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ON VARIETIES AS HYPERPLANE SECTIONS

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ABSTRACT. Here we extend to the the singular (but locally complete intersection) case a theorem of L'vovsky giving a condition ("$h^0(N_X(-1)) \leq n+1$") forcing a variety $X \subset \mathbf{P}^n$ not to be a hyperplane section (except of cones). Then we give a partial extension of this criterion to the case of subvarieties of a Grassmannian.

Several papers (both classical and recent) are devoted to the following natural question. Let $X \subset \mathbf{P}^n$ be a variety and consider $\mathbf{P}^n$ as a hyperplane H of $\mathbf{P}^{n+1}$; Is there a variety $W \subset \mathbf{P}^{n+1}$, W not a cone with base X, such that $X = W \cap H$ (scheme-theoretic intersection)? Here, stimulated from the reading of [L], we prove the following result (over an algebraically closed base field $\mathbf{K}$).

Theorem 0.1. *Let $X \subset \mathbf{P}^n$ be an integral nondegenerate locally complete intersection variety, and let N_X be its normal sheaf in $\mathbf{P}^n$; consider $\mathbf{P}^n$ as a hyperplane H of $\mathbf{P}^{n+1}$. Assume $\deg(X) > 2$ and $h^0(X, N_X(-1)) \leq n+1$. Let $W \subset \mathbf{P}^{n+1}$ be a variety with $X = H \cap W$ (as schemes). Assume W locally a complete intersection along X. Then W is a cone with base X.*

In [L] Theorem 0.1 was proved (in a different way) under the assumption that X is smooth and $\text{char}(\mathbf{K}) = 0$. Our proof of Theorem 0.1 depends on a result of Kleppe on deformation theory (see [K1–K3, P]); here we use only an elementary part of [K1]; we would happy if other mathematicians find applications for the very powerful methods of Kleppe. We stress that the assumption "$h^0(X, N_X(-1)) \leq n+1$" seems to be extremely restrictive (and Steps 1 and 2 of the proof of Theorem 0.1 support this feeling). However, this condition seems very natural and to find a nontrivial class of examples would be nice; of course, the first two steps of the proof of Theorem 0.1 rule out many situations as candidates for the examples. For instance, for numerical reasons it is not satisfied by curves in $\mathbf{P}^3$ (except the rational normal curve); for curves in $\mathbf{P}^n$ this condition is connected with other open questions (see [BC, problem 12]). The first two sections of this paper are devoted to the proof of Theorem 0.1. Then in the last section we collect a few definitions trying to find the natural

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extension of this type of result to the case of higher rank vector bundles, i.e., to the case in which the ambient variety is a Grassmannian; Proposition 3.3 is a weak version of Theorem 0.1.

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Here we collect a few remarks which will be used in the next section for the proof of Theorem 0.1 (and Lemma 1.1 is nice).

Lemma 1.1. *Fix an integral nondegenerate variety V in a projective space U and a general hyperplane L. Set $E := \{g \in \mathrm{Aut}(U)\colon g(V) = V$ and $g(P) = P$ for every $P \in L\}$. Assume $\dim(E) > 0$. Then V is a cone.*

Proof. Assume $V \neq U$. Let F be the connected component of the identity of E.

(a) Fix L and a point $P \notin L$ such that V is not a cone with P in its vertex. Assume that F fixes P. Take a general line D containing P. By the assumptions on L and P, F acts on D. The identity is the only element of $\mathrm{Aut}(D)$ fixing three different points of D. Since $D \cap V$ is finite and $\mathrm{card}(D \cap (V \cup L)) \geq 3$, while F is connected, F induces the identity on D. By the generality of D we find that F is trivial, which is a contradiction.

(b) Let U^* be the dual projective space and $V^* \subset U^*$ be the dual variety of V. Set $v := \dim(V^*)$. $\mathrm{Aut}(U)$ acts faithfully on U^*, and for this action $G := \mathrm{Aut}(V) \cap \mathrm{Aut}(U)$ sends V^* into itself. For a linear space $R \subset U$, let $R^* \subset U^*$ be its dual (hence, $\dim(R) + \dim(R^*) = \dim(U) - 1$). Take as R any linear space of dimension $v - 1$ such that $R^* \cap U^*$ is finite and the corresponding hyperplanes are tangent to V at smooth points. Take as L any hyperplane containing R but with $L \notin V^*$. Since F fixes pointwise R, F acts on R^*. Since F is connected, F fixes the finitely many points (i.e., hyperplanes of U) in $R^* \cap V^*$; since $L \neq V^*$, we are sure that there is at least one such hyperplane different from L. Since this holds also for the general linear subspace R' of L with $\dim(R') = \dim(R)$, F fixes every point of V^*. Thus F acts as the identity on the linear span M of V^*. If $M = U^*$, we have a contradiction. Assume $M \neq U^*$; then $M = A^*$ for a unique linear space $A \subset U$, $A \neq \varnothing$. By the definition of V^* for every $x \in V_{\mathrm{reg}}$, the tangent space T_xV contains A. If $\mathrm{char}(\mathbf{K}) = 0$, this is sufficient to conclude that V is a cone with vertex A. But in positive characteristic we need to work more.

(c) $G := \mathrm{Aut}(V) \cap \mathrm{Aut}(U)$ sends A into itself. Hence, if A is a point, V is a cone by part (a). In particular, this is the case if V is a curve. Hence, we may use induction on $\dim(V)$. Now assume $A \cap V \neq \varnothing$. Take L sufficiently general. Fix a one-dimensional connected subgroup F' of F. By the inductive assumption every irreducible component of $(A \cap V)_{\mathrm{red}}$ is either a point or a cone. We claim that each such irreducible component contains a point fixed by F'. Since F' is connected, it stabilizes each irreducible component of $(A \cap V)_{\mathrm{red}}$. Fix one such irreducible component B with $\dim(B) > 0$. The closure of every orbit contained in B for the action of F' is a union of orbits. Since F' is affine and one dimensional while B is complete, F' fixes at least a point of B. Hence, we may assume the claim. The proof of step (a) used only that F is connected and nontrivial. Hence, we have a contradiction.

(d) We have to handle only the case with $\dim(A) > 0$ and $V \cap A = \varnothing$. This case does not occur if V is a hypersurface of U. Now we will reduce the general case to the hypersurface case. Assume $\dim(U) \geq \dim(V) + 2$. Take a general linear space $E \subset L$ with $\dim(E) = \dim(U) - \dim(V) - 2$. Let U'' be a general linear space with $\dim(U'') = \dim(V) + 1$, and let V'' (resp. L'') be the projection of V (resp. L) from E into U''. Since F fixes pointwise L, F can be seen as a subgroup of $\mathrm{Aut}(U'')$ fixing pointwise L'' and sending V'' into itself. Note that, since $E \subset L$, E is fixed. Thus U'' and V'' are fixed; hence, varying L among the linear spaces containing E, we may assume that L'' is general in U''. Thus, we are reduced to the case in which V is a hypersurface of a projective space U''. □

Remark 1.2. Fix a nondegenerate integral variety V of codimension t in a projective space U. Let $E \subset U$ be a general linear space with $\dim(E) = t$. Assume $\deg(V) \geq t+2$. Fix $g \in \mathrm{Aut}(U)$ with $g(x) = x$ for every $X \in (V \cap E)$. We claim that $g(y) = y$ for every $y \in E$. Indeed, it is easy to check (even in positive characteristic and when the trisecant lemma fails (see, e.g., [La] and/or [B])) that $V \cap E$ contains at least $t+2$ points such that any $t+1$ of them span E—hence, the claim.

In the proof of Theorem 0.1 we will need the following lemma, which handles the case of a minimal degree variety.

Lemma 1.3. *Let V be a nondegenerate integral subvariety of the projective space U. Assume that V is locally a complete intersection, that* $\deg(V) > 2$, *and that V has minimal degree, i.e.,* $\deg(V) = \mathrm{codim}(V) + 1$. *Then* $h^0(N_V(-1)) > \dim(U) + 1$.

Proof. By the classification of minimal degree varieties and the assumptions on V, we see that V is smooth and not a hypersurface. By Euler's sequence of TU, we see that $N_V(-1)$ is spanned. Hence, if $\dim(V) = 1$, we have $h^0(N_V(-1)) = \chi(N_V(-1)) = 3(\dim(U)) - 3$. Now assume $\dim(V) > 1$. Since V is a minimal degree smooth variety, it is a projective bundle $\pi\colon V \to \mathbf{P}^1$. Since $N_V(-1)$ is spanned, we see that $R^i\pi_*(N_V) = 0$ and $H^j(\mathbf{P}^1, \pi_*(N_V(-1))) = 0$ for every $i > 0$ and $j > 0$. Hence, $h^1(N_V(-1)) = 0$. Fix a general hyperplane L of U. From the exact sequence

$$(1) \qquad 0 \to N_V(-t-1) \to N_V(-t) \to N_{V\cap L, L}(-t) \oplus \mathcal{O}_{V\cap L}(-t-1) \to 0$$

and the fact that $\deg(V) > 2$, we see first by induction on $\dim(V)$ that $h^0(N_V(-2)) = 0$ and then, since $h^1(N_V(-1)) = 0$, that $h^1(N_V(-t)) = h^1(N_V(-t-1))$ for every $t \geq 2$. Hence, $h^1(N_V(-2)) = 0$. Hence by (1) and induction on $\dim(V)$, we get $h^0(N_V(-1)) = 3\,\mathrm{codim}(V) + \dim(V) - 1$. □

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Proof of Theorem 0.1. Consider $\mathbf{P}^n$ as a hyperplane H of $\mathbf{P}^{n+1}$, and fix W as in the statement of Theorem 0.1. By Lemma 1.3 we may (and will) assume that X is not a minimal degree variety. Set $\mathbf{P} = \mathbf{P}^{n+1}$. The proof is divided into seven steps.

Step 1. Let R be a general hyperplane of H; set $Y := X \cap R$. Let Φ be the functor of formal deformations of X and $\mathbf{P}$ (not in H) leaving pointwise fixed Y. By [K3] its tangent space $T_{[X]}\Phi$ at the point $[X]$ is isomorphic to $H^0(X, N_{X,\mathbf{P}}(-1))$; since $N_{X,\mathbf{P}} \cong N_{X,H} \oplus \mathcal{O}_X(1)$, the assumption on $N_X(-1)$ means that $\dim(T_{[X]}\Phi) = n + 2$. Set $g := \{g \in \mathrm{Aut}(\mathbf{P})\colon g(P) = P$ for every

$P \in Y\}$ and $G' := \{g \in G \colon g(P) = P$ for every $P \in H\}$. G acts on $\mathrm{Hilb}(\mathbf{P})$, and with this action G' fixes X. This action defines a subfunctor Φ' of Φ. By Lemma 1.3 and Remark 1.2, $G = \{g \in \mathrm{Aut}(\mathbf{P}) \colon g(P) = P$ for every $P \in R\}$.

Step 2. By Lemma 1.1 the orbit GX of X under this action of G has dimension $\dim(G/G')$ with is exactly $n+2$ (hence at least $\dim(T_{[X]}\Phi))$. Let $\mathbf{L} = \{H_t\}$, $H_0 = H$, be the pencil of hyperplanes of $\mathbf{P}$ containing R. Among the deformations of X leaving pointwise fixed Y there is the one corresponding to the family $\{W \cap H_t\}$. We see that for general t there is $g \in G$ with $g(X) = W \cap H_t$.

Step 3. By assumption W is locally a complete intersection in a neighborhood of X. By Bertini theorem a general hyperplane section of W is locally a complete intersection. By semicontinuity a general hyperplane section of W satisfies the assumption on the conormal sheaf made in Theorem 0.1 for X. Thus by Step 2 we see that for any two general hyperplanes M, M' of $\mathbf{P}$, there is $g \in \mathrm{Aut}(\mathbf{P})$ with $g(W \cap M) = w \cap M'$ and $g(P) = P$ for every $P \in (M \cap M')$; call $g_{M,M'}$ such a projective transformation.

Step 4. Fix general hyperplanes M, M' of $\mathbf{P}$ and general hyperplanes A (resp. A') of M (resp. M') with $\dim(A \cap A') = n-2$. We will use M and M' with this meaning for all the remaining steps. Let $\mathscr{U}$ be the set of hyperplanes Γ of $\mathbf{P}$ for which we may pass from M' to Γ and from Γ to M as in Step 2 obtaining respectively the projective transformations $g_{M',\Gamma} \colon M' \to \Gamma$ and $g_{\Gamma,M} \colon \Gamma \to M$. Call *good* (resp. *bad*) a hyperplane $T \in \mathscr{U}$ (resp. $T \notin \mathscr{U}$). Consider the following condition $(\otimes)$:

$(\otimes)$ There is $g \in \mathrm{Aut}(\mathbf{P})$ with $g(M) = M'$, $g(A) = A'$, and $g(W \cap M) = W \cap M'$.

In this step we will show that if condition $(\otimes)$ fails, then a very strong property (which will be called *condition* (£) (see the end of this step)) holds. Set $D := A \cap A'$. Fix a general hyperplane M'' of $\mathbf{P}$ with M'' containing D; hence $D = M \cap M' \cap M''$. By the previous step $\mathrm{Aut}(\mathbf{P})$ contains elements $g := g_{M,M''}$, $h := h_{M'',M'}$ such that: $g(W \cap M) = W \cap M''$, $g(P) = P$ for every $P \in (M \cap M'')$, $h(W \cap M'') = W \cap M'$, $h(Q) = Q$ for every $Q \in (M'' \cap M')$. Set $r = h \circ g$; r depends on the choice of M''. We have $r(x) = x$ for every $x \in D$ and $r(W \cap M) = W \cap M'$. Hence, r sends A into an element of the pencil L of hyperplanes of M' containing D. Thus condition $(\otimes)$ holds if, moving M'', the linear space $r(A)$ moves in L. We will call *condition* (£) the fact that, for all general M, M', A, A', M'' as above, r does not depend on the choice of M''. In the next step we will assume (£) and prove that W is a cone. Thus in Steps 6 and 7 we will be allowed to assume condition $(\otimes)$ (and conclude the proof of Theorem 0.1).

Step 5. Here we will assume (£) and prove that W is a cone. This step will be subdivided into two parts.

(a) Consider

($) There is $T \in \mathscr{U}$ and a point $x \in M' \cap T$ with $x \notin M$ and $g_{M',M}(x) \in T$.

Now we will show that condition ($) is never satisfied. Indeed, fix T and x satisfying ($). Since $x \in (M' \cap T)$, we have $g_{M',T}(x) = x$. Since $g_{M',M}(x) \in T$, $g_{T,M}(g_{M',T}(x))$ is defined. By (£) we have $g_{T,M}(g_{M',T}(x)) = g_{M',M}(x) \in$

M, while by (\$) we have $g_{M',T}(x) = x$ and $g_{T,M}(x) \notin M$, which is a contradiction.

(b) By part (5a) we may assume (call it condition (ç)) that (£) holds and (\$) fails. Fix two general points x and y in $(M'\backslash M)$ and a general hyperplane A'' of M' with $\{x, y\} \subset A''$. Call $\mathbf{T}$ the pencil of hyperplanes of $\mathbf{P}$ containing A''. By (ç) every hyperplane T with $\{x, g_{M',M}(x) \subset T\}$ is bad (and the same for y). Since by the generality of x, y, and A'' the pencil $\mathbf{T}$ contains only finitely many bad hyperplanes (and a pencil is irreducible), we see that a hyperplane $T \in \mathbf{T}$ contains the line $L(x)$ spanned by x and $g_{M',M}(x)$ if and only if it contains the line $L(y)$ spanned by y and $g_{M',M}(y)$. Moving A'' we see that this implies that $L(x) \cap L(y) \neq \varnothing$. Taking another general $z \in M'$ we see that this implies the existence of a point $P \in \mathbf{P}$ contained in every such line $L(x)$. By the way in which these lines are defined, we see that the map $g_{M',M}$ is induced by the projection from the point $\mathbf{P}$. Since for general M, M', and M'' we have condition (£), we see that the point P does not depend on the choice of M and M'. Thus we see very easily that W is a cone with vertex P.

Step 6. Now we may assume condition $(\otimes)$. Forget the condition on the normal bundle. The thesis of Step 4 gives that the triple $(M \cap W, M \cap W \cap A, A \cap A')$ satisfies the same conditions which were sufficient for the triple (W, X, R) to prove the thesis of Step 2 and that, if W is not a cone, we have $(\otimes)$. This means that general hyperplane sections satisfy $(\otimes)$; iterating the proof we obtain that general codimension 2 linear sections satisfy $(\otimes)$, and so on.

Step 7. By the previous step we are reduced to consider the case of a curve C, spanning a linear space Π, for which condition $(\otimes)$ gives that $B := \mathrm{Aut}(C) \cap \mathrm{Aut}(\Pi)$ acts with an open orbit on the set of hyperplane sections of C (apply the thesis of Step 4 with $M = M'$) (no assumption on $h^0(N_C(-1))$. In particular, $\dim(B) \geq \dim(\Pi^*) = \dim(\Pi)$. Two cases are a priori possible:

(i) Assume $\dim(B) \leq 2$. Then C is a plane curve and X is a hypersurface; set $d := \deg(X)$. We have $h^0(X, N_X(-1)) = h^0(X, \mathscr{O}_X(d-1)) > \dim(X) + 1$ if $d > 2$.

(ii) Assume $\dim(B) > 2$. Then C must be a smooth rational normal curve (look at the group of automorphisms of its normalization). Hence, $\deg(X) = \deg(C) \leq 2$, by Lemma 1.3. □

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Let $G(t, k)$ be the Grassmannian of all $(t-1)$-dimensional projective subspaces of $\mathbf{P}^{k-1}$. First we give a geometric definition of the cone of a subvariety of $G(t, k)$.

Definition 3.1. Consider $\mathbf{P}^{k-1}$ as a hyperplane H of $\mathbf{P} := \mathbf{P}^k$; fix a point $P \in (\mathbf{P}^k \backslash H)$. Use this embedding to consider $G(t, k)$ as a suitable Schubert cell Y of $G(t, k+1)$. Let X be a reduced subscheme of $G(t, k)$. We will define the cone $C_P(X) \subset G(t, k+1)$ with vertex P and and base X. $C_P(X)$ will be a reduced subscheme of $G(t, k+1)$. Set theoretically $C_P(X)$ is defined in this way. Take $[x] \in X$, and let $x = \mathbf{P}^{t-1} \subset \mathbf{P}^k$ be the corresponding subspace; let $x' := [P, x] \subset \mathbf{P}^k$ be the t-dimensional projective subspace spanned by P and

x. Let x^* be the set of hyperplanes of x'. Define $C_P(X)$ as the union of x^* for all $[x] \in X$.

Now we collect a few properties of the cone $C_P(X)$. We will use here all the notations of Definition 3.1.

Properties 3.2. (a) *Since* $\Gamma := \{g \in \mathrm{Aut}(\mathbf{P}^{k+1})\colon g(y) = y$ *for every* $y \in H\}$ *acts transitively on* $\mathbf{P} \setminus H$, *we see that, for all* P, $P' \in (\mathbf{P} \setminus H)$, $C_P(X)$ *and* $C_{P'}(X)$ *are projectively equivalent.*

(b) $X = Y \cap C_P(X)$.

(c) *If* $g \in \mathrm{Aut}(\mathbf{P}) = \mathrm{Aut}^0(G(t, k+1))$ *is such that* $P \notin g(H)$, *then* $Y \cap (g(CP(X))$ *is projectively equivalent of* X. *Proof. For every* $[x] \in X$, *send it into* $x^* \cap g(H)$; *in this way we obtain the projective equivalence sending* X *onto* $Y \cap (g(C_P(X))$.

From now on we fix an integer $r \geq 1$ and write $G(t)$ instead of $G(r, t)$. For a subscheme X of $G(t)$, $N_{X,t}$ will denote the normal sheaf of X in $G(t)$ and $\mathcal{I}_{X,t}$ its ideal sheaf. We fix a complex flag $\cdots \subset \mathbf{P}^t \subset \mathbf{P}^{t+1} \subset \mathbf{P}^{t+2} \subset \cdots$ of a big projective space and use this flag to fix an embedding of $G(t)$ into $G(t+1)$ (and so on) as a fixed suitable Schubert cycle. Let Q_t be the tautological universal rank-r quotient bundle on $G(t)$. Note that $Q_t \cong Q_{t+1}|G(t)$ and that $N_{G(t),t+1} \cong Q_t$. We have $h^0(G(t), N_{G(t),t+1} \otimes \mathcal{I}_{G(t-1),t}) = 1$ as in the case "$r = 1$". Now fix integral varieties $X \subset G(t)$, $Y \subset G(t+1)$ with $X = Y \cap G(t)$ (as schemes), X locally a complete intersection, and Y locally a complete intersection in a neighborhood of X. Set $\Gamma := \Gamma(t) := \mathrm{Aut}^0(G(t)) = \mathrm{Aut}(\mathbf{P}^{t-1})$. Set $\gamma(X) = \dim\{g \in G(t)\colon g(X) = X$ and $g(y) = y$ for every $y \in (X \cap G(t-1))\}$ (in the definition of $\gamma(X)$ first we fix X and only after we choose a general embedding of $G(t-1)$ into $G(t)$). Set $Z := X \cap G(t-1)$ (for the sufficiently general fixed embedding used in the definition of $\gamma(X)$). The next proposition is the higher rank extension of Step 1 in the proof of Theorem 0.1 (and it has the same proof).

Proposition 3.3. *With the notation just introduced, assume* $h^0(N_{X,t} \otimes \mathcal{I}_{Z,X}) \leq t + 1 - \gamma(X)$. *Then for a general* $g \in \mathrm{Aut}^0(G(t+1))$ *there is* $h \in \mathrm{Aut}^0(G(t+1))$ *such that* $Y \cap (g(G(t))) = h(X)$.

Proof. We have

$$\begin{aligned} h^0(N_{X,t+1} \otimes \mathcal{I}_{Z,X}) &\leq h^0(N_{X,t} \otimes \mathcal{I}_{Z,X}) + h^0(X, (Q_{t+1}|X) \otimes \mathcal{I}_{Z,X}) \\ &= h^0(N_{X,t} \otimes \mathcal{I}_{Z,X}) + h^0(X, (Q_t|X) \otimes \mathcal{I}_{Z,X}). \end{aligned}$$

By [K3] $H^0(N_{X,t+1} \otimes \mathcal{I}_{Z,X})$ is the tangent space to the deformation functor Φ of all deformations of X into $G(t+1)$ which leave fixed each point of Z. By the definition of $\gamma(X)$ and the assumption of Proposition 3.3, we see the smoothness of the functor Φ and that, if $g \in \Gamma(t+1)$ is near the identity and $g(y) = y$ for every $y \in Z$, then the variety $Y \cap g(G(t))$ is of the form $h(X)$ with $h \in \Gamma(t+1)$, $h(y) = y$ for every $y \in Z$, and h near to the identity. □

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