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EXTENDIBILITY CRITERION FOR A PROJECTIVE MODULE OF RANK ONE OVER $R[T]$ AND $R[T, T^{-1}]$

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ABSTRACT. In this note we give a criterion for a finitely generated projective module $\mathscr{P}$ of constant rank one over $R[T]$ or $R[T, T^{-1}]$ to be extended from R in terms of invertible ideals, when R is an integral domain. We show that if I is an invertible ideal of $R[T]$ or $R[T, T^{-1}]$ such that $I \cap R \neq 0$, then I is extended from R if and only if $I \cap R$ is an invertible ideal of R.

1. INTRODUCTION

Let R be a commutative ring, and let A be the polynomial algebra $R[T]$ or the Laurent polynomial algebra $R[T, T^{-1}]$. Let $\mathscr{P}$ be a finitely generated projective A-module. We say that "$\mathscr{P}$ is *extended* from R" if there exists an R-module $\mathscr{Q}$ such that $\mathscr{P} \simeq \mathscr{Q} \otimes_R A$ as A-modules. In this note we investigate the question: when is a finitely generated projective module $\mathscr{P}$ of (constant) rank one over A extended from R? It is easy to see that for this question we can assume without loss of generality that R is a reduced ring. Hence, throughout the paper we will assume that R is a *reduced commutative* ring.

If R has only finitely many minimal prime ideals (e.g., R is an integral domain or R is a noetherian ring) then $Q(R)$, the *total quotient ring* of R, is a finite direct product of fields. In this case, since all finitely generated projective modules of (constant) rank one over $Q(R)[T]$ and $Q(R)[T, T^{-1}]$ are free, it is easy to see that there exists an invertible ideal I of A such that

(1) $I \cap R$ contains a non-zero-divisor of R,
(2) $I \simeq \mathscr{P}$ as A-modules.

See [1, Chapter II, §5] for details. Therefore, in this situation, one is reduced to consider the following question:

Question. Let R be a reduced ring with only finitely many minimal prime ideals. Let A denote the polynomial algebra $R[T]$ or the Laurent polynomial algebra $R[T, T^{-1}]$. Let I be an invertible ideal of A such that $I \cap R$ contains a non-zero-divisor of R. Then, when is I extended from R?

In this paper we settle this question as follows:

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Theorem (A). *Let R be a reduced ring with only finitely many minimal prime ideals. Let I be an invertible ideal of $R[T, T^{-1}]$ such that $J = I \cap R$ contains a non-zero-divisor of R. Then I is extended from R if J is an invertible ideal of R. Moreover, if R is an integral domain and I is extended from R, then $J = I \cap R$ is an invertible ideal of R.*

Theorem (B). *Let R be a reduced ring with only finitely many minimal prime ideals. Let I be an invertible ideal of $R[T]$ such that $J = I \cap R$ contains a non-zero-divisor of R. Then I is extended from R if and only if J is an invertible ideal of R.*

We also give an example of a reduced noetherian ring and an invertible ideal I of $A = R[T, T^{-1}]$ such that $J = I \cap R$ contains a non-zero-divisor of R, I is extended from R as an A-module, but J is not an invertible ideal of R.

In case of an ideal I of $A = R[T]$ or $R[T, T^{-1}]$, there are naturally two notions of extendibility, namely,

(1) *ideal-extendibility*, i.e., $I = \mathcal{J} A$ for some ideal $\mathcal{J}$ of R,
(2) *module-extendibility*, i.e., there exists an R-module M such that $I \approx M \otimes_R A$ as A-modules.

Obviously ideal-extendibility implies module-extendibility, but the converse need not be true.

Theorem (A) and Theorem (B) are proved by first showing that if $A = R[T, T^{-1}]$ (R a domain) or $A = R[T]$ (R a reduced ring), then for an ideal I of A the two notions of extendibility are equivalent if $I \cap R$ contains a non-zero-divisor (Lemmas 2.4 and 2.8). Example 2.7 shows that for $A = R[T, T^{-1}]$, the two notions need not be the same even for an invertible ideal I of A containing a non-zero-divisor of R if R is not a domain.

2. Extendibility criterion

In this section we will prove Theorems (A) and (B) stated above (Theorems 2.11 and 2.13, respectively). We begin with the following definition:

Definition 2.1. Let A be a reduced ring, and let $Q(A)$ denote the total quotient ring of A. An A-submodule M of $Q(A)$ is said to be *invertible* if there exists an A-submodule N of $Q(A)$ such that $MN = A$.

We note that such an N is unique and we denote it by M^{-1}. If an ideal I of A is invertible, we say that I is an *invertible ideal* of A.

Let B be an A-subalgebra of $Q(A)$, and let I be an invertible ideal of A. Then it follows immediately from the definition that IB is an invertible ideal of B.

Now we state a lemma, a proof of which can be found in [1, Chapter II, §5].

Lemma 2.2. *Let A be a reduced ring and S be a multiplicative set of non-zero-divisors of A. Let $B = S^{-1}A$. If all finitely generated projective B-modules of constant rank one are free, then given a finitely generated projective A-module $\mathcal{P}$ of constant rank one there exists an invertible ideal I of A such that $I \cap S \neq \varnothing$ and $I \simeq \mathcal{P}$ as A-modules.*

As a consequence of the above lemma we have the following:

Lemma 2.3. *Let R be a reduced ring with only finitely many minimal prime ideals. Let S denote the set of all non-zero-divisors of R. Let $A = R[T]$ or $R[T, T^{-1}]$, and let $\mathscr{P}$ be a finitely generated projective A-module of constant rank one. Then there exists an invertible ideal I of A such that $I \cap S \neq \varnothing$ and $I \simeq \mathscr{P}$.*

Now we prove Theorems (A) and (B) stated in the introduction. For the proof of these theorems we need some lemmas.

Lemma 2.4. *Let R be a domain, and let I be a finitely generated ideal of $R[T, T^{-1}]$. Assume that $J = I \cap R$ is nonzero. Then the following statements are equivalent*:

(1) $I = JR[T, T^{-1}]$.
(2) $I \simeq J \otimes_R R[T, T^{-1}]$ *as* $R[T, T^{-1}]$*-modules.*
(3) *There exists an R-module M such that $I \simeq M \otimes_R R[T, T^{-1}]$ as $R[T, T^{-1}]$-modules.*

Proof. The implications $1 \Rightarrow 2$ and $2 \Rightarrow 3$ hold for any ring R (not necessarily a domain). So it remains to prove the implication $3 \Rightarrow 1$.

Let M be an R-module such that $I \simeq M \otimes_R R[T, T^{-1}]$ as $R[T, T^{-1}]$-modules. Since I is a finitely generated nonzero ideal of $R[T, T^{-1}]$ and $R[T, T^{-1}]$ is a free R-module, it follows that M is a finitely generated torsion free R-module of rank one. Hence there exists a finitely generated nonzero ideal $\mathscr{I}$ of R such that $M \simeq \mathscr{I}$ as R-modules. Thus

$$\mathscr{I}R[T, T^{-1}] \simeq \mathscr{I} \otimes_R R[T, T^{-1}] \simeq M \otimes_R R[T, T^{-1}] \simeq I$$

as $R[T, T^{-1}]$-modules. Let $\theta\colon \mathscr{I}R[T, T^{-1}] \to I$ be an isomorphism. Let $b \in \mathscr{I}$ be a nonzero element of R. Then we claim that $bI = \theta(b)\mathscr{I}R[T, T^{-1}]$.

Let $g \in I$ and $h \in \mathscr{I}R[T, T^{-1}]$ be such that $\theta(h) = g$. Then $bg = b\theta(h) = \theta(bh) = h\theta(b)$ and this proves the claim.

Let $c \in I$ be a nonzero element of R. Then $cb = \theta(b)f$ for some $f \in \mathscr{I}R[T, T^{-1}]$. But since R is a domain, this shows that $\theta(b)T^n = a \in R$ for some integer n. Now the equality $bI = \theta(b)\mathscr{I}R[T, T^{-1}] = a\mathscr{I}R[T, T^{-1}]$ gives that $bJ = b(I \cap R) = a\mathscr{I}$. Therefore $bI = bJR[T, T^{-1}]$ and hence $I = JR[T, T^{-1}]$. $\square$

Remark 2.5. Lemma 2.4 is true if R is a finite direct product of domains and $I \cap R$ contains a non-zero-divisor of R.

Remark 2.6. The following example shows that Lemma 2.4 need not be true if R is not a direct product of domains.

Example 2.7. Let $R = k[\![X, Y]\!]/(XY) = k[\![x, y]\!]$. Let $f = x + yT$ be an element of $R[T, T^{-1}]$ and let $I = fR[T, T^{-1}]$. Then it is easy to see that $I \cap R = (x^2, y^2)$, which contains a non-zero-divisor $x^2 - y^2$. Moreover, since f is a non-zero-divisor of $R[T, T^{-1}]$, the ideal I is a free module of rank one over $R[T, T^{-1}]$ and hence it is extended from R as an $R[T, T^{-1}]$-module. But obviously $fR[T, T^{-1}] \neq (x^2, y^2)R[T, T^{-1}]$ as (x^2, y^2) is not an invertible ideal of R. $\square$

In the case of a polynomial algebra $R[T]$ we get the following generalisation of Lemma 2.4.

Lemma 2.8. *Let R be a reduced ring, and let I be a finitely generated ideal of $R[T]$. Assume that $J = I \cap R$ contains a non-zero-divisor. Then the following statements are equivalent*:

(1) $I = JR[T]$.
(2) $I \simeq J \otimes_R R[T]$ *as* $R[T]$*-modules.*
(3) *There exists an* R*-module* M *such that* $I \simeq M \otimes_R R[T]$ *as* $R[T]$*-modules.*

Proof. As above the implications $1 \Rightarrow 2$ and $2 \Rightarrow 3$ are obvious. Thus it remains to prove $3 \Rightarrow 1$.

Let $s \in J$ be a non-zero-divisor of R. Then it is easy to see that M is a finitely generated torsion free R-module and M_s is a free R_s-module of rank one. Therefore there exists a finitely generated ideal $\mathcal{J}$ of R such that $\mathcal{J} \simeq M$ as R-modules. Moreover, since $\mathcal{J}_s$ is a free R_s-module of rank one, without loss of generality we can assume that $t = s^n \in \mathcal{J}$ for some positive integer n. Let $\theta: \mathcal{J}R[T] \to I$ be an isomorphism of $R[T]$-modules. Then we claim that $tI = \theta(t)\mathcal{J}R[T]$.

Let $g \in I$ and $f \in \mathcal{J}R[T]$ be such that $\theta(f) = g$. Then $tg = t\theta(f) = \theta(tf) = f\theta(t)$. This proves the claim.

Since $s \in I$, the equality $tI = \theta(t)\mathcal{J}R[T]$ shows that $ts = \theta(t)g$ for some $g \in \mathcal{J}R[T]$. Now by Lemma 2.9 (stated below) we have $\theta(t) \in R$. Therefore

$$tJ = tI \cap R = \theta(t)\mathcal{J}R[T] \cap R = \theta(t)\mathcal{J}.$$

Hence $tI = tJR[T]$. But t is a non-zero-divisor of R. Therefore $I = JR[T]$. □

Lemma 2.9. *Let R be a reduced ring and s be a non-zero-divisor of R. Let $f \in R[T]$ be such that $s \in fR[T]$. Then $f \in R$.*

Proof. Let $s = f(T)g(T)$ for some $g(T) \in R[T]$. Write $f(T) = a_0 + a_1T + \cdots + a_nT^n$ for some $a_i \in R$, $0 \le i \le n$, with $a_n \neq 0$. We want to show that $n = 0$.

Since $s = f(0)g(0) = a_0g(0)$ and s is a non-zero-divisor, a_0 is either a unit or a non-zero-divisor in R. If $n > 0$ then since R is reduced there exists a minimal prime ideal p of R such that $a_n \notin p$. Since a_0 is a unit or a non-zero-divisor, obviously $a_0 \notin p$. Let 'bar' denote "modulo p". Then we have $\overline{s} = \overline{f}(T)\overline{g}(T)$ in $\overline{R}[T]$. But since $\overline{f}(T)$ is a polynomial of positive degree this is absurd and hence $n = 0$. □

When R is reduced (but not necessarily a domain) one has the following weaker version of Lemma 2.4.

Lemma 2.10. *Let R be a reduced ring, and let I be a finitely generated ideal of $R[T, T^{-1}]$ such that $J = I \cap R$ contains a non-zero-divisor of R. If I is extended from R as a module, then there exists an element f of $R[T, T^{-1}]$ such that f is not a zero-divisor of $R[T, T^{-1}]$ and fI is extended from R as an ideal.*

This easily follows from the proof of Lemma 2.4.

Now we prove the main theorems.

Theorem 2.11. *Let R be a reduced ring with only finitely many minimal prime ideals. Let I be an invertible ideal of $R[T, T^{-1}]$ such that $J = I \cap R$ contains a non-zero-divisor of R. Then I is extended from R if J is an invertible ideal of R. Moreover, if R is an integral domain and I is extended from R, then $J = I \cap R$ is an invertible ideal of R.*

Proof. Let I be an invertible ideal of $R[T, T^{-1}]$ such that $J = I \cap R$ contains a non-zero-divisor s of R and J is an invertible ideal of R. If $I = R[T, T^{-1}]$ then clearly $I = JR[T, T^{-1}]$, where $J = I \cap R = R$. So we can assume that I is a proper ideal of $R[T, T^{-1}]$. Now we will show that $I = JR[T, T^{-1}]$. Clearly $JR[T, T^{-1}] \subseteq I$ and to show the equality it is enough to show that for every maximal ideal $\mathfrak{I}$ of R, $J_{\mathfrak{I}}R_{\mathfrak{I}}[T, T^{-1}] = I_{\mathfrak{I}}$. But then $I_{\mathfrak{I}} \cap R_{\mathfrak{I}} = J_{\mathfrak{I}}$, and as $J_{\mathfrak{I}}$ is an invertible ideal in the local ring $R_{\mathfrak{I}}$, $J_{\mathfrak{I}}$ is principal and hence $J_{\mathfrak{I}} = tR_{\mathfrak{I}}$ for some $t \in R$ which is not a zero-divisor of R. Thus by replacing R with $R_{\mathfrak{I}}$ we are reduced to proving the following:

Let R be a reduced local ring with only finitely many minimal prime ideals. Let I be an invertible ideal of $R[T, T^{-1}]$ which is a proper ideal such that $I \cap R = tR$ for some non-zero-divisor t of R. Then $I = tR[T, T^{-1}]$.

Since I is an invertible ideal of $R[T, T^{-1}]$ containing a non-zero-divisor t of R, it is easy to see that the canonical epimorphism $I/(T-1)I \to I + (T-1)/(T-1)$ is an isomorphism and hence

$$(*) \qquad I/(T-1)I \simeq I + (T-1)/(T-1) \simeq I/I \cap (T-1).$$

This shows that $I + (T-1)/(T-1)$ is an invertible ideal of R which contains the element t of R.

If $I \neq tR[T, T^{-1}]$ then there exists an element $g_1 \in R[T, T^{-1}]$ such that $g_1 \in I \setminus tR[T, T^{-1}]$. Without loss of generality we may assume that g_1 is a polynomial in $R[T]$ and is of least degree (among such elements of I). Let us write g_1 as $g_1 = a_0 + a_1(T-1) + \cdots + a_r(T-1)^r$ with $a_i \in R$ and $a_r \neq 0$. Obviously $t \nmid a_0$. Otherwise $a_0 = ta$ for some $a \in R$. Hence $g_1 - a_0 = g_1 - ta = (T-1)f$ for some $f \in R[T]$. As $T-1$ is a non-zero-divisor modulo I (by $(*)$) we have $f \in I \setminus tR[T, T^{-1}]$. But $\deg f < \deg g_1$, contradicting the minimality of degree of g_1. Hence $t \nmid a_0$.

Let $\{t, g_1, g_2, \ldots, g_n\} \subseteq R[T, T^{-1}]$ be a set of generators of I, where $g_i \in R[T]$ for $1 \leq i \leq n$. Then $I + (T-1)/(T-1) = (t, g_1(1) = a_0, g_2(1), \ldots, g_n(1))$. Since $I + (T-1)/(T-1)$ is an invertible ideal of R and R is local, $I + (T-1)/(T-1)$ is a principal ideal of R generated, say, by b and $b \in \{t, a_0, g_2(1), \ldots, g_n(1)\}$. But since $t \nmid a_0$ we have $t \nmid b$. So $b = g_i(1)$ for some i, $1 \leq i \leq n$. Moreover $t/b = d$ belongs to the maximal ideal of R.

Let $\overline{R}$ be the normalisation of R in its total quotient ring. Note that $\overline{R}$ is a finite direct product of domains. Since $\overline{R}$ is normal and $I\overline{R}[T, T^{-1}]$ is invertible, it is extended from $\overline{R}$ as a module. Therefore, as $I\overline{R}[T, T^{-1}]$ contains a non-zero-divisor of $\overline{R}$, namely t, by Lemma 2.4 and Remark 2.5, $I\overline{R}[T, T^{-1}] = L\overline{R}[T, T^{-1}]$, where $L = I\overline{R}[T, T^{-1}] \cap \overline{R}$.

Let $\{a_1, a_2, \ldots, a_m\} \subseteq \overline{R}$ be a set of generators for L. Recall that $\{t = g_0, g_1, \ldots, g_n\}$ is a set of generators for $I\overline{R}[T, T^{-1}]$. Then we get the following relations:

$$g_i = \sum_{j=1}^{m} h_{ij} a_j \qquad \text{for } i = 0, 1, \ldots, n,$$

$$a_k = \sum_{l=0}^{n} f_{kl} g_l \qquad \text{for } k = 1, 2, \ldots, m,$$

for some h_{ij} and f_{kl} in $\overline{R}[T, T^{-1}]$. Let R' be the R-subalgebra of $\overline{R}$ generated by $\{a_i\} \cup$ coefficients of $\{h_{ij}, f_{kl}\}$, and let L' be the ideal of R' generated by $\{a_1, a_2, \ldots, a_m\}$. Clearly R' is a finitely generated R-subalgebra of $\overline{R}$. Therefore R' is a finite R-module and hence R' is semilocal. Then the equality $IR'[T, T^{-1}] = L'R'[T, T^{-1}]$ shows that L' is an invertible ideal of R' and hence (R' being semilocal) is a principal ideal, generated by, say, r. Thus we have $IR'[T, T^{-1}] = (t, g_1, \ldots, g_n)R'[T, T^{-1}] = rR'[T, T^{-1}]$. Therefore $rR' = (t, g_1(1), \ldots, g_n(1))R' = bR'$. Hence without loss of generality we can assume that $r = b$.

Now we claim that there exists a finitely generated R-subalgebra $\widetilde{R}$ of R' such that

(1) $I\widetilde{R}[T, T^{-1}] = b\widetilde{R}[T, T^{-1}]$,

(2) d^l $(d = t/b)$ $\in \mathscr{C}_{\widetilde{R}/R}$ for some positive integer l, where $\mathscr{C}_{\widetilde{R}/R}$ denotes the conductor ideal of $\widetilde{R}$ in R.

We will now complete the proof of the theorem by assuming this claim.

Since $(t = g_0, g_1, \ldots, g_n)\widetilde{R}[T, T^{-1}] = b\widetilde{R}[T, T^{-1}]$, we can write $b = \sum_{i=0}^{n} h_i g_i$, where $h_i \in \widetilde{R}[T, T^{-1}]$. Then for $c \in \mathscr{C} = \mathscr{C}_{\widetilde{R}/R}$, we have $cb = c\sum_{i=0}^{n} h_i g_i = \sum_{i=0}^{n}(ch_i)g_i$. Since $c \in \mathscr{C}$, we have $ch_i \in R[T, T^{-1}]$ and hence $cb \in I \cap R = tR$. This shows that $(b/t)\mathscr{C} = \mathscr{I}$ is an ideal of R. Clearly $\mathscr{I}$ is an ideal of $\widetilde{R}$ and hence $\mathscr{I} \subseteq \mathscr{C}$. This shows that $\mathscr{C} \subseteq d\mathscr{C}$ and therefore $\mathscr{C} = d\mathscr{C}$. Hence $\mathscr{C} = d\mathscr{C} = d^2\mathscr{C} = \cdots = d^l\mathscr{C} \subseteq d^l R \subseteq \mathscr{C}$. Therefore $\mathscr{C} = d^l R = d^{l+1}R$, which is absurd since d is an element of the maximal ideal of R which is a non-zero-divisor.

Therefore $I = tR[T, T^{-1}]$ as required.

Proof of the claim. Since $(t = g_0, g_1, \ldots, g_n)R'[T, T^{-1}] = bR'[T, T^{-1}]$, $g_i = bg_i'$ $(0 \le i \le n)$, where $g_i' \in R'[T, T^{-1}]$. In fact, since $g_i \in R[T]$ and b is not a zero-divisor of R', we have $g_i' \in R'[T]$. Moreover $g_0' = d$.

Let $K = b^{-1}I$. Then K is an invertible $R[T, T^{-1}]$-submodule of $R'[T, T^{-1}]$ generated by $\{g_0', g_1', \ldots, g_n'\}$. Since $KR'[T, T^{-1}] = R'[T, T^{-1}]$, we have $K^{-1} \subseteq R'[T, T^{-1}]$ and $K^{-1}R'[T, T^{-1}] = R'[T, T^{-1}]$.

Let $K^{-1} = (u_0, u_1, \ldots, u_n)R[T, T^{-1}]$, where $u_i \in R'[T, T^{-1}]$ for $0 \le i \le n$. Let $\widetilde{R}$ denote the finitely generated R-subalgebra of R' generated by the coefficients of $\{u_i\}_{i=0}^{n}$. Then $u_i \in \widetilde{R}[T, T^{-1}]$ for all i. Since R' is integral over $\widetilde{R}$ and $K^{-1}R'[T, T^{-1}] = R'[T, T^{-1}]$, we get that $K^{-1}\widetilde{R}[T, T^{-1}] = \widetilde{R}[T, T^{-1}]$. This shows that $K \subseteq \widetilde{R}[T, T^{-1}]$ and $K\widetilde{R}[T, T^{-1}] = \widetilde{R}[T, T^{-1}]$. Since $K = b^{-1}I$, we get that $I\widetilde{R}[T, T^{-1}] = b\widetilde{R}[T, T^{-1}]$. This proves the first part of the claim.

Since $\widetilde{R}$ is generated as an R-algebra by coefficients of u_i, generators of $\widetilde{R}$ as an R-module can be chosen to be elements which are monomials in coefficients of u_i. Since $\widetilde{R}$ is a finite R-module, finitely many such monomials will generate $\widetilde{R}$ as an R-module. Since $d = g_0' \in K$, we have $du_i \in R[T, T^{-1}]$ for all i, $i = 0, 1, \dots, n$. This shows that $d^l\widetilde{R} \subseteq R$ for some positive integer l. Thus the proof of the claim is complete.

Now assume that R is a domain. If I is extended from R as a module, then by Lemma 2.4, $I = JR[T, T^{-1}]$, since $J = I \cap R \neq 0$. Therefore J is an invertible ideal of R. □

Remark 2.12. If R is not necessarily a domain and if I is extended from R as a module, then under the hypothesis of the theorem we get, by Lemma 2.10, that there exists an element $f \in R[T, T^{-1}]$ such that $fI \cap R$ is an invertible ideal of R.

Theorem 2.13. *Let R be a reduced ring with only finitely many minimal prime ideals. Let I be an invertible ideal of $R[T]$ such that $J = I \cap R$ contains a non-zero-divisor of R. Then I is extended from R as a module if and only if J is an invertible ideal of R.*

Proof. If I is extended from R then, by Lemma 2.8, $I = JR(T)$. Therefore it follows that J is an invertible ideal of R. To prove the converse, without loss of generality, we can assume that I is a proper ideal.

Let us assume that J is an invertible ideal of R. Then as I contains a non-zero-divisor, say s, of R, it is easy to see that the canonical epimorphism $I/TI \to I+(T)/(T) \simeq I/I\cap(T)$ is an isomorphism. This implies that T is not a zero-divisor of $R[T]/I$. Therefore $IR[T, T^{-1}] \cap R[T] = I$. Now the ideal $IR[T, T^{-1}]$ is an invertible ideal of $R[T, T^{-1}]$ such that $IR[T, T^{-1}]\cap R = I\cap R = J$. Therefore, by Theorem 2.11, we get that $IR[T, T^{-1}] = JR[T, T^{-1}]$. This implies that for any $f \in I$, $T^n f \in JR[T]$ for some positive integer n. This shows that $f \in JR[T]$ and hence $I = JR[T]$ as required. □

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