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CONNECTEDNESS OF THE SPACE OF MINIMAL 2-SPHERES IN $S^{2m}(1)$

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ABSTRACT. Loo's theorem asserts that the space of all branched minimal 2-spheres of degree d in $S^4(1)$ is connected. The main theorem in this paper is that the assertion is still true for $S^{2m}(1)$. It is shown that any branched minimal 2-sphere in $S^{2m}(1)$ can be deformed, preserving its degree, to a meromorphic function.

0. INTRODUCTION

After the celebrated research on minimal 2-spheres in the unit sphere $S^N(1)$ by Calabi [Ca], there was much attention given not only to the study of individual minimal spheres but also to the structure of the space of all minimal 2-spheres in $S^N(1)$. Calabi proved that if a minimal 2-sphere is immersed fully in $S^N(1)$, then N must be even. The simplest case is the space of all minimal 2-spheres of degree d in $S^2(1)$. This space has two connected components. One component is identified with the space of all meromorphic functions of degree d; the other is its conjugate. These two components are connected in the space of all minimal 2-spheres of degree d in $S^3(1)$.

Recently, Loo [L] determined the space of all minimal 2-spheres S^2 of degree d in the unit 4-sphere $S^4(1)$. In particular, he proved that this space is connected.

In this paper, we prove that the space of all branched minimal 2-spheres S^2 of degree d in the unit N-sphere $S^N(1)$ is connected for $N \geq 3$. We shall see that any branched minimal 2-sphere $g : S^2 \longrightarrow S^{2m}(1)$ of degree d can be deformed to a nonfull minimal sphere of degree d. By repeating this process, g is deformed eventually to a $\pm$ meromorphic function $S^2 \longrightarrow S^2(1)$ of degree d. In other words, every element in the space of all minimal spheres $S^2 \longrightarrow S^{2m}(1)$ of degree d is connected to a $\pm$ meromorphic function of degree d. Every two $\pm$ meromorphic functions are connected as we noted above. From these facts, we can prove that the space of all $g : S^2 \longrightarrow S^{2m}(1)$ of degree d is connected.

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Theorem. *The space of all branched minimal 2-spheres in $S^{2m}(1)$ of degree d is connected for $m > 1$. In the case of $m = 1$, the space has two connected components.*

Since the image of any minimal sphere $S^2 \longrightarrow S^{2m+1}$ sits in a great sphere, isometric to $S^{2m}(1)$, the theorem can easily be generalized into the following:

Corollary. *The space of all branched minimal 2-spheres in $S^N(1)$ of degree d is connected for $N \geq 3$.*

In the forthcoming paper [FGKO], the fundamental groups of the spaces are determined.

Throughout this paper, when we refer to a *minimal sphere*, we mean the sphere can be either immersed or branched.

1. Preliminaries

In this section we review the investigation of minimal spheres in $S^{2m}(1)$ by Barbosa in [Ba], where he associates minimal spheres in $S^{2m}(1)$ with isotropic holomorphic curves in $\mathbf{C}P^{2m}$ as their directrix curves. Let $g : S^2 \longmapsto S^{2m}(1) \subset \mathbf{R}^{2m+1}$ be a full minimal sphere with the isothermal metric $ds^2 = \rho^2 dz d\overline{z}$ induced from g. Then g is isotropic, that is,

Proposition 1.1 (Calabi [Ca]).

$$\left(\frac{\partial^k g}{\partial z^k}, \frac{\partial^l g}{\partial z^l}\right) = 0 \quad \textit{for all } k + l > 0.$$

As g is isotropic, the space

$$V_k(x) = \operatorname{span}\left\{\frac{\partial g}{\partial z}, \frac{\partial^2 g}{\partial z^2}, \dots, \frac{\partial^k g}{\partial z^k}\right\}$$

is perpendicular to its own conjugate $\overline{V_k(x)}$ for all $k \leq m$.

Let $G_m = \partial^m g/\partial \overline{z}^m \cap V_{m-1}^{\perp}$ and $\xi = G_m/|G_m|$.

Lemma 1.2. *G_m has only isolated zeros and ξ is holomorphic except for zeros $G_m = 0$, where ξ has at most poles.*

An immediate consequence of this lemma is that ξ defines a holomorphic curve, say, $\Psi : S^2 \longrightarrow \mathbf{C}P^{2m}$ extending up to the zeros of G_m. Ψ is called the directrix curve of g.

Lemma 1.3. *Let $g : S^2 \longrightarrow S^{2m}(1)$ be a full minimal sphere in $S^{2m}(1)$ and ξ be a local representation of the directrix curve Ψ of g. Then ξ satisfies*

$$(1.1) \qquad (\xi, \xi) = (\xi', \xi') = \cdots = (\xi^{m-1}, \xi^{m-1}) = 0,$$

where $\xi^r = \partial^r \xi/\partial z^r$ is the rth derivative by the isothermal coordinate z.

A full holomorphic curve $\Psi : S^2 \longrightarrow \mathbf{C}P^{2m}$ is called *isotropic* if its local representation ξ satisfies the equations (1.1).

Next we construct a minimal sphere $g : S^2 \longrightarrow S^{2m}(1)$ from an arbitrary holomorphic curve $\Psi : S^2 \longrightarrow \mathbf{C}P^{2m}$, which is isotropic. Let $\Psi : S^2 \longrightarrow \mathbf{C}P^{2m}$ be a holomorphic isotropic curve and ξ be its local representation by a polynomial. Define $T = \xi \wedge \xi' \wedge \cdots \wedge \xi^{m-1}$ and $g = \varepsilon(T \wedge \overline{T})/|T \wedge \overline{T}|$ for $\varepsilon^2 = (-1)^m$. Then we can check that g is a minimal sphere in $S^{2m}(1)$ whose directrix curve is Ψ.

Theorem [Ba]. *There exists a canonical* 1-1 *correspondence between the set* $\mathbf{S}_m$ *of all minimal spheres in* $S^{2m}(1)$ *and the set* $\mathscr{H}_m$ *of all holomorphic isotropic curves* $\Psi : S^2 \longrightarrow \mathbf{C}P^{2m}$. *Moreover,* $\mathrm{SO}(2m+1, \mathbf{C})$ *acts on* $\mathscr{H}_m$.

2. Deformation

Let $\mathscr{H}_m$ be the space of all holomorphic isotropic curves $\Psi : S^2 \longrightarrow \mathbf{C}P^{2m}$. As we saw in §1, $\mathrm{SO}(2m+1, \mathbf{C})$ acts on this space. Take $\Psi \in \mathscr{H}_m$, which is full. Consider a smooth deformation

$$\Psi(t) = A(t)\Psi$$

defined by a smooth 1-parameter action

$$A(t) \in \mathrm{SO}(2, \mathbf{C}) \subset \mathrm{SO}(2m+1, \mathbf{C}) \quad \text{with } A(0) = I.$$

More concretely, let

$$E_1 = e_1 - \sqrt{-1}e_2,\ E_2 = e_3 - \sqrt{-1}e_4,\ \dots,\ E_m = e_{2m-1} - \sqrt{-1}e_{2m},$$
$$\overline{E_1} = e_1 + \sqrt{-1}e_2,\ \dots,\ \overline{E_m} = e_{2m-1} + \sqrt{-1}e_{2m},\ e_{2m+1}$$

be eigenvectors of $A(t)$ in $\mathrm{SO}(2m+1, \mathbf{C})$. That is,

$$A(t)E_1 = e^t E_1, \qquad A(t)\overline{E_1} = e^{-t}\overline{E_1};$$
$$A(t)E_i = E_i, \qquad A(t)\overline{E_i} = \overline{E_i} \qquad \text{for } i \neq 1;$$
$$A(t)e_{2m+1} = e_{2m+1}.$$

Then the local representation $\xi(t)$ of $\Psi(t)$ is given by

$$\xi(t) = e^t c_1 E_1 + \sum_{i=2}^{m} c_i E_i + e^{-t} c_{\overline{1}} \overline{E_1} + \sum_{j=2}^{m} c_{\overline{j}} \overline{E_j} + c_{2m+1} e_{2m+1}$$

for some functions c_i, $c_{\overline{j}}$ in z, and this extends globally to S^2. We have a 1-parameter family $g(t) : S^2 \longrightarrow S^{2m}(1)$ given by

$$g(t) = \varepsilon \frac{T(t) \wedge \overline{T(t)}}{|T(t) \wedge \overline{T(t)}|},$$

where

$$T(t) = \xi(t) \wedge \xi(t)' \wedge \cdots \wedge \xi^{m-1}.$$

$g(t)$ is full in $S^{2m}(1)$ for every $t < \infty$, and $g(\infty) = \lim_{t\to\infty} g(t)$ is contained in a smaller sphere, say, $S^{2k}(1)$ for $k < m$. It can be seen by using Plücker coordinates P_I for m-uple multi-indices $I = (i_1, \dots, i_m)$ that

$$g(t) = \{P_I \mid \text{for all } m\text{-uple multi-indices } I\}$$

and that

$$g(\infty) = \{P_{1,J}\}$$

for all $(m-1)$-uple multi-indices J containing neither 1 nor $\overline{1}$.

By the Plücker coordinates, we mean

$$P_I = \det \begin{pmatrix} c_{i_1} & c_{i_2} & \cdots \\ c'_{i_1} & c'_{i_2} & \cdots \\ \vdots & \ddots & \vdots \\ c_{i_1}^{(m-1)} & c_{i_2}^{(m-1)} & \cdots \end{pmatrix}.$$

The degree of the deformation $g(t)$ remains constant, when $A(t)$ satisfies

(1) $\max\{\deg P_{1,J}\} = \max\{\deg P_I\}$ and

(2) $\deg\{\text{common factor } Q \text{ of } \{P_{1,J}\}\} = \deg\{\text{common factor } P \text{ of } \{P_I\}\}$.

This is so because

$$\deg g(t) = \max\ \deg P_I - \deg P,$$

$$\deg g(\infty) = \max \deg\ P_{1,J} - \deg Q.$$

The condition (1) holds if and only if the degree of $c_1(z)$ attains

$$\max_{i=1,\dots,2m+1} \deg\{c_i(z)\}.$$

Lemma 2.1. *There exists* $A(t) \in \mathrm{SO}(2, \mathbf{C}) \subset \mathrm{SO}(2m+1, \mathbf{C})$ *such that the common factor* Q *of* $\{P_{1,J}\}$ *divides the common factor* P *of* $\{P_I\}$.

Proof. We shall prove all P_I have Q as a factor if all $P_{1,J}$ have common factor Q. For simplicity we put $\mathbf{c}_i = (c_i, c'_i, \dots, c_i^{(m-1)})$. Without loss of generality, we may assume that $\mathbf{c}_1$ never vanishes.

Step 1. We see that

$$\sum_{k=0}^{m} \mathbf{c}_{i_k} P_k (-1)^{k-1} = 0$$

for all $(m+1)$-uple multi-indices $I = (i_1, \dots, i_{m+1})$, where P_k is the Plücker coordinate for the m-uple multi-index $\{i_1, \dots, i_{m+1}\} - i_k$.

Consider the $(m+1) \times (m+1)$ matrix

$$\begin{pmatrix} \mathbf{c}_{i_1} & \mathbf{c}_{i_2} & \cdots & \mathbf{c}_{i_{m+1}} \\ c_{i_1} & c_{i_2} & \cdots & c_{i_{m+1}} \end{pmatrix}$$

and calculate its determinant developed along the last column. Then we obtain

$$0 = c_{i_1} P_1 - c_{i_2} P_2 + \cdots + (-1)^m c_{i_{m+1}} P_{m+1}.$$

In the same way, considering the determinants of the $(m+1) \times (m+1)$ matrices

$$\begin{pmatrix} \mathbf{c}_{i_1} & \mathbf{c}_{i_2} & \cdots & \mathbf{c}_{i_{m+1}} \\ c_{i_1}^{(k)} & c_{i_2}^{(k)} & \cdots & c_{i_{m+1}}^{(k)} \end{pmatrix}$$

for $k = 0, \dots, m-1$, we obtain Step 1.

Step 2. Let Q be a common factor of $\{P_{1,J}\}$. Then

$$\{P_K \mid \text{all } m\text{-uple multi-indices } K \text{ containing neither } 1 \text{ nor } \bar{1}\}$$

has Q as a common factor.

This is an immediate consequence of Step 1, applied to the index $I = (1, k_1, \dots, k_m)$ and using the assumption $\mathbf{c}_1 \neq 0$.

Step 3. Interchanging $A(t)$ with $A^s(t)$, which has eigenvectors E_i^s given by

$$E_1 = \cos s E_1^s - \sin s E_2^s, \qquad E_2 = \sin s E_1^s + \cos s E_2^s,$$
$$E_j = E_j^s \quad \text{for all } j = 3, \ldots, m,$$
$$e_{2m+1} = e_{2m+1}^s,$$

we see that there exists s such that all Plücker coordinates P_I^s have a common factor P (may differ from Q), except for indices $I = (\overline{1}, \overline{2}, J)$ with $(m-2)$-uple multi-indices J not containing $1, \overline{1}, 2, \overline{2}$.

With respect to this new basis E_i^s and $\overline{E_j^s}$, ξ is given by

$$\begin{aligned}\xi = {} & (c_1 \cos s + c_2 \sin s)E_1^s + (-c_1 \sin s + c_2 \cos s)E_2^s \\ & + (c_{\overline{1}} \cos s + c_{\overline{2}} \sin s)\overline{E_1} + (-c_{\overline{1}} \sin s + c_{\overline{2}} \cos s)\overline{E_2} \\ & + \sum_{k\neq 1} c_k E_k^s + \sum_{k \neq 1} c_{\overline{k}} \overline{E_k^s} + c_{2m+1} e_{2m+1}^s,\end{aligned}$$

and the Plücker coordinates P_I^s are given by

$$\begin{aligned}
P_K^s &= P_K; \\
P_{1,K}^s &= \cos s P_{1,K} + \sin s P_{2,K}, \\
P_{\overline{1},K}^s &= \cos s P_{\overline{1},K} + \sin s P_{\overline{2},K}, \\
P_{2,K}^s &= -\sin s P_{1,K} + \cos s P_{2,K}, \\
P_{\overline{2},K}^s &= -\sin s P_{\overline{1},K} + \cos s P_{\overline{2},K}; \\
P_{1,2,K}^s &= P_{1,2,K}, \\
P_{\overline{1},\overline{2},K}^s &= P_{\overline{1},\overline{2},K}, \\
P_{1,\overline{1},K}^s &= \cos s^2 P_{1,\overline{1},K} + \sin s^2 P_{2,\overline{2},K} + \sin s \cos s (P_{1,\overline{2},K} + P_{2,\overline{1},K}), \\
P_{2,\overline{2},K}^s &= \sin s^2 P_{1,\overline{1},K} + \cos s^2 P_{2,\overline{2},K} - \sin s \cos s (P_{1,\overline{2},K} + P_{2,\overline{1},K}), \\
P_{1,\overline{2},K}^s &= \cos s^2 P_{1,\overline{2},K} - \sin s^2 P_{2,\overline{1},K} + \sin s \cos s (P_{2,\overline{2},K} - P_{1,\overline{1},K}), \\
P_{2,\overline{1},K}^s &= \cos s^2 P_{2,\overline{1},K} - \sin s^2 P_{1,\overline{2},K} + \sin s \cos s (P_{2,\overline{2},K} - P_{1,\overline{1},K}); \\
P_{1,2,\overline{1},K}^s &= \cos s P_{1,2,\overline{1},K} + \sin s P_{1,2,\overline{2},K}, \\
P_{1,2,\overline{2},K}^s &= -\sin s P_{1,2,\overline{1},K} + \cos s P_{1,2,\overline{2},K}; \\
P_{\overline{1},\overline{2},1,K}^s &= \cos s P_{\overline{1},\overline{2},1,K} + \sin s P_{\overline{1},\overline{2},1,K}, \\
P_{\overline{1},\overline{2},2,K}^s &= -\sin s P_{\overline{1},\overline{2},1,K} + \cos s P_{\overline{1},\overline{2},2,K}; \\
P_{1,2,\overline{1},\overline{2},K}^s &= P_{1,2,\overline{1},\overline{2},K},
\end{aligned}$$

where K is a multi-index not containing 1, $\overline{1}$, 2, $\overline{2}$.

By Step 2, we know that all P_J^s have common factor Q^s if the multi-indices J do not contain $\overline{1}$. Noting that the P_K^s are the same for all s, we conclude that the common factor of Q^s does not depend on s. We denote this factor by P. Then we can see that all coefficients of $\cos s$, $\sin s$, etc., of P_J^s have P as a factor. It implies that P_I has common factor P unless the multi-index contains both $\overline{1}$ and $\overline{2}$ together. This gives Step 3.

Step 4. All $P_{\bar{1},\bar{2},J}$ have P as a factor.

Proof. Let a be a point such that $P^{(l)}(a) = 0$ for $l \le N$ and for some index J. Then we will see that

$$\dim\{\mathbf{c}_k(a) | k \neq \bar{1}, \bar{2}\} \le m - 2.$$

If there exist $(m-1)$ linearly independent vectors

$$\{\mathbf{c}_{j_1}(a), \dots, \mathbf{c}_{j_{m-1}}(a)\},$$

then we obtain that

$$\mathbf{c}_j(z) = f_1^j(z)\mathbf{c}_{j_1}(z) + \cdots + f_{m-1}^j(z)\mathbf{c}_{j_{m-1}} + (z-a)^N V_j,$$

where $f_i^j(z)$ are polynomials in z and V_j is a vector, from the fact that $P_{j, j_1, \dots, j_{m-1}}$ has a as a zero of order N. In particular, putting $\mathbf{c}_{\bar{1}}, \mathbf{c}_{\bar{2}}$, $\mathbf{c}_j$ into $P^{(N)}_{\bar{1},\bar{2},J}$, we see that it has a as a zero of order N. This contradicts the assumption. We prove that

$$\dim\{\mathbf{c}_k(a) | k \neq \bar{1}, \bar{2}\} = r \le m - 2.$$

Hence, that there exist r constant vectors $F_1, \dots, F_r$ such that

$$\xi(a) = c_{\bar{1}}(a)E_{\bar{1}} + c_{\bar{2}}(a)E_{\bar{2}} + \sum_{j=1}^{r} c_j(a)F_j,$$

$$\xi'(a) = c'_{\bar{1}}(a)E_{\bar{1}} + c'_{\bar{2}}(a)E_{\bar{2}} + \sum_{j=1}^{r} c'_j(a)F_j,$$

$$\vdots$$

$$\xi^{(m-1)}(a) = c^{(m-1)}_{\bar{1}}(a)E_{\bar{1}} + c^{(m-1)}_{\bar{2}}(a)E_{\bar{2}} + \sum_{j=1}^{r} c^{(m-1)}_j(a)F_j.$$

Using the isotropic property of ξ, i.e.,

$$(\xi^k, \xi^l) = 0 \quad \text{for all } k,\ l = 1, \dots, m-1,$$

we see

$$(E_{\bar{1}}, F_j) = 0 \quad \text{for all } j = 1, \dots, r,$$

which contradicts the fact that

$$E_1 \in \operatorname{span}\{F_1, \dots, F_r\}.$$

So there is no such point $z = a$ with $P^{(l)}(a) = 0$ for $l \le N$ and $P^N_{\bar{1},\bar{2},J}(a) \neq 0$. In other words, all $P_{\bar{1},\bar{2},J}$ have P as a common factor.

The case $m = 3$ *and* $N = 1$. We prove Step 4 more precisely when $m = 3$. If there is some point $z = a$ such that $P(a) = 0$ and $P_{\bar{1},\bar{2},k}(a) \neq 0$ for some index $k \neq 1, 2, \bar{1}, \bar{2}$, then, since

$$P_{\bar{1},\bar{2},k}(a) = \det(\mathbf{c}_{\bar{1}}(a), \mathbf{c}_{\bar{2}}(a), \mathbf{c}_k(a)) \neq 0,$$

the three vectors $\mathbf{c}_{\bar{1}}(a)$, $\mathbf{c}_{\bar{2}}(a)$, $\mathbf{c}_k(a)$ are linearly independent. On the other hand, $P_{\bar{1},1,k}(a) = 0$ implies that $\mathbf{c}_{\bar{1}}(a)$, $\mathbf{c}_1(a)$, $\mathbf{c}_k(a)$ are linearly dependent and $\mathbf{c}_{\bar{1}}(a)$, $\mathbf{c}_k(a)$ are independent. Therefore, we can write

$$\mathbf{c}_1(a) = \alpha\mathbf{c}_{\bar{1}}(a) + \beta\mathbf{c}_k(a).$$

In the same way we can write

$$\mathbf{c}_1(a) = \gamma\mathbf{c}_{\bar{2}}(a) + \delta\mathbf{c}_k(a)$$

by $P_{1,\bar{2},k}(a) = 0$. But since $\mathbf{c}_{\bar{1}}(a)$ and $\mathbf{c}_{\bar{2}}(a)$ are linearly independent, we conclude that

$$\mathbf{c}_1(a) \text{ is parallel to } \mathbf{c}_k(a).$$

By the same argument we see

$$\mathbf{c}_2(a) \text{ is parallel to } \mathbf{c}_k(a),$$

$$\mathbf{c}_l(a) \text{ is parallel to } \mathbf{c}_k(a) \text{ for } l \neq 1,\ 2,\ \bar{1},\ \bar{2},$$

by using

$$P_{\bar{1},2,k}(a) = P_{\bar{2},2,k}(a) = 0,$$
$$P_{\bar{1},l,k}(a) = P_{\bar{2},l,k}(a) = 0.$$

By the assumption that $\mathbf{c}_1(a) \neq 0$, we put $\mathbf{c}_j(a) = a_j\mathbf{c}_1(a)$ for all $j \neq \bar{1},\ \bar{2}$. Let $F = E_1 + a_2E_2 + a_3E_3 + a_{\bar{3}}\overline{E_3} + a_7e_7$. Then we obtain

$$\xi(a) = c_1(a)F + c_{\bar{1}}(a)\overline{E_1} + c_{\bar{2}}(a)\overline{E_2},$$
$$\xi'(a) = c_1'(a)F + c_{\bar{1}}'(a)\overline{E_1} + c_{\bar{2}}'(a)\overline{E_2},$$
$$\xi''(a) = c_1''(a)F + c_{\bar{1}}''(a)\overline{E_1} + c_{\bar{2}}''(a)\overline{E_2}.$$

The isotropic property of ξ says that

$$0 = (\xi(a), \xi(a)) = 2c_1(a)\{c_{\bar{1}}(a)(F, \overline{E_1}) + c_{\bar{2}}(a)(F, \overline{E_2})\},$$
$$0 = (\xi'(a), \xi'(a)) = 2c_1'(a)\{c_{\bar{1}}'(a)(F, \overline{E_1}) + c_{\bar{2}}'(a)(F, \overline{E_2})\},$$
$$0 = (\xi''(a), \xi''(a)) = 2c_1''(a)\{c_{\bar{1}}''(a)(F, \overline{E_1}) + c_{\bar{2}}''(a)(F, \overline{E_2})\},$$
$$0 = (\xi(a), \xi'(a)) = c_1(a)\{c_{\bar{1}}'(a)(F, \overline{E_1}) + c_{\bar{2}}'(a)(F, \overline{E_2})\} + c_1'(a)\{c_{\bar{1}}(a)(F, \overline{E_1}) + c_{\bar{2}}(a)(F, \overline{E_2})\},$$
$$0 = (\xi(a), \xi''(a)) = c_1(a)\{c_{\bar{1}}''(a)(F, \overline{E_1}) + c_{\bar{2}}''(a)(F, \overline{E_2})\} + c_1''(a)\{c_{\bar{1}}(a)(F, \overline{E_1}) + c_{\bar{2}}(a)(F, \overline{E_2})\},$$
$$0 = (\xi'(a), \xi''(a)) = c_1''(a)\{c_{\bar{1}}'(a)(F, \overline{E_1}) + c_{\bar{2}}'(a)(F, \overline{E_2})\} + c_1'(a)\{c_{\bar{1}}''(a)(F, \overline{E_1}) + c_{\bar{2}}''(a)(F, \overline{E_2})\}.$$

The matrix $X = (\xi(a), \xi'(a), \xi''(a))$ satisfies the equation $X^tX = O$, and

$$(\mathbf{c}_1(a), \mathbf{c}_{\bar{1}}(a), \mathbf{c}_{\bar{2}}(a)) \begin{pmatrix} (F, F) & (F, \overline{E_1}) & (F, \overline{E_2}) \\ (F, \overline{E_1}) & (\overline{E_1}, \overline{E_1}) & (\overline{E_2}, \overline{E_1}) \\ (F, \overline{E_2}) & (\overline{E_1}, \overline{E_2}) & (\overline{E_2}, \overline{E_2}) \end{pmatrix} \begin{pmatrix} \mathbf{c}_1(a) \\ \mathbf{c}_{\bar{1}}(a) \\ \mathbf{c}_{\bar{2}}(a) \end{pmatrix} = O$$

implies that

$$(F, \overline{E_1}) = (F, \overline{E_2}) = 0,$$

which contradicts $F = E_1 + \cdots$.

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