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CYCLICALLY PRESENTED GROUPS EMBEDDED IN ONE-RELATOR PRODUCTS OF CYCLIC GROUPS

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ABSTRACT. We consider the groups defined by the presentations

$$\langle a, b : a^2 = b^n = ab^{-1}ab(abab^{-1})^{\alpha-1}ab^2ab^{-2} = 1\rangle$$

and investigate their structure for small values of α. This forms part of a general investigation into the structure of groups defined by presentations of the form

$$\langle a, b : a^2 = b^n = w(a, b) = 1\rangle .$$

Connections between these groups and the Fibonacci groups are also explored.

1. INTRODUCTION

A group G defined by a presentation of the form

$$\langle a, b : a^m = b^n = w(a, b) = 1\rangle$$

is called a *one-relator product* of cyclic groups in that it is formed from the free product of two cyclic groups by imposing a single extra relator. There has been a great deal of interest in recent years in the structure of such groups and it is known (see [1] or [14]) that if $w(a, b)$ is of the form $u(a, b)^k$ with $k \geq 2$ and $\frac{1}{m} + \frac{1}{n} + \frac{1}{k} \leq 1$, then G is infinite; for further results on such groups, see [15–19, 24]. If this condition is not satisfied (in particular, if $w(a, b)$ is not a proper power), however, then G may be finite. In [10] the structures of all groups with $m = 2$, $n = 3$, and $w(a, b)$ of length at most 24 in $\{a, b, b^{-1}\}$ were determined, and results on groups defined by presentations of the form $\langle a, b : a^2 = b^n = w(a, b) = 1\rangle$ may be found in [3, 5–8, 13, 27].

The groups $G = G(\alpha, n)$, where $\alpha \geq 1$ and $n \geq 1$, are defined by the presentations

$$\langle a, b : a^2 = b^n = ab^{-1}ab(abab^{-1})^{\alpha-1}ab^2ab^{-2} = 1\rangle .$$

Clearly $[G : G'] = 2n$, and it was shown in [5] that $[G' : G''] = v_n(\alpha)$, where $v_n = v_n(\alpha)$ is defined by $v_0 = 0$, $v_1 = 1$, and $v_n = \alpha v_{n-1} + v_{n-2} + 1 + (-1)^{n-1}$ for $n \geq 2$. If $\alpha = 1$ then $v_n(1) = g_n - 1 - (-1)^n$ (see [4, Corollary 5]),

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where (g_n) is the *Lucas sequence* of numbers defined by $g_1 = 1$, $g_2 = 3$, and $g_n = g_{n-2} + g_{n-1}$ for $n \geq 3$. The structure of the groups $G(\alpha, n)$ for small values of n was determined in [5] and further details may be found in [27]. The purpose of this paper is to investigate the structure of the $G(\alpha, n)$ for $\alpha \leq 5$. The pattern that appears in the theorem below does not extend to the case $\alpha \geq 6$, so there is no natural generalization to arbitrary α. We prove

Theorem 1.1. (i) *$G(1, n)$ is metabelian of order $2ng_n$ for odd n, elementary abelian of order 4 for $n = 2$, metabelian of order 40 for $n = 4$, and infinite for $n = 2m \geq 6$;*

(ii) *$G(2, n)$ is metabelian of order $2nv_n(2)$;*

(iii) *$G(3, n)$ is metabelian of order $2nv_n(3)$ if 3 does not divide n and soluble with derived length 3 and order $4nv_n(3)$ if 3 does divide n;*

(iv) *$G(4, n)$ is metabelian of order $2nv_n(4)$ if 8 does not divide n and finite and soluble with derived length 3 or 4 if 8 does divide n;*

(v) *$G(5, n)$ is metabelian of order $2nv_n(5)$ if 3 does not divide n, finite and soluble of derived length at most 3 if $(n, 12) = 3$, finite and soluble of derived length 3 or 4 if $(n, 12) = 6$, and infinite if $(n, 12) = 12$.*

Theorem 1.1(i) was proved in [6], and also in [8], and is therefore included for completeness only. In fact, the structure of any group defined by any presentation of the form

$$\langle a, b : a^2 = b^n = ab^h ab^i ab^j ab^k = 1 \rangle,$$

with $n \geq 1$, $h + i + j + k = 0$, and $h, i, j, k \in \{\pm 1, \pm 2\}$, was determined in [8], and further results on such groups may be found in [7, 13].

The *Fibonacci group* $F = F(2, n)$ is defined by the presentation

$$\langle x_1, x_2, \dots, x_n : x_1x_2 = x_3, x_2x_3 = x_4, \dots, x_nx_1 = x_2 \rangle.$$

These groups were introduced in [11], and it was quickly determined that $F(2, 1)$ and $F(2, 2)$ are trivial, $F(2, 3)$ is the quaternion group Q_8, $F(2, 4)$ is cyclic of order 5, $F(2, 5)$ is cyclic of order 11, and $F(2, 6)$ is infinite. $F(2, 7)$ was shown to be cyclic of order 29 using a computer (see [2, 9, 21]), and $F(2, 8)$ and $F(2, 10)$ to be infinite [2]. $F(2, n)$ is infinite for $n \geq 11$ [25], and, lastly, $F(2, 9)$ has also been shown to be infinite [26]; alternative proofs that $F(2, n)$ is infinite for n even, $n \geq 8$, may be found in [23, 29]. The groups $F(2, n)$, and other related groups, have provoked a lot of interest, partly because of the problems they represent in determining whether or not they are finite, but also because they tend to arise in a natural way, such as the fundamental group of a 3-manifold for even n [23]; see [30] for a recent survey of these groups.

A group such as $F(2, n)$ is said to be *cyclically presented* in that the set of relations is invariant when the generators $x_1, x_2, \dots, x_n$ are permuted in a cycle of length n. This general class of groups has been the subject of much interest, particularly in that they provide many of the known examples of finite groups of *deficiency zero* (that is, finite groups with a presentation in which there are equal numbers of generators and relators). Various connections between the groups $G(\alpha, n)$ and $F(2, n)$ were pointed out in [5], and we make use of these connections in this paper also; we spell out one such connection as Proposition 2.2.

In a more general context, it was shown in [28] that a group defined by a presentation of the form

$$(\dagger) \qquad \langle x_1, x_2, \dots, x_m : w(x_1, x_2) = w(x_2, x_3) = \cdots = w(x_m, x_1) = 1\rangle$$

with $m \geq 4$ is either cyclic or infinite, and the problem is posed there of determining what happens if $m = 2$ or 3. If we adjoin the automorphism a of order dividing m permuting the x_i in a cycle of length m, we get the presentation $\langle a, b : a^m = w(b, a^{-1}ba) = 1\rangle$, where $b = x_1$. We may rewrite $w(b, a^{-1}ba) = 1$ in the form $u(a, b) = b^n$, where b has exponent sum zero in $u(a, b)$, to get $\langle a, b : a^m = 1, b^n = u(a, b)\rangle$, which has homomorphic image $\langle a, b : a^m = b^n = u(a, b) = 1\rangle$. The results presented here and in [5, 27] indicate that the structure of finite groups of the form $(\dagger)$ can be much more complicated when $m = 2$ as opposed to the case $m \geq 4$.

2. Preliminary results

For the convenience of the reader, we recall two results from [5]. If $G = G(\alpha, n)$, then it was shown in [5, §2] that G has a normal subgroup N of index 2 with presentation

$$\langle b, d, e : b^n = (eb)^n = [d, e^{\alpha-1}] = 1, b^{-1}db = e, b^{-1}eb = e^{\alpha}d\rangle$$

and that $G' = \langle d, e\rangle$. Theorem 1.1(ii) follows immediately from this and the following result [5, Proposition 2.2].

Proposition 2.1. $[G' : G''] = v_n(\alpha)$.

A crucial observation, which we use extensively, is that certain cyclically presented groups $M(\alpha, n)$ are embedded in the $G(\alpha, n)$; this gives rise to the following [5, Proposition 4.6].

Proposition 2.2. *G is finite if and only if the homomorphic image $M = M(\alpha, n)$ of $F(2, n)$ with presentation*

$$\left\langle x_1, x_2, \dots, x_n : \prod_{j=1}^{n} x_j = x_i^{\alpha-1} = 1, \; x_i = x_{i+1}x_{i+2} \right\rangle$$

is finite. Moreover, if M is soluble of derived length t, then G is soluble of derived length $t+1$ or $t+2$. If n is even, then M has presentation

$$\langle x_1, x_2, \dots, x_n : x_i^{\alpha-1} = 1, x_i = x_{i+1}x_{i+2}\rangle.$$

In particular, if G is infinite then $F(2, n)$ is infinite.

3. Proof of Theorem 1.1

In this section we prove Theorem 1.1. First suppose that $G = G(3, n)$ with presentation

$$\langle a, b, : a^2 = b^n = ab^{-1}ab(abab^{-1})^2ab^2ab^{-2} = 1\rangle.$$

As in §2, let N be the normal subgroup of index 2 in G with presentation

$$\langle b, d, e : b^n = (eb)^n = [d, e^2] = 1, \; b^{-1}db = e, \; b^{-1}eb = e^3d\rangle,$$

where $G' = \langle d, e\rangle$. Let $z = [d, e]$. Then we have

Lemma 3.1. *z is central in G'.*

Proof. Note that since $[d, e^2] = [e, e^2] = 1$, e^2 is central in G', and so $b^{-1}e^2b$ and be^2b^{-1} are central in G', i.e., $(e^3d)^2$ and d^2 are central in G'. Thus

$$\begin{aligned} d^{-1}(e^{-4}(e^3d)^2e^{-2}d^{-2})d &= d^{-1}e^{-1}de^3de^{-2}d^{-1} \\ &= d^{-1}e^{-1}dede^2e^{-2}d^{-1} = d^{-1}e^{-1}de \end{aligned}$$

is central in G' as required. □

In particular, $\langle z\rangle = \langle d^{-1}e^{-1}de\rangle$ is normal in G', and so we have also proved

Lemma 3.2. $G'' = \langle z\rangle$.

Since e^2 is central in G', we now have

$$ze^{-1}ze = (d^{-1}e^{-1}de)e^{-1}(d^{-1}e^{-1}de)e = [d, e^2] = 1,$$

and Lemma 3.1 then yields that $z^2 = 1$. Proposition 2.1 and Lemma 3.2 give that $[G' : \langle z\rangle] = v_n(3)$, and so $|G'| = v_n(3)$ or $2v_n(3)$. If 3 does not divide n, then $v_n(3)$ is odd, and so $e^p \in \langle z\rangle$ for some odd p. So $(e^2)^{(p+1)/2} = e^{p+1}$ $(= e$ or $ez)$ is central in N, and thus e is central in N and N is abelian; so, if 3 does not divide n, then $G(3, n)$ is metabelian of order $2nv_n(3)$. On the other hand, if 3 does divide n, then, since $G(3, 3)$ has derived length 3 by [5, Theorem A(ii)], $G(3, n)$ has derived length 3, and so we have proved Theorem 1.1(iii).

We now prove Theorem 1.1(iv) concerning the groups $G = G(4, n)$ by investigating the groups $M = M(4, n)$ and then applying Proposition 2.2. Now M has the presentation

$$\left\langle x_1, x_2, \ldots, x_n : \prod_{j=1}^{n} x_j = x_i^3 = 1, \ x_i = x_{i+1}x_{i+2} \right\rangle.$$

We shall show that $x_i = x_{i+8}$ $(1 \le i \le 8)$ in M, which means that we need only consider eight cases, namely, the eight possible values of $n \pmod 8$. We first prove

Proposition 3.3. (i) $(x_ix_{i+1})^3 = 1$ $(1 \le i \le n)$; (ii) $(x_ix_{i+1}^{-1})^3 = 1$ $(1 \le i \le n)$.

Proof. To prove (i), note that $x_ix_{i+1} = x_{i-1}$ and $x_{i-1}^3 = 1$; to prove (ii), note that $x_ix_{i+1}^{-1} = x_ix_{i+1}^{-1}x_i^{-1}x_i = x_ix_{i-1}^{-1}x_i = x_ix_{i-1}^{-1}(x_{i-1}x_i)^{-1}x_{i-1}x_i^{-1}$, so that (ii) now follows from (i). □

Using Proposition 3.3, we have

$$\begin{aligned} x_n &= x_1x_2, \qquad x_{n-1} = x_nx_1 = x_1x_2x_1, \\ x_{n-2} &= x_{n-1}x_n = x_1x_2x_1^2x_2 = x_1x_2x_1^{-1}x_2, \\ x_{n-3} &= x_{n-2}x_{n-1} = x_1x_2x_1^{-1}x_2x_1x_2x_1 \\ &= x_1x_2x_1^{-1}(x_2x_1)^{-1} = (x_1x_2)^2x_2 = x_2^{-1}x_1^{-1}x_2. \end{aligned}$$

Continuing in this way yields

$$x_{n-4} = x_1^{-1}x_2^{-1}, \quad x_{n-5} = x_2^{-1}x_1, \quad x_{n-6} = x_2, \quad x_{n-7} = x_1.$$

So $x_i = x_{i+8}$ for all i; but we also have $x_i = x_{i+n}$, so that $x_i = x_{i+(n,8)}$ for all i. If n is odd, we immediately deduce that $x_i = x_j$ for any i and j, so that M is trivial and G is metabelian of order $2nv_n(4)$ by Propositions 2.1 and 2.2.

If $n \equiv 2$ or $n \equiv 6 \pmod 8$, then we have $x_i = x_{i+2}$ for all i, and so $x_i = x_{i+1}x_{i+2} = x_{i+1}x_i$ for all i, and M is trivial; if $n \equiv 4 \pmod 8$ then $x_i = x_{i+4}$ for all i, and we have that

$$x_1^3 = x_2^3 = x_3^3 = x_4^3 = 1, \qquad x_1 = x_2x_3,$$
$$x_2 = x_3x_4, \qquad x_3 = x_4x_1, \qquad x_4 = x_1x_2.$$

We now have $x_1 = x_2x_3 = x_3x_4x_3 = x_4x_1x_4^2x_1$, so that $x_4x_1x_4^{-1} = 1$, and hence $x_1 = 1$; similarly, $x_2 = x_3 = x_4 = 1$ and M is trivial; so, again, G is metabelian of order $2nv_n(4)$.

If $n \equiv 0 \pmod 8$, we have the following presentation for M:

$$\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 : x_1^3 = x_2^3 = x_3^3 = x_4^3 = x_5^3 = x_6^3 = x_7^3 = x_8^3 = 1,$$
$$x_1 = x_2x_3, x_2 = x_3x_4, \ x_3 = x_4x_5, \ x_4 = x_5x_6, \ x_5 = x_6x_7,$$
$$x_6 = x_7x_8, \ x_7 = x_8x_1, \ x_8 = x_1x_2\rangle.$$

We may eliminate generators and modify the relations to obtain

$$\langle x_1, x_2 : x_1^3 = x_2^3 = (x_1x_2^{-1})^3 = (x_1x_2)^3 = 1\rangle,$$

which is a presentation for a metabelian group of order 27. Thus M has derived length 2, and so G has derived length 3 or 4 in this case by Proposition 2.2.

In order to investigate $G = G(5, n)$, we consider the group $M = M(5, n)$ with presentation

$$\left\langle x_1, x_2, \ldots, x_n : \prod_{j=1}^{n} x_j = x_i^4 = 1, x_i = x_{i+1}x_{i+2} \right\rangle.$$

We shall show that $x_i = x_{i+12}$ in M, so that we will only have to consider twelve cases, corresponding to the possible values of $n \pmod{12}$. In fact, since $x_i = x_{i+n}$, we have $x_i = x_{i+(12,n)}$, so we only need to consider the cases $n \equiv 0, 1, 2, 3, 4$, and $6 \pmod{12}$. First we have

Proposition 3.4. (i) $(x_ix_{i+1})^4 = 1$;
(ii) $(x_{i+1}x_i^2)^4 = 1$;
(iii) $(x_ix_{i+1}^{-1})^4 = 1$;
(iv) $(x_{i+1}^2x_i)^4 = 1$.

Proof. Part (i) follows from $x_ix_{i+1} = x_{i-1}$ and $x_{i-1}^4 = 1$; since $(x_{i-1}x_i)^4 = 1$ and $x_{i-1}x_i = x_ix_{i+1}x_i$, we have part (ii). Part (iii) follows from $x_{i+2}^4 = 1$ and $x_{i+2} = x_{i+1}^{-1}x_i$; since $(x_{i+2}^{-1}x_{i+1})^4 = 1$ and $x_{i+2}^{-1} = x_i^{-1}x_{i+1}$, we have $(x_i^{-1}x_{i+1}^2)^4 = 1$, and then part (iv) follows from $x_{i+1}^4 = 1$. □

Using Proposition 3.4 and arguing similarly to the case $\alpha = 4$ gives $x_n = x_1x_2$, $x_{n-1} = x_1x_2x_1$, $x_{n-2} = x_1x_2x_1^2x_2$, $x_{n-3} = x_1x_2x_1^2x_2x_1x_2x_1$, $x_{n-4} = x_1^{-1}x_2^{-1}x_1x_2x_1^{-1}x_2x_1$, $x_{n-5} = x_1^{-1}x_2^{-1}x_1x_2x_1x_2^{-1}x_1^{-1}x_2x_1$, $x_{n-6} = x_1^{-1}x_2^{-1}x_1x_2^2x_1$, $x_{n-7} = x_2^2x_1x_2^{-1}x_1$, $x_{n-8} = x_1^{-1}x_2^2$, $x_{n-9} = x_2^{-1}x_1$, $x_{n-10} = x_2$, and $x_{n-11} = x_1$. So $x_i = x_{i+12}$ for all i, and, as remarked above, $x_i = x_{i+(12,n)}$ for all i. We now consider the cases $n \equiv 0, 1, 2, 3, 4$, and 6 (mod 12) separately.

If $n \equiv 1, 2$ or 4 (mod 12), a straightforward argument shows that M is trivial, and so G is metabelian of order $2nv_n(5)$ by Propositions 2.1 and 2.2. If $n \equiv 3$ (mod 12), say $n = 3m$ where m is odd, then the relation $x_1x_2\cdots x_n = 1$ gives that $(x_1x_2x_3)^m = 1$, and then, since $x_2x_3 = x_1$, that $x_1^{2m} = 1$. Since m is odd and $x_1^4 = 1$, we have that $x_1^2 = 1$ and get the presentation

$$\langle x_1, x_2, x_3 : x_1 = x_2x_3,\ x_2 = x_3x_1,\ x_3 = x_1x_2,\ x_1^2 = x_2^2 = x_3^2 = 1\rangle$$

for M, which is readily seen to define the elementary abelian group of order 4. So M is abelian, and hence G is soluble of derived length 2 or 3 by Proposition 2.2.

If $n \equiv 6$ (mod 12) we have the presentation

$$\langle x_1, x_2, x_3, x_4, x_5, x_6 : x_1 = x_2x_3,\ x_2 = x_3x_4,\ x_3 = x_4x_5,\ x_4 = x_5x_6,\\ x_5 = x_6x_1,\ x_6 = x_1x_2,\ x_1^4 = x_2^4 = x_3^4 = x_4^4 = x_5^4 = x_6^4 = 1\rangle$$

for M. We may eliminate generators and modify the relations to get

$$\langle x_1, x_2 : x_1x_2^2 = x_2^2x_1,\ x_2x_1^2 = x_1^2x_2,\ x_1^4 = x_2^4 = (x_1x_2)^4 = 1\rangle.$$

We see that $Z = \langle x_1^2, x_2^2\rangle$ is a central subgroup of order 4 and M/Z has presentation

$$\langle x_1, x_2 : x_1^2 = x_2^2 = (x_1x_2)^4 = 1\rangle,$$

so that M/Z is dihedral of order 8 and M is therefore metabelian of order 32. So, by Proposition 2.2, G is a finite soluble group of derived length 3 or 4.

If $n \equiv 0$ (mod 12) we may use the programs described in [20, 22] to show that M is infinite; in fact, $M/M' \cong (C_4)^4$, $M'/M'' \cong C_4 \times (C_2)^4$, and $M''/M''' \cong C_4 \times (C_\infty)^4$, and so G is infinite.

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References

1. G. Baumslag, J. W. Morgan, and P. B. Shalen, *Generalized triangle groups*, Math. Proc. Cambridge Philos. Soc. **102** (1987), 25–31.

2. A. M. Brunner, *The determination of Fibonacci groups*, Bull. Austral. Math. Soc. **11** (1974), 11–14.

3. C. M. Campbell, H. S. M. Coxeter, and E. F. Robertson, *Some families of finite groups having two generators and two relations*, Proc. Roy. Soc. London Sect. A **357** (1977), 423–438.

4. C. M. Campbell, P. M. Heggie, E. F. Robertson, and R. M. Thomas, *One-relator products of cyclic groups and Fibonacci-like sequences*, Applications of Fibonacci Numbers (G. E. Bergum et al., eds.), Kluwer Academic Publishers, 1991, pp. 63–68.

5. ——, *Finite one-relator products of two cyclic groups with the relator of arbitrary length*, J. Austral. Math. Soc. (to appear).

6. C. M. Campbell, E. F. Robertson, and R. M. Thomas, *Fibonacci numbers and groups*, Applications of Fibonacci Numbers (A. F. Horadam, A. N. Philippou, and G. E. Bergum, eds.), D. Reidel, 1987, pp. 45–59.

7. ——, *On groups related to Fibonacci groups*, Group Theory (K. N. Cheng and Y. K. Leong, eds.) de Gruyter, 1989, pp. 323–331.

8. C. M. Campbell and R. M. Thomas, *On* $(2, n)$*-groups related to Fibonacci groups*, Israel J. Math. **58** (1987), 370–380.

9. C. P. Chalk and D. L. Johnson, *The Fibonacci groups*. II, Proc. Royal Soc. Edinburgh Sect. A **77** (1977), 79–86.

10. M. D. E. Conder, *Three-relator quotients of the modular group*, Quart. J. Math. **38** (1987), 427–447.

11. J. H. Conway, *Advanced problem* 5327, Amer. Math. Monthly **72** (1965), 915.

12. J. H. Conway et al., *Solution to advanced problem* 5327, Amer. Math. Monthly **74** (1967), 91–93.

13. H. Doostie, *Fibonacci-type sequences and classes of groups*, PhD Thesis, University of St Andrews, 1988.

14. B. Fine, J. Howie, and G. Rosenberger, *One relator quotients and free products of cyclics*, Proc. Amer. Math. Soc. **102** (1988), 249–254.

15. ——, *Ree-Mendelsohn pairs in generalized triangle groups*, Comm. Algebra **17** (1989), 251–258.

16. B. Fine, F. Levin, and G. Rosenberger, *Free subgroups and decompositions of one relator products of cyclics. Part* 1: *The Tits alternative*, Arch. Math. **50** (1988), 97–109.

17. ——, *Free subgroups and decompositions of one relator products of cyclics. Part* 2: *Normal torsion-free subgroups and fpa decompositions*, J. Indian Math. Soc. **49** (1988), 237–247.

18. B. Fine and G. Rosenberger, *A note on generalized triangle groups*, Abh. Math. Sem. Univ. Hamburg **56** (1986), 233–244.

19. ——, *Complex representations and one relator products of cyclics*, Contemp. Math., vol. 74, Amer. Math. Soc., Providence, RI, 1988, pp. 131–147.

20. G. Havas, *A Reidemeister-Schreier program*, Proc. Second Internat. Conf. Theory of Groups, Canberra 1973 (M. F. Newman, ed.), Lecture Notes in Math., vol. 372, Springer-Verlag, Berlin and New York, 1974, pp. 347–356.

21. ——, *Computer-aided determination of a Fibonacci group*, Bull. Austral. Math. Soc. **15** (1976), 297–305.

22. G. Havas, P. E. Kenne, J. S. Richardson, and E. F. Robertson, *A Tietze transformation program*, Computational Group Theory (M. D. Atkinson, ed.), Academic Press, 1984, pp. 69–73.

23. H. Helling, A. C. Kim, and J. L. Mennicke, *On Fibonacci groups*, preprint.

24. G. Kern-Isberner and G. Rosenberger, *Normalteiler vom Geschlecht eins in freien Produkten endlicher zyklischer Gruppen*, Results in Math. **11** (1987), 272–288.

25. R. C. Lyndon, *On a family of infinite groups introduced by Conway*, unpublished.

26. M. F. Newman, *Proving a group infinite*, Arch. Math. **54** (1990), 209–211.

27. M. F. Newman and E. A. O'Brien, *A computer-aided analysis of some finitely-presented groups*, J. Austral. Math. Soc. (to appear).

28. S. J. Pride, *Groups with presentations in which each defining relator involves exactly two generators*, J. London Math. Soc. (2) **36** (1987), 245–256.

29. R. M. Thomas, *The Fibonacci groups* $F(2, 2m)$, Bull. London Math.. Soc. **21** (1989), 463–465.

30. ——, *The Fibonacci groups revisited*, Groups-St Andrews 1989 (C. M. Campbell and E. F. Robertson, eds.), vol. 2, London Math. Soc. Lecture Note Ser., vol. 160, Cambridge Univ. Press, 1991, pp. 445–454.

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