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DERIVATIONS WITH INVERTIBLE VALUES ON A MULTILINEAR POLYNOMIAL

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ABSTRACT. Let R be a semiprime K-algebra with unity, d a nonzero derivation of R, and $f(x_1, \dots, x_t)$ a monic multilinear polynomial over K such that $d(f(a_1, \dots, a_t)) \neq 0$ for some $a_1, \dots, a_t \in R$. It is shown that if for every $r_1, \dots, r_t$ in R either $d(f(r_1, \dots, r_t)) = 0$ or $d(f(r_1, \dots, r_t))$ is invertible in R, then R is either a division ring D or $M_2(D)$, the ring of 2×2 matrices over D, unless $f(x_1, \dots, x_t)$ is a central polynomial for R.

Moreover, if $R = M_2(D)$, where $2R \neq 0$ and $f(x_1, \dots, x_t)$ is not a central polynomial for D, then d is an inner derivation of R.

In [4] Bergen, Herstein, and Lanski proved that if R is a ring with unity and $d \neq 0$ is a derivation of R such that for every $x \in R$, $d(x) = 0$ or $d(x)$ is invertible in R, then except for a special case which occurs when $2R = 0$, R must be a division ring D or $M_2(D)$, the ring of 2×2 matrices over a division ring D. In [5] Bergen and Carini gave a generalization of this result to the case of a Lie ideal. More precisely, for the semiprime case they proved: Let R be a semiprime ring with 1, U a noncentral Lie ideal of R such that $d(U) \neq 0$, and $d(u) = 0$ or $d(u)$ is invertible for every $u \in U$. Then R is either D or $M_2(D)$ for some division ring D. Moreover, if $R = M_2(D)$, where D is not commutative and $2R \neq 0$, then d must be inner.

Since by [9, Theorem 1.5] every noncentral Lie ideal of a simple ring R must contain all commutators $xy-yx$ with $x, y \in R$ except if R is of characteristic 2 and is 4-dimensional over its center, it is natural to examine what happens when the Lie ideal in Bergen and Carini's theorem is replaced by a multilinear polynomial.

Throughout this paper R always denotes a semiprime K-algebra with unity where K is a commutative ring with 1. A polynomial $f(x_1, \dots, x_t)$ in $K\{x_1, x_2, \dots\}$, the free K-algebra with indeterminates x_i, is called *monic* if $f(x_1, \dots, x_t)$ contains some monomial with coefficient 1. In this paper we shall prove the following

Main Theorem. *Let R be a semiprime K-algebra with unity, d a nonzero derivation of R, and $f(x_1, \dots, x_t)$ a monic multilinear polynomial over K such that $d(f(a_1, \dots, a_t)) \neq 0$ for some $a_i \in R$. Suppose that for every*

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$r_1, \ldots, r_t$ in R either $d(f(r_1, \ldots, r_t)) = 0$ or $d(f(r_1, \ldots, r_t))$ is invertible in R. Then R is either a division ring D or $M_2(D)$, the ring of 2×2 matrices over D, unless $f(x_1, \ldots, x_t)$ is a central polynomial for R. Moreover, if $R = M_2(D)$, where $2R \neq 0$ and $f(x_1, \ldots, x_t)$ is not a central polynomial for D, then d must be an inner derivation of R.

Given two elements $a, b \in R$, $[a, b]$ will denote the element $ab - ba$; also for two subsets A, B of R, $[A, B]$ is then the additive subgroup of R generated by all $[a, b]$ for $a \in A$ and $b \in B$. $Z(R)$ (or Z in brief) stands for the center of R. We also recall that a polynomial $f(x_1, \ldots, x_t) \in K\{x_1, x_2, \ldots\}$ is called a *central polynomial* for R if $f(r_1, \ldots, r_t) \in Z(R)$ for all $r_1, \ldots, r_t \in R$. For any subset S of R, denote by $l_R(S)$ the left annihilator of S in R; that is, $l_R(S) = \{x \in R | xS = 0\}$. We define $r_R(S)$ similarly.

We begin this paper with the following

Theorem 1. *Let R be a semiprime K-algebra with unity, d a nonzero derivation of R, and $f(x_1, \ldots, x_t)$ a monic polynomial without constant term, not necessarily multilinear, over K such that $d(f(a_1, \ldots, a_t)) \neq 0$ for some $a_i \in R$ and $f(x_1, \ldots, x_t)$ is not a central polynomial for R. Suppose that for every $r_1, \ldots, r_t \in R$ either $d(f(r_1, \ldots, r_t)) = 0$ or $d(f(r_1, \ldots, r_t))$ is invertible in R. Then R is either*

(I) *a division ring D,*
(II) *$M_2(D)$, the ring of 2×2 matrices over a division ring D, or*
(III) *$M_n(\Delta)$ for some finite-dimensional central division algebra Δ and some positive integer n.*

Moreover, if $f(b_1, \ldots, b_t)$ is an element in R of rank $m \geq 1$ for some $b_i \in R$, then R assumes (III) *only if $n \leq 2m$.*

Proof. We first claim that R is a simple ring with 1. Indeed, let I be a proper ideal of R and $y_1, \ldots, y_t \in I^2$. Then it is clear that $d(f(y_1, \ldots, y_t)) \in I$. Since either $d(f(y_1, \ldots, y_t)) = 0$ or $d(f(y_1, \ldots, y_t))$ is invertible, we have $d(f(y_1, \ldots, y_t)) = 0$ for all $y_i \in I^2$. By [12, Theorem 4] we have $d(f(x_1, \ldots, x_t))y = 0$ for all $x_i \in R$ and all $y \in I^2$. But by hypothesis $d(f(a_1, \ldots, a_t))$ is invertible; this implies $I^2 = 0$ and hence $I = 0$ by the semiprimeness of R. Thus R is a simple ring. We divide the proof into two cases.

Case 1. Assume that there exists a nonzero right ideal ρ of R such that $d(f(x_1, \ldots, x_t)) = 0$ for all $x_i \in \rho$.

Denote by $g(x_1, \ldots, x_m)$ the multilinearization of $f(x_1, \ldots, x_t)$. Since $f \neq 0$ we have $g \neq 0$. By assumption $d(g(x_1, \ldots, x_m)) = 0$ is an identity for ρ. Thus for any $u \in \rho$ we have

$$(1) \qquad \sum_{i=1}^{m} g(ux_1, \ldots, ud(x_i) + d(u)x_i, \ldots, ux_m) = 0$$

for all $x_i \in R$. If d is an outer derivation of R, by Kharchenko's theorem [11] (1) is reduced to $\sum_{i=1}^{m} g(ux_1, \ldots, uy_i + d(u)x_i, \ldots, ux_m) = 0$ for all $x_i, y_i \in R$. In particular, $g(ux_1, \ldots, ux_m) = 0$ for all $x_i \in R$. If $\rho \subseteq Z(R)$, then R is just a field and hence we are done. So we may assume $\rho \nsubseteq Z(R)$. Choose an element $u \in \rho \backslash Z(R)$. Then $g(ux_1, \ldots, ux_m) = 0$ is a nontrivial generalized polynomial identity for R. Thus by Martindale's theorem [14] R

is a strongly primitive ring. Since R is a simple ring with 1, R is a finite-dimensional central simple algebra. That is, R assumes the form (III). Thus we may assume that $d = ad(b)$, the inner derivation induced by some $b \in R$. That is, $d(x) = bx - xb$ for all $x \in R$. In this case, we have

$$(2) \qquad [b, g(x_1, \ldots, x_m)] = 0 \quad \text{for all } x_i \in \rho.$$

Assume first that $(b - \alpha)\rho = 0$ for some $\alpha \in Z(R)$. Choosing $u \in \rho \backslash Z(R)$ and using (2) we obtain that $g(ux_1, \ldots, ux_m)(b - \alpha) = 0$ for all $x_i \in R$. Thus R assumes the form (III) as before. So we assume that $(b - \alpha)\rho \neq 0$ for any $\alpha \in Z(R)$. Then there exists an element $u \in \rho$ such that bu and u are linearly independent over $Z(R)$. Now by (2) we yield that $bg(ux_1, \ldots, ux_m) - g(ux_1, \ldots, ux_m)b = 0$ is a nontrivial generalized polynomial identity for R. As before, R assumes the form (III).

Case 2. Assume that $d(f(\rho)) \neq 0$ for all nonzero right ideals ρ of R.

The proof of this case is essentially that of [4, Lemma 4]. For any nonzero right ideal ρ of R we have $0 \neq d(f(\rho)) \subseteq d(\rho)\rho + \rho \subseteq d(\rho) + \rho$ since f has no constant term and $d(\rho) + \rho$ is a right ideal of R. But $0 \neq d(f(\rho))$ contains invertible values; this implies $d(\rho) + \rho = R$. Let ρ_1, ρ_2 be right ideals of R such that $0 \neq \rho_1 \subset \rho_2$. We want to prove that $\rho_2 = R$. Indeed, this will imply that R always assumes either (I) or (II). Note that $d(\rho_1) + \rho_1 = R = d(\rho_2) + \rho_2$. Choose an element $t \in \rho_2 \backslash \rho_1$. Write $t = a + d(b)$ for some $a, b \in \rho_1$. Then $d(b) \neq 0$ and $d(b) = t - a \in \rho_2$. Since bR is a nonzero right ideal of R, we have $bR + d(bR) = R$. But $d(bR) \subseteq bd(R) + d(b)R \subseteq \rho_2$, thus we have $R = bR + d(bR) \subseteq \rho_2$, and hence $R = \rho_2$ as desired. This completes the first part of the theorem.

Finally, suppose that $R = M_n(\Delta)$ as given in (III) and that $\operatorname{rank}(f(b_1, \ldots, b_t)) = m \geq 1$ for some $b_i \in R$. We want to prove $n \leq 2m$. Assume on the contrary that $n > 2m$. Since $\operatorname{rank}(f(b_1, \ldots, b_t)) = m$, $f(b_1, \ldots, b_t) = gx$ for some $x \in R$ and some idempotent $g \in R$ with rank m. Thus $d(f(b_1, \ldots, b_t)) = d(gx) = gd(gx) + d(g)gx$ and hence $\operatorname{rank}(d(f(b_1, \ldots, b_t))) \leq 2m$. So $d(f(b_1, \ldots, b_t)) = 0$. Now consider the additive subgroup A of R generated by elements of rank m assuming the form $f(u_1, \ldots, u_t)$ for some $u_i \in R$. Since $f(b_1, \ldots, b_t) \in A$, A is a noncentral additive subgroup of R. Clearly, A is invariant under special automorphisms in the sense of [7]. Thus by [7, Theorem 1] A contains a noncentral Lie ideal of R, i.e., $A \supseteq [R, R]$. But $d(A) = 0$, thus we get $d([R, R]) = 0$. Also, $\dim_Z R > 4m^2$, which implies $d = 0$, a contradiction. So $n \leq 2m$ as desired. This completes the proof.

To prove the Main Theorem we need some notation from [13]. Let S be a ring with 1 and let e_{ij} be the usual matrix units in the $n \times n$ matrix ring $M_n(S)$. Recall that for a sequence $u = (A_1, \ldots, A_k)$ in $M_n(S)$ the value of u is defined to be the product $|u| = A_1 A_2 \cdots A_k$ and u is nonvanishing if $|u| \neq 0$. For a permutation σ of $\{1, 2, \ldots, k\}$ we write $u^\sigma = (A_{\sigma(1)}, \ldots, A_{\sigma(k)})$. We call u *simple* if it has the form $u = (a_1 e_{i_1 j_1}, \ldots, a_k e_{i_k j_k})$, where $a_i \in S$, $i = 1, \ldots, k$. Finally, a simple sequence u is called *even* if for some σ, $|u^\sigma| = be_{ii} \neq 0$, and *odd* if for some σ, $|u^\sigma| = be_{ij} \neq 0$, where $i \neq j$.

Before giving the proof of the Main Theorem we need the following result which is interesting in itself. The proof of the following lemma is implicit in [13, Lemma 2, proof of Lemma 3].

Lemma. *Let S be a K-algebra with 1 and let $R = M_n(S)$, $n \geq 2$. Suppose that $h(x_1, \dots, x_t)$ is a multilinear polynomial over K such that $h(u) = 0$ for all odd simple sequences u. Then $h(x_1, \dots, x_t)$ is a central polynomial for R.*

Proof. Let $B_1, \dots, B_t \in R$ be arbitrary. Since $h(x_1, \dots, x_t)$ is multilinear, we have $h(B_1, \dots, B_t) = \sum_{l=1}^m h(u^{(l)})$, where the $u^{(l)}$ are even simple sequences because $h(u) = 0$ for any odd simple sequence u. By [13, Lemma 2] each $h(u^{(l)})$ assumes a diagonal form. Thus $h(B_1, \dots, B_t)$ always assumes a diagonal form. Write

$$h(B_1, \dots, B_t) = \sum_{j=1}^n \beta_j e_{jj},$$

where $\beta_j \in S$. For $1 < k \leq n$ and any $\delta \in S$ we have that

$$h((1+\delta e_{1k})B_1(1+\delta e_{1k})^{-1}, \dots, (1+\delta e_{1k})B_t(1+\delta e_{1k})^{-1})$$

still assumes a diagonal form. However,

$$\begin{aligned} h((1+\delta e_{1k})&B_1(1+\delta e_{1k})^{-1}, \dots, (1+\delta e_{1k})B_t(1+\delta e_{1k})^{-1}) \\ &= (1+\delta e_{1k})h(B_1, \dots, B_t)(1+\delta e_{1k})^{-1} \\ &= \sum_{j=1}^n \beta_j e_{jj} + (\delta\beta_k - \beta_1\delta)e_{1k}, \quad \text{since } (1+\delta e_{1k})^{-1} = 1 - \delta e_{1k}. \end{aligned}$$

Thus $\delta\beta_k = \beta_1\delta$ for all $\delta \in S$. In particular, set $\delta = 1$; then $\beta_1 = \beta_k$. So $\delta\beta_1 = \beta_1\delta$ for all $\delta \in S$, which implies $\beta_1 \in Z(S)$. Now $h(B_1, \dots, B_t) = \beta_1 \cdot \sum_{j=1}^n e_{jj} \in Z(R)$ as desired. This completes the proof.

Proof of Main Theorem. Assume that $f(x_1, \dots, x_t)$ is not a central polynomial for R. By Theorem 1, $R = M_n(\Delta)$ for some division ring Δ, and to prove $n \leq 2$ it suffices to show that $\operatorname{rank}(f(b_1, \dots, b_t)) = 1$ for some $b_i \in R$. By the previous lemma there exists an odd simple sequence u such that $f(u) \neq 0$. But by [13, Lemma 2] $f(u) = \mu e_{ij} \neq 0$ for some $\mu \in \Delta$, $i \neq j$; we get $\operatorname{rank}(f(u)) = 1$ as claimed.

Suppose next that $R = M_2(D)$, where $2R \neq 0$ and $f(x_1, \dots, x_t)$ is not a central polynomial for the division ring D. We want to prove that d is inner. To do this we will refer to some arguments given in [4, Lemma 8; 5, Lemma 10]. Since d is a derivation of R, d has the form:

$$d\begin{pmatrix} a & b \\ c & e \end{pmatrix} = \begin{pmatrix} \phi(a) - b\beta - \alpha c & \phi(b) + a\alpha + b\gamma - \alpha e \\ \phi(c) + \beta a - e\beta - \gamma c & \phi(e) + e\gamma - \gamma e + \beta b + c\alpha \end{pmatrix} \tag{1}$$

for all $a, b, c, e \in D$, where $\alpha, \beta, \gamma \in D$ and ϕ is a derivation of D. Furthermore, by [4, Lemma 7] d is inner on $M_2(D)$ if and only if ϕ is inner on D. Thus the aim is to prove that ϕ is inner on D. Suppose that $\alpha = 0$. Then for $\beta_1, \dots, \beta_t \in D$ we have

$$\begin{aligned} &d\left(f\left(\begin{pmatrix} \beta_1 & 0 \\ 0 & 0 \end{pmatrix}, \dots, \begin{pmatrix} \beta_t & 0 \\ 0 & 0 \end{pmatrix}\right)\right) \\ &\qquad = d\begin{pmatrix} f(\beta_1, \dots, \beta_t) & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \phi(f(\beta_1, \dots, \beta_t)) & 0 \\ \beta f(\beta_1, \dots, \beta_t) & 0 \end{pmatrix}, \end{aligned}$$

which is zero or invertible. Thus $\phi(f(\beta_1, \dots, \beta_t)) = 0$ for all $\beta_j \in D$. Let T denote the subdivision ring of D generated by all elements $f(\beta_1, \dots, \beta_t)$, where $\beta_i \in D$. Thus $\phi(T) = 0$. Since f is noncentral on D, T is then a noncentral subdivision ring of D invariant under all automorphisms. By a result of Brauer-Cartan-Hua [6], $T = D$ follows. Thus $\phi(D) = 0$, implying that ϕ is inner. So we assume from now on that $\alpha \neq 0$. By (1) we have for $a \in D$ that

$$d\begin{pmatrix} a & 0 \\ \alpha^{-1}\phi(a) & \alpha^{-1}a\alpha \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ u & v \end{pmatrix}, \tag{2}$$

where

$$u = \phi(\alpha^{-1}\phi(a)) + \beta a - \alpha^{-1}a\alpha\beta - \gamma\alpha^{-1}\phi(a),$$
$$v = \phi(\alpha^{-1}a\alpha) + \alpha^{-1}a\alpha\gamma - \gamma\alpha^{-1}a\alpha + \alpha^{-1}\phi(a)\alpha.$$

Note that for $a, b \in D$ we have

$$\begin{pmatrix} a & 0 \\ \alpha^{-1}\phi(a) & \alpha^{-1}a\alpha \end{pmatrix}\begin{pmatrix} b & 0 \\ \alpha^{-1}\phi(b) & \alpha^{-1}b\alpha \end{pmatrix} = \begin{pmatrix} ab & 0 \\ \alpha^{-1}\phi(ab) & \alpha^{-1}ab\alpha \end{pmatrix}.$$

Thus for $\beta_1, \dots, \beta_t \in D$ we have

$$\begin{aligned} d&\begin{pmatrix} f(\beta_1, \dots, \beta_t) & 0 \\ \alpha^{-1}\phi(f(\beta_1, \dots, \beta_t)) & \alpha^{-1}f(\beta_1, \dots, \beta_t)\alpha \end{pmatrix} \\ &= d\left(f\left(\begin{pmatrix} \beta_1 & 0 \\ \alpha^{-1}\phi(\beta_1) & \alpha^{-1}\beta_1\alpha \end{pmatrix}, \dots, \begin{pmatrix} \beta_t & 0 \\ \alpha^{-1}\phi(\beta_t) & \alpha^{-1}\beta_t\alpha \end{pmatrix}\right)\right), \end{aligned}$$

which is either 0 or invertible. By (2) it must be zero. Now using the same calculations given in [4, Lemma 8] we have

$$\phi(f(\beta_1, \dots, \beta_t)) = \tfrac{1}{2}[\phi(\alpha)\alpha^{-1} + \alpha\gamma\alpha^{-1},\ f(\beta_1, \dots, \beta_t)]$$

for all $\beta_1, \dots, \beta_t \in D$. Assume on the contrary that ϕ is outer on D. Since f is multilinear, we have

$$\sum_{j=1}^{t} f(\beta_1, \dots, \phi(\beta_j), \dots, \beta_t) = \tfrac{1}{2}[\phi(\alpha)\alpha^{-1} + \alpha\gamma\alpha^{-1},\ f(\beta_1, \dots, \beta_t)] \tag{3}$$

for all $\beta_i \in D$. Applying Kharchenko's theorem [11] we have that

$$\sum_{j=1}^{t} f(x_1, \dots, y_j, \dots, x_t) = \tfrac{1}{2}[\phi(\alpha)\alpha^{-1} + \alpha\gamma\alpha^{-1},\ f(x_1, \dots, x_t)] \tag{4}$$

for all $x_i, y_i \in D$. In particular, taking $y_1 = \cdots = y_t = 0$ we get

$$\tfrac{1}{2}[\phi(\alpha)\alpha^{-1} + \alpha\gamma\alpha^{-1}, f(x_1, \dots, x_t)] = 0 \quad \text{for all } x_i \in D.$$

Thus (4) is reduced to $\sum_{j=1}^{t} f(x_1, \dots, y_j, \dots, x_t) = 0$ for all $x_i, y_i \in D$. So in particular $f(x_1, \dots, x_t) = 0$ for all $x_i \in D$, a contradiction. This completes the proof of the theorem.

With the Main Theorem in hand, the following question is naturally raised: *Let R be a semiprime K-algebra and $f(x_1, \dots, x_t)$ a monic polynomial over K. Suppose that d is a derivation of R such that $d(f(x_1, \dots, x_t)) = 0$ for all $x_i \in R$. Then what can we say about the structure of R?*

We conclude this paper with a precise description for the above question.

Theorem 2. *Let R be a semiprime K-algebra with center Z, Q the Martindale two-sided quotient ring of R, and $f(x_1, \ldots, x_t)$ a monic polynomial over K. Suppose that $d(f(x_1, \ldots, x_t)) \in Z$ for all $x_1, \ldots, x_t \in R$. Then there is a ring decomposition $Q = Q_1 \oplus Q_2 \oplus Q_3$ satisfying*

(I) $d(Q_1) = 0$,
(II) Q_2 *satisfies* S_4, *the standard polynomial of degree* 4, *and*
(III) $f(x_1, \ldots, x_t)$ *is a central polynomial for* Q_3.

Proof. Denote by C the extended centroid of R; then $Z(Q) = C$. It is well known that d can be uniquely extended to Q. By [12, Theorem 3] Q and R satisfy the same differential identities. Thus we have $d(f(x_1, \ldots, x_t)) \in C$ for all $x_i \in Q$. Let $\mathscr{M}$ be any maximal ideal of B, the complete Boolean algebra of idempotents of C [2]. Then $\mathscr{M}Q$ is a d-invariant prime ideal of Q. Let $\overline{d}$ denote the canonical derivation of $\overline{Q} = Q/\mathscr{M}Q$ induced by d. Note that $Z(\overline{Q}) = (C + \mathscr{M}Q)/\mathscr{M}Q \cong C/\mathscr{M}C$. Thus $\overline{d}(f(x_1, \ldots, x_t)) \in (C + \mathscr{M}Q)/\mathscr{M}Q$ for all $x_i \in \overline{Q}$. It follows from [8, Theorem 3; Lemma 6; 10, Lemma 2] that either $f(x_1, \ldots, x_t)$ is a central polynomial for $\overline{Q}$, or $\overline{Q}$ satisfies S_4, or $\overline{d} = 0$. Thus we have $d(Q)QS_4(z_1, z_2, z_3, z_4)Q[f(x_1, \ldots, x_t), y] \subseteq \mathscr{M}Q$ for all $x_i, y, z_i \in Q$. But since $\bigcap\{\mathscr{M}Q \mid \mathscr{M}$ is any maximal ideal of $B\} = 0$, we obtain

$$d(Q)QS_4(z_1, z_2, z_3, z_4)Q[f(x_1, \ldots, x_t), y] = 0 \quad \text{for all } x_i, y, z_i \in Q.$$

By [2, Point 2] there exists an idempotent $h \in C$ such that $\{\alpha \in C \mid \alpha d(Q) = 0\} = hC$. Then $d(hQ) = hd(Q) = 0$ and

$$S_4(z_1, z_2, z_3, z_4)(1 - h)Q[f(x_1, \ldots, x_t), y] = 0$$

for all $x_i, z_i, y \in (1 - h)Q$. But $(1 - h)Q$ is still an orthogonally complete ring; there exists an idempotent $g \in (1 - h)C$ such that

$$\{\beta \in (1 - h)C \mid \beta S_4(z_1, z_2, z_3, z_4) = 0 \text{ for all } z_i \in (1 - h)Q\} = gC.$$

So gQ satisfies S_4 and $(1 - h)(1 - g)Q$ satisfies $[f(x_1, \ldots, x_t), y]$. Now set $Q_1 = hQ$, $Q_2 = gQ$, and $Q_3 = (1 - h)(1 - g)Q$. Then $Q = Q_1 \oplus Q_2 \oplus Q_3$ as desired. This completes the proof.

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