = \frac{1}{3}(1 - \gamma_G)\varepsilon$. *Then for every* $F \in \mathfrak{M}$ *with* $\rho(F, G) < \delta$, *we have*

$$\omega(D_F(\delta)) \leq \varepsilon .$$

Proof. Let $G \in \mathfrak{N}$, and let $\varepsilon > 0$. Let $x \in D_F(\delta)$. We have $x \in F(x) + 2\delta S \subset G(x) + 3\delta S$, and so $D_F(\delta) \subset G(D_F(\delta)) + 3\delta S$. By Proposition 2.1(iii), (v), (vi), we have

$$\omega(D_F(\delta)) \leq \gamma_G \omega(D_F(\delta)) + 3\delta ;$$

whence the result follows.

Theorem 3.1. *Let* $\mathfrak{M}_0$ *be the set of all* $F \in \mathfrak{M}$ *such that the problem* (1.1) *is weakly well posed. Then* $\mathfrak{M}\backslash\mathfrak{M}_0$ *is a* σ*-porous subset of* $(\mathfrak{M}, \rho)$. *In particular* $\mathfrak{M}_0$ *is a residual subset of* $(\mathfrak{M}, \rho)$.

Proof. Let $\{\varepsilon_k\}$ be a decreasing sequence of positive numbers converging to zero. Define

$$\widetilde{\mathfrak{M}} = \bigcap_{k=1}^{\infty} \bigcup_{G\in\mathfrak{N}} S_{\mathfrak{M}}(G, \delta_G(\varepsilon_k)), \tag{3.1}$$

where $\delta_G(\varepsilon_k) = \frac{1}{3}(1 - \gamma_G)\varepsilon_k$.

We claim that $\widetilde{\mathfrak{M}} \subset \mathfrak{M}_0$. Indeed, let $F \in \widetilde{\mathfrak{M}}$. For every $k \in \mathbb{N}$ there is $G_k \in \mathfrak{N}$ such that $F \in S_{\mathfrak{M}}(G_k, \delta_{G_k}(\varepsilon_k))$. By Lemma 3.2 we have $\omega(D_F(\delta_{G_k}(\varepsilon_k))) \leq \varepsilon_k$. It follows that $\lim_{\sigma \to 0} \omega(D_F(\sigma)) = 0$. By Proposition 3.1 the problem (1.1) is weakly well posed. Consequently $\widetilde{\mathfrak{M}} \subset \mathfrak{M}_0$.

We have

$$\mathfrak{M} \backslash \widetilde{\mathfrak{M}} = \bigcup_{k=1}^{\infty} \mathscr{Z}_k, \text{ where } \mathscr{Z}_k = \mathfrak{M} \backslash \bigcup_{G \in \mathfrak{N}} S_{\mathfrak{M}}(G, \delta_G(\varepsilon_k)). \tag{3.2}$$

We prove that for every $k \in \mathbb{N}$, $\mathscr{Z}_k$ is a porous set in $\mathfrak{M}$ with $r_0 = \min\{1, M\}$ and $\alpha = \min\{1/2, \varepsilon_k/6M\}$, where $M = \operatorname{diam}(C)$. Indeed, let $F \in \mathscr{Z}_k$ and $0 < r \leq r_0$. Take $x_0 \in C$ and define $G: C \to \mathfrak{C}_\omega(C)$ by

$$G(x) = \frac{r}{2M} \cdot x_0 + \left(1 - \frac{r}{2M}\right) F(x).$$

Clearly $G \in \mathfrak{N}$ $(\gamma_G \leq 1 - r/2M)$ and $\rho(F, G) \leq r/2$. Observe that for every $H \in S_{\mathfrak{M}}(G, \alpha r)$ we have

$$\rho(H, F) \leq \rho(H, G) + \rho(G, F) < \alpha r + r/2 \leq r$$

and so $S_{\mathfrak{M}}(G, \alpha r) \subset S_{\mathfrak{M}}(F, r)$. On the other hand, $S_{\mathfrak{M}}(G, \alpha r) \subset S_{\mathfrak{M}}(G, \delta_G(\varepsilon_k))$ because

$$\alpha r \leq \frac{\varepsilon_k}{6} \frac{r}{M} \leq \frac{1}{3}(1 - \gamma_G)\varepsilon_k = \delta_G(\varepsilon_k).$$

Consequently $S_{\mathfrak{M}}(G, \alpha r) \subset S_{\mathfrak{M}}(F, r) \cap (\mathfrak{M} \backslash \mathscr{Z}_k)$, which proves that $\mathscr{Z}_k$ is porous in $(\mathfrak{M}, \rho)$. Since $\mathfrak{M} \backslash \mathfrak{M}_0 \subset \mathfrak{M} \backslash \widetilde{\mathfrak{M}}$, the proof is complete.

Corollary 3.1. *Let $\mathfrak{M}^0$ be the set of all $F \in \mathfrak{M}$ such that the set $\mathscr{S}_F$ is nonempty and weakly compact. Then $\mathfrak{M} \backslash \mathfrak{M}^0$ is σ-porous in $(\mathfrak{M}, \rho)$.*

Corollary 3.2 [8]. *The set $\mathfrak{M}^0$ of all $F \in \mathfrak{M}$, which possesses a fixed point, is residual in $(\mathfrak{M}, \rho)$.*

This result has been proved in [8] by using a rather complicated argument essentially due to Butler [1]. Our argument can be also used to prove Butler's result [1].

Let $\mathscr{M}$ be the set of all ω-nonexpansive single-valued continuous mappings from (C, τ_ω) into (C, τ_ω), endowed with the metric of uniform convergence (in norm topology).

Theorem 3.2. *Let $\mathscr{M}_0$ be the set of all $f \in \mathscr{M}$ such that* (i) *the set $\mathscr{S}_F$ of all fixed points of f is nonempty and weakly compact and,* (ii) *every sequence $\{x_n\} \subset C$ such that $\|fx_n - x_n\| \to 0$ as $n \to +\infty$ is weakly compact. Then $\mathscr{M} \backslash \mathscr{M}_0$ is σ-porous in $\mathscr{M}$. In particular, $\mathscr{M}_0$ is a residual subset of $\mathscr{M}$.*

Proof. As that of Theorem 3.1.

4. Weak properness

Lemma 4.1. *Let $F: C \to \mathfrak{C}_\omega(E)$ be ω-condensing. Then the map $I - F$, where I is the identity mapping, is weakly proper.*

Proof. Suppose for a contradiction that there is a weakly compact subset K of E such that $\overline{K_1}^{\tau_\omega}$, where $K_1 = (I - F)^{-}(K)$, is not weakly compact. It is routine to see that $K_1 \subset F(K_1) + K$. By Proposition 2.1(iv), (i) we have

$$\omega(K_1) \le \omega(F(K_1)) + \omega(K) < \omega(K_1),$$

which furnishes a contradiction. Thus $I - F$ is weakly proper.

Lemma 4.2. *Let $F: C \to \mathfrak{C}_\omega(E)$ be weakly-weakly u.s.c. and weakly proper. Then $F(A)$ is weakly sequentially closed for every weakly closed subset A of C.*

Proof. Let $A \subset C$ be weakly closed. Let $\{y_n\} \subset F(A)$ be such that $y_n \to y_0$. Set $K = \{y_n\} \cup \{y_0\}$. For each $n \in \mathbb{N}$, choose $x_n \in A$ such that $y_n \in F(x_n)$. Since K is weakly closed and weakly compact, hence $F^-(K)$ is weakly closed and weakly compact too. By Eberlein–Šmulian Theorem, $F^-(K)$ is weakly sequentially compact. Hence there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ weakly converging, say to x_0. Clearly $x_0 \in A$ and $y \in F(x_0)$. This completes the proof.

Lemma 4.3. *Let $G \in \mathfrak{N}$, and let $\varepsilon > 0$. Let $\delta = (1 - \gamma_G)\varepsilon$. Then for every $F \in \mathfrak{M}$ with $\rho(F, G) < \delta$, and every weakly compact subset K of E, we have $\omega((I - F)^-(K)) \le \varepsilon$.*

Proof. Let $G \in \mathfrak{N}$ and $\varepsilon > 0$. Let $F \in \mathfrak{M}$ be such that $\rho(F, G) < \delta$, and let $K \subset E$ be weakly compact. Set $A = (I - F)^-(K)$. Clearly $A \subset F(A) + K \subset G(A) + K + \delta S$. By Proposition 1.1 we have $\omega(A) \le \gamma_G \omega(A) + \delta$, whence the result follows.

Theorem 4.1. *Let $\mathfrak{M}^*$ be the set of all $F \in \mathfrak{M}$ such that $I - F$ is weakly proper. Then $\mathfrak{M} \backslash \mathfrak{M}^*$ is a σ-porous subset of $(\mathfrak{M}, \rho)$.*

Proof. Let $\{\varepsilon_k\}$ be a decreasing sequence of positive numbers converging to zero. Define $\widetilde{\mathfrak{M}}$ by (3.1) with $\delta_G(\varepsilon_k) = (1 - \gamma_G)\varepsilon_k$.

Observe that $\widetilde{\mathfrak{M}} \subset \mathfrak{M}^*$. Indeed, let $F \in \widetilde{\mathfrak{M}}$. Let K be a weakly compact subset of E. By definition of $\widetilde{\mathfrak{M}}$, for every $k \in \mathbb{N}$ there is $G_k \in \mathfrak{N}$ such that $\rho(G_k, F) < \delta_G(\varepsilon_k)$. By Lemma 4.3 we have $\omega((I - F)^-(K)) \le \varepsilon_k$. It follows that $\omega((I - F)^-(K)) = 0$. Since K is an arbitrary weakly compact subset of E, this means that $I - F$ is weakly proper.

To prove that $\mathfrak{M} \backslash \mathfrak{M}^*$ is σ-porous, consider (3.2) with $\delta_G(\varepsilon_k) = (1 - \gamma_G)\varepsilon_k$. As in the proof of Theorem 3.1, one can show that for every $k \in \mathbb{N}$ the set $\mathscr{Z}_k$ is porous with $\alpha = \min\{1/2, \varepsilon_k/\varepsilon M\}$ and $r_0 = \min\{1, M\}$ $(M = \mathrm{diam}(C))$. This completes the proof.

Remark 4.1. Using Theorem 4.1 and Lemmas 4.1 and 4.2, one can given an alternative proof of Corollary 3.1.

Theorem 4.2. *Let $\mathscr{M}$ be as in Theorem 3.2. Let $\mathscr{M}^*$ be the set of all $f \in \mathscr{M}$ such that $I - f$ is weakly proper. Then $\mathscr{M} \backslash \mathscr{M}^*$ is a σ-porous set in $\mathscr{M}$.*

Proof. As that of Theorem 4.1.

5. Concluding remarks

Note that the results of this note remain true if "ω-measure of noncompactness in the weak topology," "weak convergence," and "weak compactness" are replaced by "α-measure of noncompactness" (i.e. Kuratowski measure of noncompactness), "convergence in norm," and "compactness," respectively. In such framework the result corresponding to Corollary 3.1 (resp. Corollary 3.2, Theorem 4.2) generalizes the result by F. S. De Blasi [3] (resp. J. Butler, [1], Dominguez Benavides [6]). The results corresponding to Theorem 3.1 are given in [4].

REFERENCES

1. G. J. Butler, *Almost all* 1*-set contractions have a fixed point*, Proc. Amer. Math. Soc. **74** (1979), 353–357.
2. F. S. De Blasi, *On a property of the unit sphere in a Banach space*, Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.) **21** (1977), 259–262.
3. ——, *Some generic properties in fixed point theory*, J. Math. Anal. Appl. **71** (1979), 161–166.
4. F. S. De Blasi and J. Myjak, *Sur la porosite de l'ensemble des contractions sans point fixe*, C. R. Acad. Sci. Paris, Sér. I Math., **308** (1989), 51–52.
5. T. Dominguez Benavides, *Generic existence of a nonempty compact set of fixed points*, J. Math. Anal. Appl. **90** (1982), 421–430.
6. ——, *Some generic properties of α-nonexpansive mappings*, J. Math. Anal. Appl. **105** (1985), 176–187.
7. N. Dunford and J. T. Schwartz, *Linear operators, Part* I, Interscience, New York, 1964.
8. J. Ewert, *Multivalued contractions with respect to the weak measure of noncompactness*, Bull. Acad. Polon. Sci. Math. **34** (1986).
9. J. Myjak, *Orlicz type category theorems for functional and differential equations*, Dissertationes Math. **206** (1983), 1–82.

DIPARTAMENTO MATEMATICAS PURA ED APPLICATA, UNIVERSITÀ DELL'AQUILA, 67100 L'AQUILA, ITALY