

ADDENDUM TO  
"REMARKS ON WEAK COMPACTNESS OF OPERATORS DEFINED  
ON CERTAIN INJECTIVE TENSOR PRODUCTS"

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The idea (used in [1]) of embedding  $X \otimes_{\varepsilon} Y$  into  $X^{**} \otimes_{\varepsilon} Y$ , when  $X$  is an  $\mathcal{L}_{\infty}$  space can also be used to get the following results.

**Theorem 1.** *Let  $X$  be an  $\mathcal{L}_{\infty}$  space and  $Y$  a Banach space. Then  $X \otimes_{\varepsilon} Y$  contains a complemented copy of  $l^1$  iff  $Y$  does the same.*

*Proof.* If  $l^1$  embeds complementably into  $X \otimes_{\varepsilon} Y$ , then  $c_0$  embeds into  $(X \otimes_{\varepsilon} Y)^*$ , a closed subspace of  $(X^{**} \otimes_{\varepsilon} Y)^*$ ; then  $l^1$  embeds complementably into  $X^{**} \otimes_{\varepsilon} Y$ , that is, a complemented subspace of some  $C(K, Y)$  space (see Theorem 2 of [1]). Hence, it is enough to apply the main result of [2] to get our thesis.

**Theorem 2.** *Let  $X$  be a Banach space with  $X^*$  isometric to an  $L^1$  space. If  $Y$  is a reflexive Banach space, then  $X \otimes_{\varepsilon} Y$  has property (V) of Pelczynski.*

*Proof.* Let  $T$  be an unconditionally converging operator on  $X \otimes_{\varepsilon} Y$ . The results of [3] imply that  $T^{**}$  is unconditionally converging, too. Hence  $T^{**}$  restricted to  $X^{**} \otimes_{\varepsilon} Y$  is unconditionally converging; since  $X^{**} \otimes_{\varepsilon} Y$  is complemented in some  $C(K, Y)$  space, it inherits property (V) of Pelczynski. Hence  $T^{**}$ , and so  $T$ , are weakly compact. We are done.

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