LAYERS OF COMPONENTS OF $\beta([0, 1] \times N)$ ARE INDECOMPOSABLE

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ABSTRACT. We examine the structure of certain subcontinua of the Stone-Cech compactification of the reals. Let $N$ denote the integers, let $X = [0, 1] \times N$, and let $C$ be a component of $X^* = \beta X - X$. It is known that $C$ admits an upper semicontinuous decomposition $G$ into maximal nowhere dense subcontinua of $C$ so that $C/G$ is a Hausdorff arc. The elements of $G$ are called layers. It has been shown that the layers of $C$ that contain limit points of a countable increasing or decreasing sequence of cut points of $C$ are nondegenerate indecomposable continua (various forms of this fact have been proven by Bellamy and Rubin, Mioduszewski, and Smith). We show that all the layers of $C$ are indecomposable.*

Let $A = [0, \infty)$. We wish to examine certain subcontinua of $A^* = \beta A - A$ the remainder of the Stone-Cech compactification of $A$. Let $N$ denote the integers. Then $X = [0, 1] \times N$ is a subspace of $A$ and $X^*$ is a subspace of $A^*$. It has been shown that certain layers of components of $X^*$ are indecomposable. (See Mioduszewski [M], Bellamy and Rubin [BR], and Smith [S4].) It is the purpose of this paper to show that every “layer” of components of $X^*$ is indecomposable. This answers a question posed by Mioduszewski in [M]. A layer of a component $C$ of $X^*$ is a maximal nowhere dense (in $C$) subcontinuum of $C$.

First we introduce some notation. Let $Z(X)$ denote the zero sets of $X$ (the closed subsets of $X$). Elements of $\beta X$ are identified with ultrafilters of zero sets of $X$ (closed sets of $X$). Let $I_n$ denote $[0, 1] \times \{n\}$ for each $n \in N$. If $x \in X^*$, let $u_x = \{\{n\}I_n \cap H \neq \emptyset \} | H \in x\}$. It can be verified that if $x \in X^*$ then $u_x \in N^*$ and if $u \in N^*$ then $C(u) = \{x|u_x = u\}$ is a component of $X^*$.

We wish to state some properties of the layers of components of $X^*$ and a useful characterization.

There is a natural order induced by the topology on the layers of $C(u)$. Let $p, q \in C(u)$; then we say that $p <_u q$ if and only if there exist $H \in p, K \in q$, and $D \in u$ so that every element of $H \cap I_n$ precedes every element of $K \cap I_n$ with respect to the usual order on $I_n$ for all $n \in D$ (i.e., $H \cap I_n <_n K \cap I_n \forall n \in D$).

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* The theorem was recently discovered independently by J. Zhu.
The subscript "\( u \)" of the symbol "\(<_u \)" may be omitted if the meaning is clear from the context. Let \( \Omega \) denote the set of all sequences \( S = \{S(n)\}_{n=1}^{\infty} \) so that \( S(n) \in I_n \). If \( u \in N^* \) and \( S \in \Omega \) then define \( A(u, S) = \{H \in Z(X) | \exists D \in u \text{ so that } \{S(n) | n \in D\} \subset H\} \). \( A(u, S) \) is an ultrafilter and hence an element of \( X^* \). In fact \( A(u, S) \in C(u) \). If \( \{i | S(i) = 1\} \in u \) then \( A(u, S) \) is an end point of \( C(u) \). Similarly if \( \{i | S(i) = 0\} \in u \) then \( A(u, S) \) is an end point of \( C(u) \). Except for these two cases, if \( S \in \Omega \) then \( A(u, S) \) is a cut point of \( C(u) \). (The converse cannot be assumed. See Baldwin and Smith [BS].) Note that \( \{A(u, S) | S \in \Omega\} \) is dense in \( C(u) \).

The following observations are not difficult to verify (see Baldwin and Smith [BS], Mioduszewski [M], and Smith [S4]).

Observation 1. If \( p \) and \( q \) are two points of \( C(u) \) and \( p >_u q \) then there is a cut point \( A(u, S) \) with \( S \in \Omega \) so that \( p <_u A(u, S) <_u q \).

Observation 2. If \( p = A(u, S) \) for some \( S \in \Omega \) and \( q \in C(u) \), with \( q \neq p \), then either \( q <_u p \) or \( p <_u q \).

Observation 3. If \( S_1 \) and \( S_2 \) are two elements of \( \Omega \), \( u \in N^* \), and \( A(u, S_1) <_u A(u, S_2) \) then \( \{p \in C(u) | A(u, S_1) <_u p \leq A(u, S_2)\} \) is open in \( C(u) \) and \( \{p \in C(u) | A(u, S_1) \leq p \leq A(u, S_2)\} \) is a subcontinuum of \( C(u) \), and hence is not nowhere dense.

Lemma 1. Every cut point of \( C(u) \) is a layer of \( C(u) \).

Proof. If \( x \in C(u) \) then define \( L_x \) and \( R_x \) as follows:

\[
L_x = \{p \in C(u) | p <_u x\} \quad R_x = \{p \in C(u) | x <_u p\}.
\]

If \( p \) and \( q \) are points of \( C(u) \) and \( p <_u q \) then there is a sequence \( S \in \Omega \) so that \( p <_u A(u, S) <_u q \). Therefore we have

Claim 1.1. If \( x \in C(u) \) then \( L_x \) and \( R_x \) are connected.

Also since \( \{A(u, S) | S \in \Omega\} \) is dense in \( C(u) \) we have

Claim 1.2. \( Cl_{C(u)}(R_x \cup L_x) = C(u) \).

We also need to prove

Claim 1.3. If \( x \in M \) and \( M \) is a nowhere dense subcontinuum of \( C(u) \) containing \( x \) then \( M \cap L_x = \emptyset \) and \( M \cap R_x = \emptyset \).

Proof. Suppose \( x \in C(u) \) and \( M \) is a subcontinuum of \( C(u) \) that contains \( x \) and \( M \cap R_x \neq \emptyset \). Then there exists \( y \in M \cap R_x \) so that \( x <_u y \). By Observation 1 there exist \( S_1 \) and \( S_2 \) in \( \Omega \) so that \( x <_u A(u, S_1) <_u A(u, S_2) <_u y \). Since \( A(u, S_1) \) is a cut point of \( C(u) \) and separates \( x \) from \( y \), it follows that \( A(u, S_1) \in M \). Similarly \( A(u, S_2) \in M \); and in fact, if \( A(u, S_1) <_u A(u, S) <_u A(u, S_2) \) for \( S \in \Omega \) then \( A(u, S) \in M \). Therefore since \( M \) is compact, \( \{z \in C(u) | A(u, S_1) \leq z \leq A(u, S_2)\} \subset M \). So by Observation 3, \( M \) is not nowhere dense. This is a contradiction. Therefore \( M \cap R_x = \emptyset \) and similarly \( M \cap L_x = \emptyset \), which verifies Claim 1.3.

Suppose now that \( x \) is a cut point of \( C(u) \) and \( \{x\} \) is not a layer of \( C(u) \). Then there is a nowhere dense nondegenerate subcontinuum \( M \) of \( C(u) \) containing \( x \). Let \( y \in M - \{x\} \) Since \( x \) is a cut point of \( C(u) \), \( C(u) - \{x\} \) is the union of two mutually separated point sets \( H \) and \( K \). Since
$L_x \cup R_x \subseteq C(u) - \{x\}$, by Claims 1.1 and 1.2 either $L_x \subseteq H$ and $R_x \subseteq K$ or $R_x \subseteq H$ and $L_x \subseteq K$. Without loss of generality assume $R_x \subseteq H$ and $L_x \subseteq K$. Furthermore by Claim 3 $M \cap (L_x \cup R_x) = \emptyset$.

Claim 1.4. $y$ is a limit point of $R_x$.

Proof. Let $O$ be an open set containing $y$. Let $A \in y$, $B \in x$, and $U$ be a basic open set in $\beta X$ so that

(i) $A \cap B = \emptyset$,
(ii) $A \subseteq U$,
(iii) $Cl_{\beta X} U \subseteq 0$,
(iv) $B \cap Cl_{\beta X} U = \emptyset$,
(v) the number of components of $U \cap I_n$ is finite for all $n \in N$ (this can be done since $A \cap I_n$ is compact). Furthermore choose $A$ so that $\{(0, i)\} \notin A$ for all $i$.

Let $N' = \{i \mid U \cap I_i \neq \emptyset\}$. Then $N' \in u$.

Let $v_i$ be the rightmost component of $U \cap I_i$ (with respect to the usual order on $I_i = [0, 1] \times \{i\}$). Then if $i \in N'$ let $p_i \in v_i$ be a point such that $p_i > A \cap I_i$. If $P \in \Omega$ is such that $P(i) = p_i$ then $A(u, P) \in C(u)$, $A(u, P) > y$, and $A(u, P) \in U$.

Note that $p_i | z' \in N' \cap B = \emptyset$ and $\{B \cap I_j | j \in N'\} \in x$.

Consider $B' = \{B \cap ([1, p_j] \times \{j\}) | j \in N'\}$. If $B' \in x$ then $x > u A(u, P)$ so $x > u y$. So $y \in L_x$, which contradicts Claim 1.3. Therefore $B'' = \{B \cap ([p_j, 0] \times \{j\}) | j \in N'\}$ is in $x$. Therefore $A(u, P) > x$. Therefore $O$ contains a point of $R_x$. So $y$ is a limit point of $R_x$. Thus Claim 1.4 is established.

Similarly it can be shown that $y$ is a limit point of $L_x$. So $y$ is a limit point of both $H$ and $K$. But $H$ and $K$ are mutually separated. This is a contradiction so the lemma must be true.

Lemma 2. If $p \in C(u)$ and $L = \{q \in C(u) | \text{neither } p <_u q \text{ nor } q <_u p\}$ then $L$ is a layer of $C(u)$.

Proof. Let $p \in L$ and let $L = \{q \in C(u) | \text{neither } p <_u q \text{ nor } q <_u p\}$. Then $L = \bigcap\{\{x \in C(u) | A(u, s) \leq_u x \leq_u A(u, r)\} | A(u, s) < p < A(u, r)\}$. So $L$ is a continuum. In order to verify that $L$ is nowhere dense we only need to observe that if $L$ is not nowhere dense in $C(u)$ then $L$ would contain at least two cut points $q_1$ and $q_2$ of $C(u)$. But then one of them, $q_i$, is not so either $q_i < p$ or $p < q_i$, which is a contradiction. In order to verify that $L$ is a maximal nowhere dense subcontinuum we only need to observe that if $L'$ is a subcontinuum of $C(u)$ with $L \subseteq L'$ then $L'$ contains a point $q$ so that either $p < q$ or $q < p$. In either case $L'$ contains at least two cut points of $C(u)$ and hence is not nowhere dense in $C(u)$. Therefore $L$ is a layer of $C(u)$.

Lemma 3. If $L$ is a maximal nowhere dense subcontinuum of $C(u)$ and $p \in L$ then $L = \{q | \text{neither } p <_u q \text{ nor } q <_u p\}$.

Proof. If $p$ is a cut point or an end point of $C(u)$ then the lemma follows from the observations above and Lemma 1.

Suppose that $L$ is a nondegenerate maximal nowhere dense subcontinuum of $C(u)$. Let $p \in L$. Then since $L$ is nondegenerate, it follows from the above observations that $L$ contains no cut point of $C(u)$. Therefore if $q \in L$ then
p \neq q$ and $q \neq p$. Therefore $\hat{L} = \{ x | p \neq x \text{ and } x \neq p \}$ contains $L$. But $\hat{L}$ is nowhere dense, so since $L$ is maximal, it follows that $L = \hat{L}$.

Notice that if $G$ is the collection of layers of $C(u)$ then $G$ is upper semi-continuous and $C(u)/G$ is a Hausdorff arc.

Lemmas 1, 2, and 3 (with a slight modification for the end points of $C(u)$) establish that $L$ is a layer in $C(u)$ if and only if for each point $p \in L$,

$$L = \bigcap \{ \{ x \in C(u) | A(u, s) \leq x \leq A(u, r) \} | A(u, s) < p < A(u, r); \ r, s \in \Omega \}.$$ 

We are now prepared to prove the main result.

**Theorem.** Let $u \in N^*$ and let $L$ be a layer of $C(u)$. Then $L$ is indecomposable.

**Proof.** Suppose that $M$ is an arbitrary property subcontinuum of $L$. We will prove that $M$ is nowhere dense in $L$, which implies that $L$ is indecomposable. Let $\hat{U}$ and $\hat{V}$ be disjoint open sets in $\beta X$ so that $\text{Cl}_{\beta X} \hat{U} \cap \text{Cl}_{\beta X} \hat{V} = \emptyset$, $M \cap \hat{V} \neq \emptyset$, $M \cap \text{Cl}_{\beta X} \hat{U} = \emptyset$, and $\hat{U} \cap L \neq \emptyset$. Let $U$ and $V$ be open sets in $\beta X$ so that

$$U \subset \text{Cl}_{\beta X} U \subset \hat{U}, \quad V \subset \text{Cl}_{\beta X} V \subset \hat{V},$$

$$M \cap V \neq \emptyset, \quad L \cap U \neq \emptyset,$$

and for each $n$ the number of components of $I_n \cap U$ and of $I_n \cap V$ is finite.

Let $p \in M \cap V$.

**Claim 1.** If $k$ is an integer, $H \in p$, $H \subset V \cap X$, and $J$ is the set of all integers $n$ such that there is a sequence $U_1^n, U_2^n, \ldots, U_{2k+1}^n$ of components of $U \cap I_n$ and a sequence $V_1^n, V_2^n, \ldots, V_{2k+3}^n$ of components of $V \cap I_n$ that intersect $H$ such that

$$U_1^n < V_1^n < U_2^n < V_2^n < \cdots < U_{2k+1}^n < V_{2k+3}^n$$

and $k \leq i_n$, then $J \in u$.

**Proof.** Let $k$ be a positive integer and suppose the hypothesis of the claim and that $J \notin u$. So there is a sequence \(\{s^i\}_{i=0}^{2k+3}\) of elements of $\Omega$ such that for each $n \in N - J$, $0 = s^0(n) < s^1(n) < s^2(n) < \cdots < s^{2k+1}(n) < s^{2k+2}(n) \leq s^{2k+3}(n) = 1$ and

$$U \cap I_n \subset \{ x \in I_n | s^i(n) \leq x \leq s^{i+1}(n) \text{ and } i \text{ is even} \},$$

$$V \cap I_n \subset \{ x \in I_n | s^i(n) \leq x \leq s^{i+1}(n) \text{ and } i \text{ is odd} \}. $$

Note that if the first component of $V \cap I_n$ precedes the first component of $U \cap I_n$ it may be ignored. Let $H^i = \bigcup_{n \in N - J} \{ x \in I_n | s^i(n) \leq x \leq s^{i+1}(n) \}$. Since $J \notin u$ we have $N - J \in u$ and then $\hat{H} = \{ x \in H \cap I_n | n \in N - J \} \in p$. Then $\hat{H} \subset \bigcup_{i=1, \text{ odd}} H^i$, so $H^1 \cup H^3 \cup \cdots \cup H^{2k+1} \in p$, and hence $H^i \in p$ for some odd integer $i$. But then $A(u, s^i) \leq u p \leq A(u, s^{i+1})$ and $U \cap \{ z \in C(u) | A(u, s^i) \leq u z \leq u A(u, s^{i+1}) \} = \emptyset$. This contradicts the fact that $U \cap L \neq \emptyset$, because $L \subset \{ z \in C(u) | A(u, s^i) \leq u z \leq u A(u, s^{i+1}) \}$. This establishes the claim.

For each $n$ let $B^n_1, B^n_2, \ldots, B^n_{2k_{n}}$ be the components of $I_n - U$ that intersect $V$ listed in order. If $\hat{H} \in p$ let $B(\hat{H}) = \{ B^n_1 | B^n \cap H \neq \emptyset \}$ and let $\mathcal{B} = \{ B(H) | H \in p \}$. Define $\text{Ls}(\mathcal{B}) = \bigcap_{B \in \mathcal{B}} \text{Cl}_{\beta X}(\bigcup B)$.

**Claim 2.** $\text{Ls}(\mathcal{B})$ is a continuum, $p \in \text{Ls}(\mathcal{B})$, and $\text{Ls}(\mathcal{B})$ is a component of $X^* - U$. 

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Various forms of this claim have appeared elsewhere (see Blass [Bl], Smith [S1], and van Douwen [vD]). We will outline a proof for completeness.

**Proof.** Assume the hypothesis of the claim. Then the collection \( \{ \text{Cl}_{\beta X}(\bigcup B(H)) \mid H \in p \} \) is a collection of compact sets in \( \beta X \) so the common part \( \text{Ls}(\mathcal{B}) \) is compact. Since \( p \in \text{Cl}_{\beta X}(\bigcup B(H)) \) for each \( H \in p \) we have \( p \in \text{Ls}(\mathcal{B}) \).

Suppose that \( \text{Ls}(\mathcal{B}) \) is not a continuum but is the union of two mutually exclusive compact sets \( C_1 \) and \( C_2 \). Let \( W_1 \) and \( W_2 \) be open sets in \( \beta X \) containing \( C_1 \) and \( C_2 \) respectively with disjoint closures. Then there exists an element \( B(H) \) in \( \mathcal{B} \) so that \( \text{Cl}_{\beta X}(\bigcup B(H)) \subset W_1 \cup W_2 \). But \( X \cap W_1 \cap (\bigcup B(H)) \) and \( X \cap W_2 \cap (\bigcup B(H)) \) are both compact and \( (W_1 \cap (\bigcup B(H))) \cup (W_2 \cap (\bigcup B(H))) \in p \) so \( W_i \cap (\bigcup B(H)) \in p \) for \( i = 1 \) or \( i = 2 \). (Note that each element of \( B(H) \) is connected and lies in \( W_1 \cup W_2 \).) Assume \( i = 1 \). Let \( \tilde{H} = W_1 \cap (\bigcup B(H)) \). Then \( \tilde{H} \in p \) and \( \text{Cl}_{\beta X}(\bigcup B(\tilde{H})) \subset \text{Cl}_{\beta X}(W_1) \), and hence \( \text{Cl}_{\beta X}(\bigcup B(\tilde{H})) \cap W_2 = \emptyset \) and so \( \text{Ls}(\mathcal{B}) \cap W_2 = \emptyset \), which is a contradiction.

It is easy to verify that \( \text{Ls}(\mathcal{B}) \) is a component of \( \bigcap_{i=1}^{\infty} B_i^* = X^* - U \).

Let \( \widehat{M} \) denote \( \text{Ls}(\mathcal{B}) \). Since \( \widehat{M} \) is a component of \( X^* - U \), \( p \in \widehat{M} \), \( M \subset X^* - U \), and \( p \in M \), it follows that \( M \subset \widehat{M} \).

For each \( B_i^\alpha \) let \( p_i^\alpha \in B_i^\alpha \cap V \), and let \( D = \{ p_i^\alpha \}_{i=1, n=1}^{\infty} \). If \( d \in D \) then let \( B(d) \) denote \( B_i^\alpha \) where \( d = p_i^\alpha \). Let \( D^* = \text{Cl}_{\beta X}(D) - D \).

**Claim 3.** Every component of \( X^* \cap \text{Cl}_{\beta X}(\bigcup \{ B_i^\alpha \mid n \in N, 1 \leq i \leq k_n \}) \) contains exactly one element of \( D^* \).

**Proof.** Let \( R = \{ (n, i) \mid n \in N, 1 \leq i \leq k_n \} \). If \( r \in R \) with \( r = (n, i) \) let \( B_r = B_i^\alpha \) and \( d_r = p_i^\alpha \). If \( \alpha \in R^* \) then let

\[
d\alpha = \bigcap \{ \text{Cl}_{\beta X}(d_r) \mid r \in J \} \mid J \in \alpha \}.
\]

Then \( d\alpha \) is a single point of \( D^* \) and \( d\alpha = d\gamma \) iff \( \alpha = \gamma \). Furthermore \( C \) is a component of \( X^* \cap \text{Cl}_{\beta X}(\bigcup \{ B_i^\alpha \mid n \in N, 1 \leq i \leq k_n \}) \) if and only if there is an \( \alpha \in R^* \) so that \( C = B\alpha \) where

\[
B\alpha = \bigcap \{ \text{Cl}_{\beta X}(Br) \mid r \in J \} \mid J \in \alpha \}.
\]

(See the argument to Claim 2 above and also [S1].) Furthermore \( B\alpha = B\gamma \) if and only if \( \alpha = \gamma \). Therefore \( d\alpha \in By \) if and only if \( \alpha = \gamma \); and this verifies Claim 3.

We have \( D^* \subset V \) by construction. Consider the sets \( \text{Even} = \bigcup \{ B_i^\alpha \mid i \text{ is even} \} \), \( \text{Odd} = \bigcup \{ B_i^\alpha \mid i \text{ is odd} \} \). Then either \( \text{Even} \in p \) or \( \text{Odd} \in p \). Without loss of generality assume \( \text{Even} \in p \). For \( i \) even, \( 1 \leq i \leq k_n \), and \( n \) a positive integer let \( E_i^n = \{ x \in I_n \mid x \in B_{i-1}^\alpha \cup B_i^\alpha \text{ or } B_{i-1}^\alpha < x < B_i^\alpha \} \). Then \( B_i^\alpha \subset E_i^n \). Let \( E(J) = \{ E_i^n \mid E_i^n \cap J \neq \emptyset \} \) and let \( \mathcal{E} = \{ E(J) \mid J \in p \} \). Let \( \text{Ls}(\mathcal{E}) = \bigcap_{E \in \mathcal{E}} \text{Cl}_{\beta X}(\bigcup E) \), which is a continuum containing \( p \) by the same argument used to prove Claim 2. Since \( \text{Even} \in p \) and \( \text{Even} \subset E(\text{Even}) \) we have \( \widehat{M} = \text{Ls}(\mathcal{B}) \subset \text{Ls}(\mathcal{E}) \).

**Claim 4.** \( \text{Ls}(\mathcal{E}) \subset L \).

**Proof.** Suppose that the claim is not true. Since \( p \in \text{Ls}(\mathcal{E}) \) and \( \text{Ls}(\mathcal{E}) \not\subset L \), there exists an element \( s \in \Omega \) so that \( A(u, s) \in \text{Ls}(\mathcal{E}) \) and \( A(u, s) \in V \).
But by Claim 1 $E(\{s_n|n \in N\}) \notin p$. So $p \notin Cl_{\beta X}(\cup E(\{s(n)|n \in N\}))$, and hence $p \notin Cl_{\beta X}(\cup (E^n_i|s(n) \in E^n_i, n \in N, \text{ i even}, i \leq i \leq k_n))$. So $E = \{E^n_i|s(n) \in E^n_i, n \in N, \text{ i even}, i \leq i \leq k_n\}$ is an element of $\mathcal{E}$ and $A(u, s) \notin Cl_{\beta X}(\cup E)$, which is a contradiction.

Let $z = d\alpha$ be the element of $D^*$ that lies in $\widetilde{M}$. Let $\alpha$ be the element of $R^*$ defined as follows: $\alpha = \{G \in R| \text{ there is a } J \in \alpha \text{ so that } \{(n, i-1)|(n, i) \in J\} \subset G\}$. It is not difficult to verify that $\alpha \in R^*$. But since $\{(n, i)|i \text{ is even}\} \in \alpha$ and $\{(n, i-1)|i \text{ is even}\} \in \alpha$, it follows that $\alpha \neq \alpha$, so $d\alpha \neq d\alpha$. Yet by construction since $B^n_i \cup B^n_{i-1} \subset E^n_i$ for $i$ even, $1 \leq i \leq k_n$, it is easy to see that $d\alpha \in Ls(\mathcal{E})$. Therefore $d\alpha$ is a point of $D^*$ distinct from $d\alpha$, $d\alpha \in Ls(\mathcal{E}) \subset L$ so $d\alpha \in L$, and since $\widetilde{M} \cap D^* = \{d\alpha\}$, it follows that $d\alpha \in \widetilde{M}$, and hence $d\alpha \in M$. So $d\alpha$ is a point of $L \cap V$ that is not in $M$. So it follows that $M$ is nowhere dense in $L$, and hence $L$ is indecomposable.

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