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ADDENDUM TO "FINITELY EMBEDDED COMMUTATIVE RINGS"

CARL FAITH

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The main reason for this addendum is to supply the reference to [GM] that was missing in [F1]. At the same time, there is an interesting application of the main result of [F1] using a result of [F2] that also ought to have been included, namely,

Theorem. *A commutative ring R is Artinian iff R is a Goldie quotient ring with nil Jacobson radical.*

Proof. The necessity is clear. To prove sufficiency, recall that a ring R is Goldie if it satisfies the acc on annihilators ($=\operatorname{acc}\perp$) and also the acc on direct sums of ideals ($=\operatorname{acc}\oplus$). By [F1], any finitely embedded $\operatorname{acc}\perp$ ring is Artinian, so it remains only to prove that R is finitely embedded, i.e., has finite essential socle. By [F2, Corollary 3.7, p. 1879], R has semilocal quotient ring Q, hence R is semilocal by our assumption that $R = Q$. Also, employing a theorem of Levitzki, Herstein, and Small (as in [F1]), the radical J is nilpotent, hence R is finitely embedded since any semiprimary ring R has essential socle S (since R satisfies, e.g., the dcc on principal ideals) and S is finite via $\operatorname{acc}\oplus$.

Added in proof

See [F3] for applications of this theorem: e.g., any commutative Goldie algebraic algebra is Artinian.

References

[F1] Carl Faith, *Finitely embedded commutative rings*, Proc. Amer. Math. Soc. **112** (1991), 657–659.

[F2] ——, *Annihilator ideals, associated primes, and Kasch-McCoy commutative rings*, Comm. Algebra **19** (1991), 1867–1892.

[F3] ——, *Polynomial rings over Goldie-Kerr commutative rings*, preprint.

[GM] S. M. Ginn and P. B. Moss, *Finitely embedded modules over Noetherian rings*, Bull. Amer. Math. Soc. **81** (1975), 709–710.

Department of Mathematics, Rutgers University, New Brunswick, New Jersey 08903

Permanent address: 199 Longview Drive, Princeton, New Jersey 08540-5639

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