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THE DUNFORD-PETTIS PROPERTY IN THE PREDUAL OF A VON NEUMANN ALGEBRA

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ABSTRACT. The von-Neumann algebras whose predual has the Dunford-Pettis property are characterised as being Type I finite. This answers the question asked by Chu and Iochum in *The Dunford Pettis property in C^*-algebras*, Studia Math. **97** (1990), 59–64.

The purpose of this paper is to characterise those von Neumann algebras M for which the predual M_* has the Dunford-Pettis property thereby settling the question raised in [1].

A Banach space X is said to have the Dunford-Pettis property if for each Banach space Y each weakly compact linear operator from X to Y sends weakly convergent sequences to norm convergent sequences. Classically, all L_1 spaces (Dunford and Pettis) and all $C(X)$ spaces (Gröthendieck) have the Dunford-Pettis property (see [2] for much more). A thorough study of the Dunford-Pettis property in C^*-algebras and von Neumann algebras was undertaken in [1] and [3], to which we refer the reader for any unmentioned details. But one question remained unanswered. It was proved in [1] and [3] that if M is a von Neumann algebra then

(a) if M_* has the Dunford-Pettis property then M is finite;
(b) if M is Type I finite then M_* has the Dunford-Pettis property.

An obstacle to a characterisation was that it was not known whether it was possible for the predual of a Type II_1 von Neumann algebra to have the Dunford-Pettis property. We show that is it not possible. Thus we establish the following.

Theorem. *The following are equivalent for a von Neumann algebra M.*

(i) *M_* has the Dunford-Pettis property.*
(ii) *M is Type* I *finite.*

Proof. In order to establish the validity of the converse of (b) let M be a von Neumann algebra for which M_* has the Dunford-Pettis property. In view of (a), and since the predual of every summand of M clearly inherits the property, it can be supposed that M is of Type II_1. At this point we appeal to the Jordan operator theory of spin factors contained in [4–6]. Since M contains a

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Type II_1 subfactor it certainly contains a countably infinite spin system $\{s_n\}$, consisting of nontrivial symmetries s_n (in M_{sa}) satisfying $s_n s_m + s_m s_n = 0$ whenever $n \neq m$. The real Banach subspace V of M_{sa} generated by $\{s_n\}$ is a JW-subalgebra of M_{sa} (called a spin factor). In addition, V is an infinite dimensional real Hilbert space in an equivalent norm. Evidently, then, there exists a sequence (x_n) in V such that $\|x_n\| = 1$ for all n and $x_n \to 0$ in the $\sigma(V, V^*)$ topology. Clearly $x_n \to 0$ in the $\sigma(M, M^*)$ topology. But as M_* has the Dunford-Pettis property this means that $x_n^2 \to 0$ in the $\sigma(M, M_*)$ topology, by [3, Lemma 1] or [1, Corollary 5]. Hence $x_n \to 0$ in the strong operator topology. But as proved in [5, Theorem 7.1] the latter coincides on V with the norm topology. So $x_n \to 0$ in norm, a contradiction which completes the proof.

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