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SOME FIXED POINT THEOREMS FOR COMPOSITES OF ACYCLIC MAPS

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ABSTRACT. We obtain fixed point theorems for a new class of multifunctions containing compact composites of acyclic maps defined on a convex subset of a locally convex Hausdorff topological vector space. Our new results are applied to approximatively compact, convex sets or to Banach spaces with the Oshman property.

1. INTRODUCTION

Recently there have appeared a number of new results on fixed points of so-called Kakutani factorizable multifunctions or composites of acyclic multifunctions defined on convex subsets of topological vector spaces; for example, see [1, 2, 3, 13, 14, 18]. Especially in [14] Lassonde considered a class $\mathbb{K}_c^+$ of multifunctions containing Kakutani factorizable multifunctions and obtained very general results.

In the present paper we replace the convexity of the functional values by acyclicity and, motivated by [14], consider a class $\mathbb{V}_c^+$ of multifunctions properly containing $\mathbb{K}_c^+$ and composites of acyclic maps. First we obtain generalizations of a coincidence theorem of Granas and Liu [10] and a fixed point theorem of Lassonde [14, 12]. Then we apply these theorems to generalize results of Reich [20] on approximatively compact sets or on Banach spaces with the Oshman property.

2. PRELIMINARIES

A *multifunction* $F: X \to 2^Y$ is a function with set values $Fx \subset Y$ for each $x \in X$. The set $\{(x, y): y \in Fx\}$ is called either the graph of F or, simply, F. So $(x, y) \in F$ if and only if $y \in Fx$. For any $C \subset X$ let $F(C) = \bigcup\{Fx: x \in C\}$. For any $B \subset Y$ let $F^-(B) = \{x \in X: Fx \cap B \neq \varnothing\}$. If B is a singleton $\{y\}$ in Y then $F^-(B)$ is called a *fiber* denoted F^-y.

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In this paper a *convex space* C is a nonempty convex set in a vector space with any topology that induces the Euclidean topology on the convex hulls of its finite subsets (see [12]). Such convex hulls are called *polytopes*.

For topological spaces X and Y a multifunction $F: X \to 2^Y$ is *upper semicontinuous* (u.s.c.) if, for each closed set $B \subset Y$, $F^-(B)$ is closed in X. F is *lower semicontinuous* (l.s.c.) if, for each open set $B \subset Y$, $F^-(B)$ is open in X. F is *continuous* if F is both u.s.c. and l.s.c. F is *compact* if $F(X)$ is contained in a compact subset of Y. A set $K \subset X$ is called *σ-compact* if K is the countable union of compact sets. A nonempty topological space is *acyclic* if all its reduced Čech homology groups over rationals vanish.

For a given class $\mathbb{L}$ of multifunctions define

$$\mathbb{L}(X, Y) = \{T: X \to 2^Y | T \in \mathbb{L}\}, \qquad \mathbb{L}_c = \{T = T_m T_{m-1} \cdots T_1 | T_i \in \mathbb{L}\}.$$

Using the above notation we have the following definitions:

(1) If f is continuous and single-valued, we write $f \in \mathbb{C}(X, Y)$.

(2) We say that F is a *Kakutani* map and write $F \in \mathbb{K}(X, Y)$ if Y is a convex space and F is u.s.c. with nonempty compact convex values.

(3) F is an *acyclic* map, written $F \in \mathbb{V}(X, Y)$, if F is u.s.c. with compact acyclic values.

(4) $F \in \mathbb{K}^+(X, Y)$ (resp. $\mathbb{V}^+(X, Y)$) if for any σ-compact subset K of X there is a $\Gamma \in \mathbb{K}(K, Y)$ ($\mathbb{V}(K, Y)$) such that $\Gamma x \subset Fx$ for each $x \in K$.

(5) $F \in \mathbb{K}_c^+(X, Y)$ (resp. $\mathbb{V}_c^+(X, Y)$) if for any σ-compact subset K of X there is a $\Gamma \in \mathbb{K}_c(K, Y)$ ($\mathbb{V}_c(K, Y)$) such that $\Gamma x \subset Fx$ for each $x \in K$.

(6) $F \in \mathbb{M}(X, Y)$ if Fx is convex for all $x \in X$ and $F^- y$ is open for all $y \in Y$.

(7) $F \in \mathbb{T}(X, Y)$ if Fx is convex and $\bigcup_{V \in \mathscr{V}(x)} (\bigcap_{x' \in V} Fx') \neq \varnothing$ for all $x \in X$, where $\mathscr{V}(x)$ is a neighborhood base of x in X (see Lassonde [14]).

It is known that $\mathbb{K}_c^+$ contains $\mathbb{C}$, $\mathbb{M}$, $\mathbb{K}$, $\mathbb{K}_c$, and $\mathbb{T}$ (see [14]). Moreover, it is clear that $\mathbb{V}_c^+$ includes $\mathbb{V}_c$ and $\mathbb{K}_c^+$.

Let C be a subset of a topological vector space E, and suppose $x \in E$. The *inward set* $I_C(x)$ and *outward set* $O_C(x)$ of C at x are defined, respectively, by

$$I_C(x) = \{x + r(y - x): y \in C, r \geq 0\},$$
$$O_C(x) = \{x + r(y - x): y \in C, r \leq 0\}.$$

Their closures will be denoted by $\overline{I_C(x)}$ and $\overline{O_C(x)}$.

For the remaining definitions in this section assume that C is a nonempty subset of a locally convex Hausdorff topological vector space E and that p is a continuous seminorm on E.

For $y \in E$ define $d_p(y, C) = \inf\{p(y - x): x \in C\}$ and $Q_p(y) = \{x \in C: p(y - x) = d_p(y, C)\}$. C is called *approximatively compact* (with respect to p) if for each $y \in E$ every net $\{x_\alpha: \alpha \in \Lambda\} \subset C$ such that $p(y - x_\alpha) \to d_p(y, C)$ has a subnet that converges to an element of C.

We list some *boundary conditions* for a multifunction $F: C \to 2^E$. We use ∂C to denote the boundary of C, and each condition must hold for all

$x \in \partial C \backslash Fx$:

(0) For each $y \in Fx$ and each continuous seminorm p, $p(y-x) > 0$ implies $p(y-x) > p(y-z)$ for some $z \in \overline{I_C(x)}$.

(i) For each $y \in Fx$ there exists a number λ (real or complex, depending on whether E is real or complex) such that

$$|\lambda| < 1 \quad \text{and} \quad \lambda x + (1-\lambda)y \in \overline{I_C(x)}.$$

(ii) $Fx \subset \overline{I_C(x)}$.

(iii) For each $y \in Fx$ there exists a λ (as above) such that

$$|\lambda| < 1 \quad \text{and} \quad \lambda x + (1-\lambda)y \in C.$$

(iv) $Fx \subset IF_C(x) := \{x + c(u-x) : u \in C, \operatorname{Re}(c) > \frac{1}{2}\}$.

(v) $Fx \subset C$.

(vi) $F(C) \subset C$.

It is well known that (vi) $\Rightarrow$ (v) $\Rightarrow$ (iv) $\Rightarrow$ (ii) $\Rightarrow$ (i) and that (iv) $\Leftrightarrow$ (iii) $\Rightarrow$ (i) $\Rightarrow$ (0).

Moreover, each of (0), (i), (ii), and (iv) has a corresponding outward boundary condition.

Note that for a convex-valued multifunction we can consider more general conditions than some of (i)–(vi) (see Park [17]).

3. Main results

We need the following lemma in our subsequent work.

Lemma 1. *Let P be a polytope. If $F \in \mathbb{V}_c(P, P)$ then F has a fixed point.*

This fact is quite well known in the Lefschetz fixed point theory. For details we refer the reader to [19, 8, 9].

Recently there have appeared several interesting coincidence theorems. We give the following, which unifies several known results. We use a partition of unity argument in the proof.

Theorem 1. *Let X be a convex space, Y a Hausdorff space, and $F, G: X \to 2^Y$ multifunctions satisfying:*

(1) *$F \in \mathbb{V}_c^+(X, Y)$ is compact;*

(2) *for each $y \in F(X)$, $G^- y$ is convex; and*

(3) *$\{\operatorname{Int} Gx : x \in X\}$ covers $\overline{F(X)}$.*

Then F and G have a coincidence point $x_0 \in X$; that is, $Fx_0 \cap Gx_0 \neq \varnothing$.

Proof. Since $\overline{F(X)}$ is compact and covered by $\{\operatorname{Int} Gx : x \in X\}$, there is a finite set $N = \{x_1, x_2, \dots, x_n\}$ in X such that $\overline{F(X)} \subset \bigcup\{\operatorname{Int} Gx : x \in N\}$. Let $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ be the partition of unity corresponding to this cover, and let P represent the convex hull of N, denoted $\operatorname{co} N$. Define $f: \overline{F(X)} \to P$ by

$$fy = \sum_1^n \lambda_i(y) x_i = \sum_{i \in N_y} \lambda_i(y) x_i$$

for $y \in \overline{F(X)} \subset Y$, where $i \in N_y$ if and only if $\lambda_i(y) \neq 0$, which implies that $y \in \operatorname{Int} Gx_i$. Then, for $i \in N_y$, $x_i \in G^- y$. Clearly f is continuous and by condition (2) we have $fy \in \operatorname{co}\{x_i : i \in N_y\} \subset G^- y$ for each $y \in F(X)$. Since

$P \subset X$ is compact and $F \in \mathbb{V}_c^+(X, Y)$, there is a $\Gamma \in \mathbb{V}_c(P, F(X))$ such that $\Gamma x \subset Fx$ for all $x \in P$. Therefore, by Lemma 1, $f\Gamma \in \mathbb{V}_c(P, P)$ has a fixed point $x_0 \in P \subset X$. Since $x_0 \in (f\Gamma)x_0 \subset (fF)x_0$ and $f^- x_0 \subset Gx_0$, we have

$$Fx_0 \cap Gx_0 \neq \varnothing.$$

This completes the proof.

Particular forms of Theorem 1 were given by Browder [4–6] and Reich [21] with $X = Y$ and $F = 1_X$, by Granas and Liu [10] with $\mathbb{V}$ instead of $\mathbb{V}_c^+$, and by Lassonde [13, Theorem 3 for $\mathbb{K}_c$; 14, Theorem 4 for $\mathbb{K}_c^+$]. For related results see [1, Theorems 5.1 and 5.2].

Based on Theorem 1 we give

Theorem 2. *Let X and C be nonempty convex subsets of a locally convex Hausdorff topological vector space E and $F \in \mathbb{V}_c(X, X+C)$ a compact multifunction. Suppose that one of the following conditions holds.*

(1) *X is closed and C is compact.*
(2) *C is closed and X is compact.*
(3) *$C = \{0\}$.*

Then there is an $\hat{x} \in X$ such that $F\hat{x} \cap (\hat{x} + C) \neq \varnothing$.

Proof. Let V be an open convex neighborhood of the origin 0 in E, and let Y be a compact set such that $F(X) \subset Y \subset X + C$. Define $G: X \to 2^Y$ by

$$Gx = (x + C + V) \cap Y \quad \text{for } x \in X.$$

Then each Gx is open in Y and $G^- y = (y - C - V) \cap X$ is convex for each $y \in Y$. Moreover, since $Y \subset X + C$, for every $y \in Y$ there exists an $x \in X$ such that $y \in x + C + V$; that is, $\{Gx : x \in X\}$ covers Y. Therefore, by Theorem 1, there exist $x_V \in X$ and $y_V \in Y$ such that $y_V \in Fx_V \cap Gx_V$; that is, $y_V - x_V \in C + V$. Hence for each neighborhood V of 0 in E we get that

$$(*) \qquad (F - i)(X) \cap (C + V) \neq \varnothing,$$

where $i: X \to E$ is the inclusion.

Case (1). Because X is closed, so is $(F - i)(X)$. Since C is compact and E is regular, $(*)$ implies $(F - i)(X) \cap C \neq \varnothing$; that is, there is an $\hat{x} \in X$ such that $F\hat{x} \cap (\hat{x} + C) \neq \varnothing$.

Case (2). Since $(F - i)(X)$ is compact and C is closed, the same conclusion follows as in Case (1).

Case (3). Since F is u.s.c., for each neighborhood V of 0 in E, there exist $x_V, y_V \in X$ such that $y_V \in Fx_V$ and $y_V - x_V \in V$. Since $F(X)$ is relatively compact, we may assume that y_V converges to some $\hat{x}$. Then x_V also converges to $\hat{x}$. Since the graph of F is closed in $X \times \overline{F(X)}$, we have $\hat{x} \in F\hat{x}$. This completes our proof.

Theorem 2(3) can be stated in the following, seemingly, more general form.

Corollary 1. *Let X be a nonempty convex subset of a locally convex Hausdorff topological vector space E and $F \in \mathbb{V}_c^+(X, X)$. If F is compact, then F has a fixed point.*

Proof. Let $K = \operatorname{co}\overline{F(X)}$. Then K is σ-compact by Proposition 1(3) of [14]. Since $F \in \mathbb{V}_c^+(X, K)$ there exists a $\Gamma \in \mathbb{V}_c(K, K)$ such that $\Gamma x \subset Fx$ for each $x \in K$. Moreover, since F is compact, so is Γ. Therefore, by Theorem 2(3), Γ has a fixed point $x_0 \in \Gamma x_0 \subset Fx_0$. This completes the proof.

Particular forms of Theorem 2 have been given by Lassonde [12, Theorem 1.6 and Corollary 1.18] for $\mathbb{K}$ and Park [16, Theorem 7] for $\mathbb{V}$. Corollary 1 includes Lassonde [14, Théorème 5] for $\mathbb{K}_c^+$ and [13, Theorem 4] for $\mathbb{K}_c$, Himmelberg [11, Theorem 2] for $\mathbb{K}$, and many others. Himmelberg's result extends earlier works of Schauder, Mazur, Bohnenblust and Karlin, Hukuhara, Singbal, Tychonoff, Kakutani, Fan, and Glicksberg (for the literature see [16]).

The next result uses an additional lemma given for approximatively compact sets. Recall that a compact set is approximatively compact but that the converse is false.

Lemma 2. *Let C be a subset of a Hausdorff topological vector space E and p a continuous seminorm on E. If C is approximatively compact (with respect to p), then the metric projection $Q_p\colon E \to 2^C$ is u.s.c.*

Lemma 2 is given in [22] and also in [20].

Theorem 3. *Let C be a nonempty approximatively compact, convex subset of a locally convex Hausdorff topological vector space E, and suppose that $F \in \mathbb{V}_c^+(C, E)$ is a compact multifunction. Then for each continuous seminorm p on E there exists an $(x_0, y_0) \in F$ such that*

$$p(x_0 - y_0) \le p(x - y_0) \quad \textit{for all } x \in \overline{I_C(x_0)}.$$

Proof. Consider the metric projection $Q_p\colon E \to 2^C$. Clearly $Q_p(x)$ is nonempty, compact, and convex for every $x \in E$, and Q_p is an u.s.c. multifunction by Lemma 2. Thus $Q_p \in \mathbb{K}(E, C) \subset \mathbb{V}^+(E, C)$. Since it is clear that $\mathbb{V}_c^+$ is closed under composition (cf. Lassonde [14, Proposition 2(1)]), we have $Q_pF \in \mathbb{V}_c^+(C, C)$ and Q_pF is compact. Therefore, by Corollary 1, Q_pF has a fixed point; that is, there is a $y_0 \in Fx_0$ such that

$$x_0 \in Q_p y_0 = \{x \in C\colon p(y_0 - x) = d_p(y_0, C)\}.$$

Actually, this implies the conclusion by the methods of Park in [15].

Remarks. For $F \in \mathbb{V}_c^+(C, E)$ define a multifunction $F'\colon C \to 2^E$ by $F'x = 2x - Fx$ for $x \in C$. If $F' \in \mathbb{V}_c^+(C, E)$, then $\overline{I_C(x_0)}$ in the conclusion of Theorem 3 can be replaced by $\overline{O_C(x_0)}$.

Note that for $F = f \in \mathbb{C}(C, E)$ Theorem 3 reduces to Reich [20, Corollary 2.2]. For a compact convex subset of a normed vector space E, the origins of Theorem 3 go back to the best approximation theorem of Fan [7].

For a continuous multifunction $F\colon C \to 2^E$ satisfying certain restrictions, some authors obtain the following type of variational inequality:

$$\text{there exists } x_0 \in C \text{ such that } d_p(x_0, Fx_0) = d_p(C, Fx_0).$$

It is well known that, in this case, one cannot dispense with the lower semicontinuity of F. The following example shows that lower semicontinuity is not necessary in Theorem 3.

Example. Take $E = \mathbb{R}^2$ and $C = [0, 1] \times \{0\}$. Suppose $F \in \mathbb{V}(C, E)$ is given by

$$F(a, 0) = \begin{cases} \{(0, 1)\} & \text{if } a \neq 0, \\ [(0, 1), (1, 0)] \cup [(-1, 0), (0, 1)] & \text{if } a = 0, \end{cases}$$

where $[A, B]$ stands for the closed line segment joining points A and B in the plane. Then F is not lower semicontinuous, but $x_0 = (0, 0)$ and $y_0 = (\frac{1}{2}, \frac{1}{2})$ satisfy the conclusion of Theorem 3.

The following fixed point theorem holds in normed vector spaces.

Corollary 2. *Let C be an approximatively compact, convex subset of a normed vector space E, and suppose that $F \in \mathbb{V}_c^+(C, E)$ is compact such that*

(0) *for all $(x, y) \in F$, $y \neq x$, we have $\|y - x\| > \|y - z\|$ for some $z \in \overline{I_C(x)}$.*

Then F has a fixed point.

Proof. By Theorem 3 there exists an $(x_0, y_0) \in F$ such that $\|x_0 - y_0\| \leq \|x - y_0\|$ for all $x \in \overline{I_C(x_0)}$. If $x_0 \neq y_0$, then (0) leads to a contradiction. This completes the proof.

Remarks. Suppose F' is defined as in the remarks following Theorem 3. If $F' \in \mathbb{V}_c^+(C, E)$, then the outward boundary conditions corresponding to (0), (i), (ii), and (iv) can be used instead of (0) in Corollary 2. In some of these cases we may have some surjectivity; that is, $F(C) \supset C$ (see Park [17]).

If $F \in \mathbb{K}(C, E)$ and C is compact, then Corollary 2 extends Reich [20, Theorem 3.3(a)], who used the boundary condition

$$(0') \qquad Fx \cap Q^{-1}x \subset \{x\} \quad \text{for all } x \in C.$$

Note that $(0') \Rightarrow (0)$. In fact, if $y \in Fx \cap Q^{-1}x$ and $y = x$, then F has a fixed point. If $Fx \cap Q^{-1}y = \varnothing$, then any $y \in Fx$ satisfies (0).

Recall that a reflexive Banach space has the Oshman property if the metric projection on every closed convex subset is u.s.c.

Theorem 4. *Let C be a closed convex subset of a Banach space E with the Oshman property and $F \in \mathbb{V}_c^+(C, E)$ a compact multifunction. Then there exists $(x_0, y_0) \in F$ such that*

$$\|x_0 - y_0\| \leq \|x - y_0\| \quad \textit{for all } x \in \overline{I_C(x_0)}.$$

The proof is like that of Corollary 1, and we omit it here.

Remarks. As in the remarks following Corollary 1, if $F' \in \mathbb{V}_c^+(C, E)$, then the inward set can be replaced by the outward set.

For $F = f \in \mathbb{C}(C, E)$ Theorem 4 reduces to Proposition 2.3 of Reich [20].

Corollary 3. *Let C be a closed convex subset of a Banach space E with the Oshman property, and suppose $F \in \mathbb{V}_c^+(C, E)$ is a compact multifunction satisfying boundary condition (0). Then F has a fixed point.*

The proof is like that of Corollary 2.

Remark. Reich [20, Proposition 3.2] obtained Corollary 3 for $F \in \mathbb{K}(C, E)$ satisfying boundary condition (iv). Our proof is different from his. Reich [20, Theorem 3.3(b)] is also a particular form of Corollary 3 for $F \in \mathbb{K}(C, E)$ satisfying condition $(0')$. Related results for single-valued and set-valued mappings are given in Reich [21].

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