PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 116, Number 4, December 1992

A CHARACTERIZATION OF INNER IDEALS IN JB^*-TRIPLES

C. M. EDWARDS AND G. T. RÜTTIMANN

(Communicated by Palle E. T. Jorgensen)

ABSTRACT. It is shown that a norm-closed subtriple B of a JB^*-triple A is an inner ideal if and only if every bounded linear functional on B has a unique norm-preserving extension to a bounded linear functional on A. It follows that the norm-closed subtriples B of a C^*-algebra A that enjoy this unique extension property are precisely those of the form $eA^{**}f \cap A$ where (e, f) is a pair of centrally equivalent open projections in the W^*-envelope A^{**} of A.

1. INTRODUCTION

A complex Banach space A equipped with a triple product

$$(a, b, c) \to \{a\,b\,c\} = D(a, b)c$$

that is symmetric and linear in the first and third variables and conjugate linear in the second variable is said to be a *JB^*-triple* if the following conditions hold:

(i) The mapping D is continuous from $A \times A$ into the Banach space of bounded linear operators on A;

(ii) For each element a in A, the linear operator $iD(a, a)$ is a derivation of the triple product structure, that is, satisfies the condition that for all elements b, c, and d in A,

$$iD(a, a)\{b\,c\,d\} = \{iD(a, a)b\,c\,d\} + \{b\,iD(a, a)c\,d\} + \{b\,c\,iD(a, a)d\};$$

(iii) For each element a in A, the operator $D(a, a)$ is hermitian with non-negative spectrum and such that $\|D(a, a)\| = \|a\|^2$.

An important characterization of JB^*-triples is that obtained by Kaup who showed that a complex Banach space possesses a triple product with respect to which it is a JB^*-triple if and only if its open unit ball is a bounded symmetric domain. The properties of JB^*-triples and of those JB^*-triples that are dual spaces, the *JBW^*-triples*, have been the subject of much research in recent years. For the basic theory of JB^*-triples the reader is referred to [20] and [27] whilst for that of JBW^*-triples one is referred to [3–10, 16, 18, 19, 22].

Received by the editors April 19, 1991.
1991 *Mathematics Subject Classification*. Primary 46L10.
Key words and phrases. JB^*-triple, inner ideal.
Research supported by a grant from the Schweizerischer Nationalfonds/Fonds national suisse.

Examples of JB^*-triples are JB^*-algebras [29] (Jordan C^*-algebras [28]) with triple product defined for elements a, b, and c by

$$\{a\,b\,c\} = a \circ (b^* \circ c) - b^* \circ (c \circ a) + c \circ (a \circ b^*)$$

where $(a, b) \to a \circ b$ is the Jordan product. In particular, every C^*-algebra is a JB^*-triple with triple product defined for elements a, b, and c by

$$\{a\,b\,c\} = (ab^*c + cb^*a)/2.$$

Consequently, JBW^*-algebras [29] (Jordan W^*-algebras [9]) and W^*-algebras provide examples of JBW^*-triples.

A subspace B of a JB^*-triple A is said to be a *subtriple* if $\{BBB\}$ is contained in B and is said to be an *inner ideal* in A if $\{BAB\}$ is contained in B. A norm-closed subtriple of A is itself a JB^*-triple and an inner ideal in A is clearly a subtriple of A. Some progress has been made in the classification of inner ideals in JBW^*-triples [11–15, 23, 27]. However, the algebraic characterization requires a consideration of different cases. The purpose of this paper is to provide a geometric characterization of norm-closed inner ideals in any JB^*-triple A amongst all norm-closed subtriples of A. It is shown that a norm-closed subtriple B of A is an inner ideal if and only if every bounded linear functional on B has a unique norm-preserving linear extension to A. It was shown by Phelps [25] that the closed subspaces W of a Banach space V that enjoy this unique norm-preserving linear extension property are precisely the annihilators W° of which have the property that for every element x of V^* there is a unique element of W° at which the distance from x to W° is attained. This gives a further characterization of norm-closed inner ideals in a JB^*-triple.

In recent papers [11, 13] the authors showed that there is an order isomorphism

$$(e, f) \to eA^{**}f \cap A$$

from the complete lattice of centrally equivalent pairs of open projections in the W^*-envelope A^{**} of a C^*-algebra A onto the complete lattice of norm-closed inner ideals in a C^*-algebra. Consequently, the norm closed subtriples of the C^*-algebra A the bounded linear functionals on which possess unique norm-preserving extensions to A are precisely those of the form $eA^{**}f \cap A$. Moreover, by restricting attention to *-subalgebras rather than subtriples it can be seen that the norm-closed *-subalgebras of A that enjoy the unique norm-preserving extension property are those of the form $eA^{**}e \cap A$ for some open projection e in A^{**}. These are of course the hereditary C^*-subalgebras in A. This is a slightly weaker result than that recently obtained by Kusuda [21] who showed that the hereditary C^*-subalgebras of A are precisely those whose states have unique extensions to states to A. However, as a consequence of the proofs of the main results of this paper, it can be seen that the theorem of Kusuda also holds for JB^*-algebras.

2. Main results

A Jordan *-algebra A that is also a complex Banach space such that for all elements a and b in A,

$$\|a \circ b\| \le \|a\|\,\|b\|, \quad \|a^*\| = \|a\|, \quad \text{and} \quad \|\{a\,a\,a\}\| = \|a\|^3$$

where

$$\{a\,b\,c\} = a \circ (b^* \circ c) - b^* \circ (c \circ a) + c \circ (a \circ b^*)$$

is the Jordan triple product on A, is said to be a *JB^*-algebra* [29] (*Jordan C^*-algebra* [28]). Examples of JB^*-algebras include all C^*-algebras A with Jordan product defined for elements a and b in A by

$$a \circ b = (ab + ba)/2\,.$$

A JB^*-algebra A that is the dual of a Banach space A_* is said to be a *JBW^*-algebra* (*Jordan W^*-algebra* [9]).

A real Jordan algebra A that is also a Banach space such that for all elements a and b in A,

$$\|a^2 - b^2\| \le \max\{\|a^2\|, \|b^2\|\} \quad \text{and} \quad \|a^2\| = \|a\|^2,$$

is said to be a *JB-algebra.* The set A^+ consisting of squares of elements of A forms a norm-closed cone in A. For elements a and c in A, let $U_{a,c}$ denote the bounded linear operator operator on A defined, for each element b in A, by $U_{a,c}b = \{a\,b\,c\}$. The positive bounded linear operator $U_{a,a}$ is denoted by U_a. A subspace B of a real Jordan algebra A such that $\{BAB\}$ is contained in B is alternatively referred to as an *inner ideal* or as a *quadratic ideal.*

If the JB-algebra A is the dual of a Banach space A_* then A is said to be a *JBW-algebra.* In this case A^+ is weak*-closed and monotone complete. It follows that A possesses a multiplicative unit and a unique predual. Let A_+^* denote the weak*-closed cone of elements of A_* that are positive on A^+. Then A_*^+ can be identified with the cone of positive normal linear functionals on A.

The selfadjoint part A_{sa} of a JB^*-algebra A (respective, JBW^*-algebra) is a JB-algebra (respectively, JBW-algebra) and every JB-algebra (respectively, JBW-algebra) is obtained in this manner. The correspondences so established are one-one. Moreover, for a JBW^*-algebra A, $(A_{sa})_*$ coincides with $(A_*)_{sa}$. Examples of JBW^*-algebras are W^*-algebras with the Jordan product defined above. For more details to the theory of the Jordan algebras defined above, the reader is referred to [2, 7–9, 17, 26, 28, 29].

The first two results are slight generalizations of the corresponding results for C^*-algebras.

Lemma 2.1. *Let A be a JBW-algebra with unit e and predual A_* and let B be a weak*-closed Jordan subalgebra of A. Then B is an inner (quadratic) ideal in A if and only if every element of the cone B_*^+ of positive normal linear functionals on B has a unique norm-preserving extension to an element of A_*^+.*

Proof. Let B be a weak*-closed inner ideal in A. By [7, Theorem 2.3], there exists an idempotent p in A such that B coincides with U_pA. Moreover, the predual B_* of B can be identified with $U_p^*A_*$. Under this identification the cone B_*^+ coincides with $U_p^*A_*^+$. Now let x be an element of $U_p^*A_*^+$ and let y be an element of A_*^+ such that $y|_B = x|_B$ and $\|y\| = \|x\|$. Then U_p^*y and x coincide. Since $\|U_p^*y\| = \|y\|$, it follows from [2, Lemma 4.10] that $x = y$.

Conversely, suppose that B is weak*-closed Jordan subalgebra of A with the property that every element x in B_*^+ has a unique extension $\psi(x)$ to an element A_*^+ such that $\|\psi(x)\| = \|x\|$. For each element y of A_* let $\phi(y)$ denote its restriction to B. Then ϕ is a positive norm nonincreasing linear mapping from A_* onto B_*. The adjoint ϕ^* of ϕ is the inclusion mapping from B into A. By [1, Theorem 12.6], each element x in B_* has a unique decomposition $x = x_1 - x_2$, with x_1 and x_2 in B_*^+, satisfying $\|x\| = \|x_1\| + \|x_2\|$. Moreover, there exists an idempotent q in B such that,

$$U_q^* x_1 = x_1, \qquad U_q^* x_2 = 0, \qquad U_{p-q}^* x_1 = 0, \qquad U_{p-q}^* x_2 = x_2,$$

where p denotes the unit in B. For each such element define

$$\psi(x) = \psi(x_1) - \psi(x_2).$$

Then

$$\|\psi(x)\| \leq \|x_1\| + \|x_2\| = \|x\|.$$

Moreover,

$$\|\psi(x)\| \geq \psi(x)(2q - p) = x_1(q) + x_2(p - q) = \|x_1\| + \|x_2\| = \|x\|.$$

Since the norm is additive on A_*^+, it is clear that ψ is a positive linear isometry from B_* into A_*. Therefore, the adjoint ψ^* of ψ is a positive weak*-continuous norm nonincreasing linear mapping from A into B. For each element x in B_* and b in B,

$$x(\psi^* \phi^* b) = \psi(x)(\phi^* b) = x(b)$$

and it follows that $\psi^* \phi^* b = b$. Hence, $\psi^* \phi^*$ is a positive weak*-continuous norm nonincreasing projection from A onto B. Next observe that for each element y in A_*^+,

$$\|\psi\phi(y)\| = \|\phi(y)\| = y(p) = y(U_p e) = U_p^* y(e) = \|U_p^* y\|.$$

Moreover, for b in B,

$$\begin{aligned}(\phi\psi\phi(y))(b) &= \phi(y)(\psi^*\phi^* b) = \phi(y)(b) = y(\phi^* b) \\ &= y(U_p(\phi^* b)) = U_p^* y(\phi^* b) = (\phi U_p^* y)(b).\end{aligned}$$

By uniqueness it follows that $\psi\phi(y) = U_p^* y$ and hence that $\phi^*\psi^* = U_p$. Therefore B coincides with the weak*-closed inner ideal $U_p A$ and the proof is complete.

Let A be a *JB*-algebra with dual A^* and second dual A^{**}. When endowed with the Arens multiplication, A^{**} is a *JBW*-algebra with predual A^*, which is said to be the *JBW-envelope* of A. The canonical mapping from A into A^{**} is an algebraic isomorphism. In what follows, a Banach space will always be identified with its image in its second dual under the canonical mapping.

The next result is not needed in the proof of the main theorems but provides a slight extension of Theorem 2.2 of [21].

Corollary 2.2. *Let A be a JB-algebra with dual A^* and let B be a norm closed Jordan subalgebra of A. Then B is an inner (quadratic) ideal in A if and only if every element of the cone B^{*+} of positive linear functionals in B^* has a unique norm-preserving extension to an element of A^{*+}.*

Proof. Let B be an inner ideal in A. By [8, Theorem 2.3], there exists a unique idempotent p in A^{**} such that B coincides with $U_p A^{**} \cap A$ and the weak* closure of B in A^{**} with $U_p A^{**}$. Since the preduals of A^{**} and B^{**} are respectively identified with the duals of A and B, Lemma 2.1 shows that B has the required extension property.

Conversely, if B has the given property it follows from Lemma 2.1 that B^{**} is an inner ideal in A^{**}. Since, under the usual identification, B coincides with $B^{**} \cap A$, it follows that B is an inner ideal in A.

It should be observed that the norm-closed inner ideals in a JB-algebra coincide with its hereditary norm closed Jordan subalgebras. Moreover, since the selfadjoint part of a JB^*-algebra is a JB-algebra and every JB^*-algebra arises in this way, the above result could just as easily be phrased in terms of hereditary norm-closed Jordan *-subalgebras of a JB^*-algebra, thereby generalizing [21, Theorem 2.2].

An element u in a JB^*-triple A is said to be a *tripotent* if $\{u\,u\,u\}$ is equal to u. The set of tripotents in A is denoted by $\mathscr{U}(A)$. For each tripotent u, the operators $Q(u)$, $P_j(u)$, $j = 0, 1, 2$, are defined by

$$Q(u)a = \{u\,a\,u\}, \qquad P_2(u) = Q(u)^2,$$
$$P_1(u) = 2(D(u, u) - Q(u)^2), \qquad P_0(u) = I - 2D(u, u) + Q(u)^2.$$

The linear operators $P_j(u)$, $j = 0, 1, 2$, are norm nonincreasing projections onto the eigenspaces $A_j(u)$ of $D(u, u)$ corresponding to the eigenvalues $j/2$ and

$$A = A_0(u) \oplus A_1(u) \oplus A_2(u)$$

is the *Peirce decomposition* of A relative to u. For $i, j, k = 0, 1, 2$, $A_i(u)$ is a norm-closed subtriple such that

$$\{A_i(u)A_j(u)A_k(u)\} \subseteq A_{i-j+k}(u)$$

when $i - j + k = 0, 1$, or 2 and $\{0\}$ otherwise, and

$$\{A_2(u)A_0(u)A\} = \{A_0(u)A_2(u)A\} = \{0\}.$$

Let A be a JBW^*-triple. Then the operators $D(a, b)$, $Q(u)$, $P_j(u)$, $j = 0, 1, 2$, are weak* continuous and $A_j(u)$, $j = 0, 1, 2$, are weak*-closed subtriples of A. Moreover, $A_0(u)$ and $A_2(u)$ are weak*-closed inner ideals in A and $A_2(u)$ is a JBW^*-algebra with product $(a, b) \to \{a\,u\,b\}$, unit u, and involution $a \to \{u\,a\,u\}$.

For two elements u and v in $\mathscr{U}(A)$ we write $u \leq v$ if $\{u\,v\,u\} = u$. This defines a partial ordering on $\mathscr{U}(A)$ with respect to which $\mathscr{U}(A)$ with a greatest element adjoined forms a complete lattice [10]. An element u in $\mathscr{U}(A)$ is *maximal* if and only if $A_0(u)$ is zero, or, equivalently, if and only if u is an extreme point of the unit ball A_1 in A.

For each element a in the JBW^*-triple A there exists a smallest element $r(a)$ in $\mathscr{U}(A)$, called the *support* of a, such that a is a positive element in the JBW^*-algebra $A_2(r(a))$.

Lemma 2.3. *Let A be a JBW^*-triple and let B be a weak*-closed subtriple of A. Then B is an inner ideal in A if and only if $\bigcup_{u \in \mathscr{U}(B)} A_2(u) \subseteq B$ and, in this case, $\bigcup_{u \in \mathscr{U}(B)} A_2(u) = B$.*

Proof. Let B be an inner ideal in A. Then for each element u in $\mathscr{U}(B)$, $A_2(u)$ is contained in B. Conversely, suppose that a is an element in B with support $r(a)$ in A. Since, by [10] and [16], $r(a)$ is contained in the smallest weak*-closed subtriple of A containing a, it follows that $r(a)$ lies in B. But

$A_2(r(a))$ is an inner ideal and, therefore,

$$\{a\,A\,a\} \subseteq A_2(r(a)) \subseteq B$$

by hypothesis, and it follows that B is an inner ideal in A. Furthermore, since a is contained in $A_2(r(a))$, it follows that $B \subseteq \bigcup_{u \in \mathscr{U}(B)} A_2(u)$ and the proof is complete.

Lemma 2.4. *Let A be a JBW^*-triple and let u be a tripotent in A. Then every weak*-continuous linear functional on the weak* closed inner ideal $A_2(u)$ in A admits a norm-preserving extension to an element of A_*.*

Proof. Let x be an element of $A_2(u)_*$. Since $P_2(u)$ is a weak*-continuous norm nonincreasing projection, it follows that $x \circ P_2(u)$ is an extension of the required kind.

Recall that for each element x in the predual A_* of a JBW^*-triple A there exists a unique element $e(x)$ of $\mathscr{U}(A)$, called the *support* of x such that the restriction of x to $A_2(e(x))$ is a faithful positive normal linear functional [16, Proposition 2; 10, Corollary 4.2].

We proceed to the first main result.

Theorem 2.5. *Let A be a JBW^*-triple and let B be a weak*-closed subtriple of A. Then B is an inner ideal in A if and only if every weak*-continuous linear functional on B has a unique weak*-continuous norm-preserving linear extension to A.*

Proof. Suppose that B is an inner ideal in A and x is an element of B_*. Let $e^B(x)$ be the support of x in $\mathscr{U}(B)$. The restriction of x to $B_2(e^B(x))$ is a positive normal linear functional such that $x(e^B(x)) = \|x\|$. Since B is an inner ideal in A, $B_2(e^B(x))$ coincides with $A_2(e^B(x))$. By Lemma 2.4 there exists an element y in A_* that agrees with x on $A_2(e^B(x))$ and such that $\|y\| = \|x\|$. Let z be a norm-preserving linear extension of x to A. It follows that z is a norm-preserving linear extension of the restriction of x to $A_2(e^B(x))$. By [16, Proposition 1], it follows that y and z coincide.

Conversely, suppose that B is a weak*-closed subtriple of A and u is an element of $\mathscr{U}(B)$. Let x be an element of $B_2(u)_*^+$. By Lemma 2.4, there exists an element x_1 in B_* such that $x_1|_{B_2(u)} = x$ and $\|x_1\| = \|x\|$. By hypothesis, there exists an element y in A_* such that $y|_B = x_1$ and $\|y\| = \|x_1\|$. Then

$$y(u) = x(u) = \|x\| = \|y\|.$$

By [10, Lemma 2.3], $y|_{A_2(u)}$ is a positive normal linear functional on $A_2(u)$ and a norm-preserving extension of x. Let z be any further such extension of x to $A_2(u)$. By Lemma 2.4, there exists an element z_1 in A_* that is a norm-preserving extension of z. Then z_1 and x_1 agree on $B_2(u)$ and, by [16, Proposition 1], it follows that they agree on B. Since $\|z_1\| = \|x_1\|$, from the uniqueness part of the hypothesis it can be concluded that z_1 and y are equal. By Lemma 2.1, $B_2(u)_{sa}$ is an inner ideal in the JBW-algebra $A_2(u)_{sa}$, and therefore, $B_2(u)$ and $A_2(u)$ coincide. It now follows from Lemma 2.3 that B is an inner ideal in A.

Recall that the second dual A^{**} of a JB^*-triple A possesses a triple product with respect to which it is a JBW^*-triple and the canonical mapping from A into A^{**} is a triple isomorphism [3, 6, 18].

The second main result of the paper is an almost immediate consequence of Theorem 2.5 and the above remark.

Theorem 2.6. *Let A be a JB^*-triple and let B be a norm-closed subtriple of A. Then B is an inner ideal in A if and only if every bounded linear functional on B has a unique norm-preserving linear extension to A.*

Proof. Identifying a Banach space with its image in its second dual under the canonical mapping as before, B^{**} may be identified with the weak* closure of B in A^{**}. Let B be an inner ideal in A. Then, by separate weak* continuity of the triple product, it follows that B^{**} is a weak *-closed inner ideal in A^{**}. Since the preduals of A^{**} and B^{**} are respectively identified with the duals of A and B, the result follows from Theorem 2.5.

Conversely, if every element of B^* has a unique norm-preserving extension to an element of A^* then, from Theorem 2.5, B^{**} is an inner ideal in A^{**}. Since B may be identified with $B^{**} \cap A$, it follows that B is an inner ideal in A.

Recall that a pair (e, f) of projections in a W^*-algebra is said to be *centrally equivalent* if e and f have the same central support [11]. Let A be a C^*-algebra and let A^{**} be its enveloping W^*-algebra. A projection e in A^{**} is said to be *open* if the weak* closure of $eA^{**}e \cap A$ coincides with $eA^{**}e$. Notice that the norm-closed *-subalgebras of A of this form are precisely the hereditary norm closed *- subalgebras of A. For the basic properties of C^*-algebras and W^*-algebras, the reader is referred to [24]. In [13] the authors showed that the norm closed inner ideals in a C^*-algebra A are of the form $eA^{**}f \cap A$ for some unique pair (e, f) of centrally equivalent open projections in A^{**}.

When applied to a C^*-algebra, Theorem 2.6 yields.

Corollary 2.7. *Let A be a C^*-algebra and let B be a norm closed subtriple of A. Then B is of the form $eA^{**}f \cap A$ for a pair (e, f) of centrally equivalent open projections is A^{**} if and only if every bounded linear functional on B has a unique norm-preserving linear extension to A.*

Applying the above corollary to norm-closed *-subalgebras leads to

Corollary 2.8. *Let A be a C^*-algebra and let B be a norm-closed *-subalgebra of A. Then B is a hereditary norm-closed *- subalgebra of A if and only if every bounded linear functional on B has a unique norm-preserving linear extension to A.*

Proof. Let B be a norm-closed *-subalgebra of A with the given property. Then, by Corollary 2.7, there exists a unique pair (e, f) of centrally equivalent open projections in A^{**} such that B coincides with $eA^{**}f \cap A$. Since B is selfadjoint, it follows that

$$eA^{**}f \cap A = fA^{**}e \cap A,$$

and hence that

$$eA^{**}f = (eA^{**}f \cap A)^{-w^*} = (fA^{**}e \cap A)^{-w^*} = fA^{**}e.$$

By [12] it follows that e and f are equal. Therefore, B is of the form $eA^{**}e \cap A$ for some open projection e in A^{**} and is therefore hereditary.

Conversely, if B is of the form $eA^{**}e \cap A$ then it is an inner ideal in A and the result follows from Corollary 2.6.

Notice that it is a consequence of this result and that of [21, Theorem 2.2] that a norm-closed *-subalgebra B of a C^*-algebra A has the unique norm-preserving extension property for bounded linear functionals if and only if it has the same property for positive bounded linear functionals. Of course, from Corollary 2.2 it can be seen that the same result holds for any JB^*-algebra.

References

1. E. M. Alfsen and F. W. Shultz, *Non-commutative spectral theory for affine function spaces on convex sets*, Mem. Amer. Math. Soc., vol. 172, Amer. Math. Soc., Providence, RI, 1976.
2. E. M. Alfsen, F. W. Shultz, and E. Størmer, *A Gelfand-Neumark theorem for Jordan algebras*, Adv. Math. **28** (1978), 11–56.
3. T. J. Barton and R. M. Timoney, *Weak*-continuity of Jordan triple products and its applications*, Math. Scand. **59** (1986), 177–191.
4. T. J. Barton, T. Dang, and G. Horn, *Normal representations of Banach Jordan triple systems*, Proc. Amer. Math. Soc. **102** (1987), 551–555.
5. M. Battaglia, *Order theoretic type decomposition of JBW^*-triples*, Quart. J. Math. Oxford Ser. 2 **42** (1991), 129–147.
6. S. Dineen, *Complete holomorphic vector fields in the second dual of a Banach space*, Math. Scand. **59** (1986), 131–142.
7. C. M. Edwards, *Ideal theory in JB-algebras*, J. London Math. Soc. (2) **16** (1977), 507–513.
8. ——, *On the facial structure of a JB-algebra*, J. London Math. Soc. (2) **19** (1979), 335–344.
9. ——, *On Jordan W^*-algebras*, Bull. Sci. Math. (2) **104** (1980), 393–403.
10. C. M. Edwards and G. T. Rüttimann, *On the facial structure of the unit balls in a JBW^*-triple and its predual*, J. London Math. Soc. (2) **38** (1988), 317–332.
11. ——, *Inner ideals in W^*-algebras*, Michigan Math. J. **36** (1989), 147–159.
12. ——, *On inner ideals in ternary algebras*, Math. Z. **204** (1990), 309–318.
13. ——, *Inner ideals in C^*-algebras*, Math. Ann. **290** (1991), 621–628.
14. C. M. Edwards, G. T. Rüttimann, and S. Yu. Vasilovsky, *Inner ideals in exceptional JBW^*-triples*, Michigan Math J. (to appear).
15. ——, *Invariant inner ideals in W^*-algebras*, preprint.
16. Y. Friedman and B. Russo, *Structure of the predual of a JBW^*-triple*, J. Reine Angew. Math. **356** (1985), 67–89.
17. H. Hanche-Olsen and E. Størmer, *Jordan operator algebras*, Pitman, London, 1984.
18. G. Horn, *Characterization of the predual and the ideal structure of a JBW^*-triple*, Math. Scand. **61** (1987), 117–133.
19. G. Horn and E. Neher, *Classification of continuous JBW^*-triples*, Trans. Amer. Math. Soc. **306** (1988), 553–578.
20. W. Kaup, *Riemann mapping theorem for bounded symmetric domains in complex Banach spaces*, Math. Z. **183** (1983), 503–529.
21. M. Kusuda, *Unique state extension and hereditary C^*-algebras*, Math. Ann. **288** (1990), 201–209.
22. E. Neher, *Jordan triple systems by the grid approach*, Lecture Notes in Math., vol. 1280, Springer-Verlag, Berlin, Heidelberg, and New York, 1987.
23. ——, *Jordan pairs with finite grids*, Comm. Algebra **19** (1991), 455–478.
24. G. K. Pedersen, *C^*-algebras and their automorphism groups*, Academic Press, London, 1979.

25. R. R. Phelps, *Uniqueness of Hahn-Banach extensions and unique best approximations*, Trans. Amer. Math. Soc. **95** (1960), 238–255.
26. F. W. Shultz, *On normed Jordan algebras which are Banach dual spaces*, J. Funct. Anal. **31** (1979), 360–376.
27. H. Upmeier, *Symmetric Banach manifolds and Jordan C^*-algebra*, North-Holland, Amsterdam, 1985.
28. J. D. M. Wright, *Jordan C^*-algebras*, Michigan Math. J. **24** (1977), 291–302.
29. M. A. Youngson, *A Vidav theorem for Banach Jordan algebras*, Math. Proc. Cambridge Philos. Soc. **84** (1978), 263–272.

THE QUEEN'S COLLEGE, OXFORD, 0X1 4AW UNITED KINGDOM
E-mail address: MEDWARDS@VAX.OXFORD.AC.UK

INSTITUTE OF MATHEMATICAL STATISTICS, UNIVERSITY OF BERNE, BERNE CH-3012, SWITZERLAND
E-mail address: RUETTIMANN@STAT.UNIBE.CH