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RIGIDITY THEOREMS FOR NONPOSITIVE EINSTEIN METRICS

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ABSTRACT. In this paper we study the following problem: When must a complete Einstein metric g on an n-manifold with $\mathrm{Ric} = (n-1)\lambda g$, $\lambda \leq 0$, be a metric of constant curvature λ?

1. INTRODUCTION AND MAIN RESULTS

Let g be a complete riemannian metric on an n-manifold M. Denote by R the curvature tensor of g. The Ricci curvature Ric is then defined as

$$\mathrm{Ric}(x, y) = \sum_{i=1}^{n} g(R(x, e_i)e_i, y), \qquad x, y \in T_pM,$$

where $\{e_1, \dots, e_n\}$ is an orthonormal basis for T_pM. The metric g is said to be Einstein if the Ricci curvature is constant, i.e.,

$$\mathrm{Ric} = (n-1)\lambda g$$

for some constant λ. λ is called the Einstein constant of g. It is clear that in dimension three the metric g is Einstein with $\mathrm{Ric} = (n-1)\lambda g$ if and only if it has constant curvature λ, i.e.,

$$R(x, y)z = \lambda(g(y, z)x - g(x, z)y), \qquad x, y, z \in T_pM.$$

In higher dimensions, this is not the case. One may ask if an Einstein metric has constant curvature whenever it has almost constant curvature in a certain sense. From now on we always assume that g is a complete Einstein metric with Einstein constant λ. It is natural to consider the new tensor $\mathring{R}$, defined

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$$\overset{\circ}{R}(x, y)z = R(x, y)z - \lambda(g(y, z)x - g(x, z)y), \qquad x, y, z \in T_pM.$$

Denote by σ the pointwise norm of $\overset{\circ}{R}$, defined by

$$\sigma = \sqrt{\sum_{ijkl} g(\overset{\circ}{R}(e_i, e_j)e_k, e_l)^2},$$

where $\{e_1, \dots, e_n\}$ is an orthonormal basis for T_pM. By a formula in [H], one can easily show that σ satisfies

$$\Delta\sigma + c_0(n)\sigma^2 - 2(n-1)\lambda\sigma \geq 0 \tag{1}$$

in the sense of distribution, where $c_0(n)$ is a positive constant depending only on n and Δ denotes the Laplace-Beltrami operator (in $\mathbb{R}^n$, $\Delta = \sum_{i=1}^n \partial^2/\partial x_i^2$).

In the case of $\lambda > 0$, Myers's Theorem (cf., e.g., [CE]) tells us that the manifold is closed. Thus by integrating (1), one obtains

Theorem 1 (Berger [B]). *Given n, there is a small constant $\varepsilon = \varepsilon(n) > 0$ depending only on n such that if a complete Einstein metric g, with $\lambda > 0$, on an n-manifold satisfies $\sigma \leq \lambda\varepsilon$, then $\sigma \equiv 0$, i.e., g has constant curvature λ.*

In [S] the author gives an $L^{\frac{n}{2}}$-version of Theorem 1, which says that if

$$\int \sigma^{\frac{n}{2}} \leq \lambda^{\frac{n}{2}} \mathrm{vol}(M)\varepsilon$$

for some small $\varepsilon = \varepsilon(n) > 0$, depending only on n, where $\mathrm{vol}(M)$ denotes the volume of (M, g), then $\sigma \equiv 0$.

In the case of $\lambda \leq 0$, the manifold can be compact or noncompact. First let us consider the case $\lambda = 0$. In this case, the following fact is known: There is a small constant $\varepsilon = \varepsilon(n) > 0$ depending only on n, if a Ricci-flat metric g on a closed n-manifold M satisfies

$$\sigma \cdot \mathrm{dia}(M)^2 \leq \varepsilon, \tag{2}$$

where $\mathrm{dia}(M)$ denotes the diameter of g, then $\sigma \equiv 0$, i.e., g is flat. The proof of this fact is trivial. By a theorem of Gromov [G], any almost flat manifold is aspherical, i.e., its universal cover is diffeomorphic to R^n. Thus for a sufficiently small $\varepsilon = \varepsilon(n)$, (2) implies that the universal cover $\widetilde{M}$ is diffeomorphic to $\mathbb{R}^n$. On the other hand, by the Cheeger-Gromoll's Splitting Theorem (cf. [CG]), $\widetilde{M}$ with the induced metric $\tilde{g}$ is isometric to a riemannian product $N \times \mathbb{R}^k$ for some closed riemannian manifold N. Thus N must be a point and $\tilde{g}$ is flat. Therefore g is flat. This argument in fact shows that all nonnegatively Ricci-curved aspherical manifolds are flat. By a theorem of Fukaya-Yamaguchi [FY], if $\mathrm{dia}(M) \leq D$, then condition (2) can be replaced by $-1 \leq K_g \leq \varepsilon$ for a small number $\varepsilon = \varepsilon(n, D) > 0$, where K_g denotes the sectional curvature of g.

In §2 we will prove an $L^{\frac{n}{2}}$-version of the above fact, that is,

Theorem 2. *Given n, there is a small constant $\varepsilon = \varepsilon(n) > 0$ depending only on n such that if a Ricci-flat metric g on a closed n-manifold satisfies*

$$\int \sigma^{\frac{n}{2}} \le \frac{\mathrm{vol}(M)}{\mathrm{dia}(M)^n}\varepsilon,$$

then $\sigma \equiv 0$, i.e., g is flat.

For complete Ricci-flat metrics on noncompact n-manifolds, some rigidity phenomena have been discovered (cf. [A2, Ba, S], etc.). Roughly speaking, if a complete Ricci-flat metric g has sufficiently small total curvature, i.e., there is a small $\varepsilon = \varepsilon(n) > 0$ such that if

$$\int \sigma^{\frac{n}{2}} \le \nu_M^{n+1}\varepsilon \tag{3}$$

where

$$\nu_M := \lim_{r\to+\infty} \frac{\mathrm{vol}(B(p, r))}{\sigma_n r^n} > 0,$$

where σ_n denotes the volume of the unit ball in $\mathbb{R}^n$, then $\sigma \equiv 0$. It is worth mentioning that the result of Anderson [A2] does not require (3), but requires that $\nu_M \ge 1 - \varepsilon$ for a small $\varepsilon = \varepsilon(n) > 0$.

Now let us consider the case of $\lambda < 0$. The following theorem is first proved by Ye [Y, Theorem 2].

Theorem 3 ([Y]). *Given n, $D > 0$, and $\lambda < 0$, there is a small constant $\varepsilon = \varepsilon(n, \sqrt{-\lambda}D) > 0$ such that if an Einstein metric g, with $\lambda < 0$, on a closed n-manifold satisfies $\mathrm{dia}(M) \le D$ and $\sigma \le |\lambda|\varepsilon$, then $\sigma \equiv 0$, i.e., g has constant curvature λ.*

In §3 we will prove the following $L^{\frac{n}{2}}$-version of Theorem 3.

Theorem 4. *Given n, $D > 0$, and $\lambda < 0$, there is a small constant $\varepsilon = \varepsilon(n, \sqrt{-\lambda}D) > 0$ such that if an Einstein metric g, with $\lambda < 0$, on a closed n-manifold satisfies $\mathrm{dia}(M) \le D$ and*

$$\int \sigma^{\frac{n}{2}} \le |\lambda|^{\frac{n}{2}}\,\mathrm{vol}(M)\varepsilon,$$

then $\sigma \equiv 0$, i.e., g has constant curvature λ.

Complete Einstein metrics on noncompact n-manifolds with Einstein constant $\lambda < 0$ are still not completely understood. The Sobolev inequalities do not hold on such manifolds. Instead, the Poincaré inequalities hold, which will be used to prove the following

Theorem 5. *Let g be a complete Einstein metric on a noncompact simply connected n-manifold with $\lambda < 0$. Suppose $n \ge 10$. There is a small constant $\varepsilon = \varepsilon(n) > 0$ such that if*

(i) *$\sigma \le |\lambda|\varepsilon$ and*

(ii) *for some* $p \in M$,

$$\lim_{r\to+\infty} e^{-\delta_n|\lambda|^{\frac{1}{2}}r}\int_{B(p,r)}\sigma^2 = 0,$$

where $\delta_n = \frac{1}{3}\sqrt{(n-1)(n-9)} > 0$ *and* $B(p,r)$ *denotes the geodesic ball of radius* r *around* p, *then* $\sigma \equiv 0$, *i.e., g has constant curvature* $\lambda < 0$.

The proof of Theorem 5 will be given in §4. The author does not know the case of $n \le 9$.

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2. Closed Einstein manifolds with $\lambda = 0$

In this section we will prove Theorem 2. The argument given here is quite standard and similar to that given in Lemma 2.1 of [A1].

In §1 we have shown that every Ricci-flat metric satisfying (2) for some small $\varepsilon = \varepsilon(n)$ is flat. Throughout this section $M = (M, g)$ always denotes a closed Ricci-flat manifold of dimension $n \ge 4$ and $c_i(n)$'s denote constants depending only on n. In the case of $\lambda = 0$, (1) is equivalent to

$$\Delta\sigma + c_0(n)\sigma^2 \ge 0 \tag{4}$$

in the sense of distribution. Recall that the following Sobolev inequality holds in M (cf. [Be] for references):

$$\|f\|_{\frac{2n}{n-2}} \le c_1(n)\mathrm{vol}(M)^{-\frac{1}{n}}[\mathrm{dia}(M)\|\nabla f\|_2 + \|f\|_2] \tag{5}$$

for every $f \in C^\infty(M)$.

Multiply (4) by σ^α for $\alpha \ge 1$. Integration by parts gives

$$c_0(n)\int\sigma^{\alpha+2} \ge \frac{4\alpha}{(\alpha+1)^2}\int|\nabla\sigma^{\frac{\alpha+1}{2}}|^2 \ge \frac{1}{\alpha}\int|\nabla\sigma^{\frac{\alpha+1}{2}}|^2. \tag{6}$$

Taking $f = \sigma^{\frac{\alpha+1}{2}}$ in (5), we obtain by (6)

$$\|\sigma^{\frac{\alpha+1}{2}}\|_{\frac{2n}{n-2}} \le c_2(n)\mathrm{vol}(M)^{-\frac{1}{n}}[\alpha^{\frac{1}{2}}\mathrm{dia}(M)\|\sigma\cdot\sigma^{\alpha+1}\|_1^{\frac{1}{2}} + \|\sigma^{\frac{\alpha+1}{2}}\|_2]. \tag{7}$$

Taking $\alpha + 1 = \frac{n}{2}$ in (7) and applying Hölder's inequality to $\|\sigma\cdot\sigma^{n/2}\|_1$, we have

$$\|\sigma^{\frac{n}{4}}\|_{\frac{2n}{n-2}} \le c_3(n)\mathrm{vol}(M)^{-\frac{1}{n}}[\mathrm{dia}(M)\|\sigma\|_{\frac{n}{2}}^{\frac{1}{2}}\|\sigma^{\frac{n}{4}}\|_{\frac{2n}{n-2}} + \|\sigma\|_{\frac{n}{2}}^{\frac{n}{4}}]. \tag{8}$$

It follows from (8) that there is a small constant $\varepsilon(n) > 0$, such that if for some $\varepsilon \le \varepsilon(n)$

$$\|\sigma\|_{\frac{n}{2}} \le \frac{\mathrm{vol}(M)^{\frac{2}{n}}}{\mathrm{dia}(M)^2}\varepsilon, \tag{9}$$

then

$$\begin{aligned}\|\sigma\|_{\frac{q}{2}} = \|\sigma^{\frac{n}{4}}\|_{\frac{2n}{n-2}}^{\frac{4}{n}} &\le c_4(n)\mathrm{vol}(M)^{-\frac{4}{n^2}}\|\sigma\|_{\frac{n}{2}} \\ &\le c_5(n)\mathrm{vol}(M)^{\frac{2}{n}-\frac{4}{n^2}}\mathrm{dia}(M)^{-2},\end{aligned} \tag{10}$$

where $q = \frac{n}{2} \cdot \frac{2n}{n-2}$. For general $\alpha \geq 1$, by Hölder's inequality, the interpolation inequality, and (10), we have that for all $\theta > 0$

$$\begin{aligned}(11)\qquad \|\sigma \cdot \sigma^{\alpha+1}\|_1 &\leq \|\sigma\|_{\frac{q}{2}} \|\sigma^{\frac{\alpha+1}{2}}\|^2_{\frac{2q}{q-2}} \\ &\leq c_5(n)\mathrm{vol}(M)^{\frac{2}{n}-\frac{4}{n^2}} \mathrm{dia}(M)^{-2}(\theta\|\sigma^{\frac{\alpha+1}{2}}\|_{\frac{2n}{n-2}} + \theta^{-\frac{n-2}{2}}\|\sigma^{\frac{\alpha+1}{2}}\|_2)^2.\end{aligned}$$

Thus it follows from (7) and (11) that

$$\begin{aligned}(12)\qquad \|\sigma^{\frac{\alpha+1}{2}}\|_{\frac{2n}{n-2}} \leq c_6(n)[&\alpha^{\frac{1}{2}}\mathrm{vol}(M)^{-\frac{2}{n^2}}\theta\|\sigma^{\frac{\alpha+1}{2}}\|_{\frac{2n}{n-2}} \\ &+ (\alpha^{\frac{1}{2}}\mathrm{vol}(M)^{-\frac{2}{n^2}}\theta^{-\frac{n-2}{2}} + \mathrm{vol}(M)^{-\frac{1}{n}})\|\sigma^{\frac{\alpha+1}{2}}\|_2].\end{aligned}$$

Choosing $\theta = \frac{1}{2}c_6(n)^{-1}\alpha^{-\frac{1}{2}}\mathrm{vol}(M)^{\frac{2}{n^2}}$, we obtain by (12)

$$(13)\qquad \|\sigma^{\frac{\alpha+1}{2}}\|_{\frac{2n}{n-2}} \leq c_7(n)\alpha^{\frac{n}{4}}\mathrm{vol}(M)^{-\frac{1}{n}}\|\sigma^{\frac{\alpha+1}{2}}\|_2.$$

Let $\chi = \frac{n}{n-2}$ and $\alpha + 1 = \frac{n}{2}\chi^i$, $i \geq 0$. It follows from (13) that

$$\begin{aligned}\|\sigma\|_{\frac{n}{2}\chi^{i+1}} &\leq c_8(n)^{\frac{1}{\chi^i}}\chi^{\frac{i}{\chi^i}}\mathrm{vol}(M)^{-\frac{4}{n^2}\cdot\frac{1}{\chi^i}}\|\sigma\|_{\frac{n}{2}\chi^i} \\ &\leq c_8(n)^{\frac{1}{\chi^i}+\cdots+\frac{1}{\chi^0}}\chi^{(\frac{i}{\chi^i}+\cdots+\frac{1}{\chi^1})}\mathrm{vol}(M)^{-\frac{4}{n^2}(\frac{1}{\chi^i}+\cdots+\frac{1}{\chi^0})}\|\sigma\|_{\frac{n}{2}}.\end{aligned}$$

Letting $i \to +\infty$, we obtain

$$\sigma \leq c_9(n)\mathrm{vol}(M)^{-\frac{2}{n}}\|\sigma\|_{\frac{n}{2}} \leq c_9(n)\mathrm{dia}(M)^{-2}\varepsilon,$$

i.e.,

$$\sigma \cdot \mathrm{dia}(M)^2 \leq c_9(n)\varepsilon.$$

The last inequality follows from (9). Choosing a smaller ε in (9) if necessary, by the argument in §1, we conclude that $\sigma \equiv 0$, i.e., g is flat. This completes the proof of Theorem 2.

3. Closed Einstein manifolds with $\lambda < 0$

In this section we will only give a sketch of the proof of Theorem 4. The method applied here is very standard and similar to that given in §2. Let $M = (M, g)$ be a closed Einstein n-manifold with Einstein constant $\lambda < 0$ and $\mathrm{dia}(M) \leq D$. Throughout this section $c_i(n)$'s always denote positive constants depending only on n.

First one has the following Sobolev inequality in M (cf., e.g., [Be] for references): for every $f \in C^\infty(M)$

$$(14)\qquad \|f\|_{\frac{2n}{n-2}} \leq c_1(n)C(\sqrt{|\lambda|}D)^{-1}\mathrm{vol}(M)^{-\frac{1}{n}}(|\lambda|^{-\frac{1}{2}}\|\nabla f\|_2 + \|f\|_2),$$

where $C(x)$, $x > 0$, is the unique positive root of the equation

$$y\int_0^x (\cosh t + y\sinh t)^{n-1}\,dt = \int_0^\pi \sin^{n-1} t\,dt.$$

Similarly, by (1) and (14) we obtain that there is a constant $\varepsilon(n) > 0$ if for some $\varepsilon \le \varepsilon(n)C(\sqrt{|\lambda|}D)^2$

$$\|\sigma\|_{\frac{n}{2}} \le |\lambda|\mathrm{vol}(M)^{\frac{2}{n}}\varepsilon \tag{15}$$

then for $q = \frac{n}{2}\cdot\frac{2n}{n-2}$

$$\begin{aligned}\|\sigma\|_{\frac{q}{2}} &\le c_2(n)C(\sqrt{|\lambda|}D)^{-\frac{4}{n}}\mathrm{vol}(M)^{-\frac{4}{n^2}}\|\sigma\|_{\frac{n}{2}}\\ &\le c_3(n)C(\sqrt{|\lambda|}D)^{2-\frac{4}{n}}|\lambda|\mathrm{vol}(M)^{\frac{2}{n}-\frac{4}{n^2}}\end{aligned}$$

and for $\alpha \ge 1$,

$$\|\sigma^{\frac{\alpha+1}{2}}\|_{\frac{2n}{n-2}} \le c_4(n)C(\sqrt{|\lambda|}D)^{-1}\alpha^{\frac{n}{4}}\mathrm{vol}(M)^{-\frac{1}{n}}\|\sigma^{\frac{\alpha+1}{2}}\|_2.$$

Then the last argument in §2 carries over to give

$$\sigma \le c_5(n)C(\sqrt{-\lambda}D)^{-2}|\lambda|\varepsilon.$$

Choosing a smaller ε in (15) if necessary, by Theorem 3 (Ye), one concludes that $\sigma \equiv 0$, i.e., g has constant curvature $\lambda < 0$.

4. Proof of Theorem 5

Let $M = (M, g)$ be a complete n-manifold. Denote by $\lambda_1(M, g)$ the first eigenvalue of M, defined as

$$\lambda_1(M, g) = \inf\frac{\int|\nabla f|^2}{\int f^2},$$

where the infumum is taken over all $f \in C_0^\infty(M)$ with compact support in M. It is proved in [M] that if M is simply connected with sectional curvature $K_g \le -\Lambda^2$ $(\Lambda > 0)$,

$$\lambda_1(M, g) \ge \frac{(n-1)^2}{4}\Lambda^2,$$

i.e., for every $f \in C_0^\infty(M)$,

$$\frac{1}{4}(n-1)^2\Lambda^2\int f^2 \le \int|\nabla f|^2. \tag{16}$$

From now on (M, g) always denotes a complete Einstein n-manifold with Einstein constant $\lambda < 0$ and $c_i(n)$'s denote positive constants depending only on n. Clearly, there is a small constant $\varepsilon(n) > 0$ such that if for some $\varepsilon \le \varepsilon(n)$, $\sigma \le |\lambda|\varepsilon$, then the sectional curvature satisfies

$$K_g \le -(1 - c_1(n)\varepsilon)|\lambda| < 0. \tag{17}$$

By (1) and (17) we have

$$\Delta\sigma + (c_0(n)\varepsilon + 2(n-1))|\lambda|\sigma \ge 0. \tag{18}$$

Multiply (18) by $\sigma\eta^2$, where η is a cut off function of compact support in M. Integration by parts gives

$$(c_0(n)\varepsilon + 2(n-1))|\lambda|\int(\sigma\eta)^2 \ge \int|\nabla(\sigma\eta)|^2 - \int|\nabla\eta|^2\sigma^2. \tag{19}$$

Taking $f = \eta\sigma$ in (16), by (17) we have

$$\frac{1}{4}(n-1)^2(1-c_1(n)\varepsilon)|\lambda| \int (\sigma\eta)^2 \le \int |\nabla(\sigma\eta)|^2;$$

so that by (19)

$$\int |\nabla\eta|^2\sigma^2 \ge \left[\frac{1}{4}(n-1)(n-9) - c_2(n)\varepsilon\right] |\lambda| \int (\sigma\eta)^2. \tag{20}$$

Now suppose $n \ge 10$. Take $\delta(n) = \frac{1}{3}\sqrt{(n-1)(n-9)}$. One can choose a smaller ε in (17) if necessary, such that

$$\frac{1}{4}(n-1)(n-9) - c_2(n)\varepsilon \ \ge\ \frac{1}{4}e^2\delta(n)^2.$$

Choosing $\eta(x) = \eta(d(p, x))$, where

$$\eta(t) = \begin{cases} 1 & \text{if } t \le r, \\ \dfrac{R-t}{R-r} & \text{if } r \le t \le R, \\ 0 & \text{if } t \ge R, \end{cases}$$

we obtain by (20)

$$\frac{1}{(R-r)^2}\int_{B(p, R)} \sigma^2 \ge \frac{1}{4}e^2\delta(n)^2|\lambda| \int_{B(p, r)} \sigma^2. \tag{21}$$

For any $r_0 > 0$, take $r_j = 2\delta(n)^{-1}|\lambda|^{-\frac{1}{2}}j + r_0$, $j \ge 0$. It then follows from (21) that

$$\int_{B(p, r_j)} \sigma^2 \ge e^2 \int_{B(p, r_{j-1})} \sigma^2 \ge e^{2j} \int_{B(p, r_0)} \sigma^2 = e^{\delta(n)|\lambda|^{\frac{1}{2}}(r_j - r_0)} \int_{B(p, r_0)} \sigma^2.$$

Thus it is easy to see that

$$\int_{B(p, r_0)} \sigma^2 \le e^{-\delta(n)|\lambda|^{\frac{1}{2}}(r_j - r_0)} \int_{B(p, r_j)} \sigma^2.$$

Letting $r_j \to +\infty$, by Theorem 5(ii), one obtains $\sigma \equiv 0$ on $B(p, r_0)$. Since r_0 is arbitrary, one concludes that $\sigma \equiv 0$ on M, i.e., g has constant curvature $\lambda < 0$. This completes the proof of Theorem 5.

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